ADBI Working Paper Series

Debt Policy and Economic Growth in a Small Open Economy Model with Productive Government Spending

Koichi Futagami, Takeo Hori, and Ryoji Ohdoi

No. 212
April 2010

Asian Development Bank Institute

Koichi Futagami is a professor at Osaka University in Osaka, Japan. Takeo Hori is a lecturer at Hitotsubashi University in Tokyo. Ryoji Ohdoi is an associate professor at Osaka City University in Osaka, Japan.

The views expressed in this paper are the views of the authors and do not necessarily reflect the views or policies of ADBI, the Asian Development Bank (ADB), its Board of Directors, or the governments they represent. ADBI does not guarantee the accuracy of the data included in this paper and accepts no responsibility for any consequences of their use. Terminology used may not necessarily be consistent with ADB official terms.

The Working Paper series is a continuation of the formerly named Discussion Paper series; the numbering of the papers continued without interruption or change. ADBI's working papers reflect initial ideas on a topic and are posted online for discussion. ADBI encourages readers to post their comments on the main page for each working paper (given in the citation below). Some working papers may develop into other forms of publication.

Suggested citation:

Futagami, K., T. Hori, and R. Ohdoi. 2010. Debt Policy and Economic Growth in a Small Open Economy Model with Productive Government Spending. ADBI Working Paper 212. Tokyo: Asian Development Bank Institute. Available: http://www.adbi.org/working-paper/2010/04/15/3656.debt.policy.economic.growth.model/

Please contact the author(s) for information about this paper.

Koichi Futagami: futagami@econ.osaka-u.ac.jp

Asian Development Bank Institute
Kasumigaseki Building 8F
3-2-5 Kasumigaseki, Chiyoda-ku
Tokyo 100-6008, Japan

Tel: +81-3-3593-5500
Fax: +81-3-3593-5571
URL: www.adbi.org
E-mail: info@adbi.org

© 2010 Asian Development Bank Institute

Abstract

In this paper, we examine the effects of introducing constraints on government borrowing using a continuous-time overlapping generations model of a small open economy. We consider government placing constraints on the amount of government bonds outstanding by establishing an upper limit, or target level, for the ratio of government bonds to gross domestic product. We first show that there exist multiple steady states in the model small open economy. One is a steady state with high growth, the other a steady state with low growth. We next examine how changes in the target level for bonds affect economic growth rates at the steady states. If the economy has a positive amount of asset holdings, we obtain the following results. When the government runs budget surpluses, an increase in the target level for government bonds reduces the growth rate of the low-growth economy, but raises the growth rate of the high-growth economy. However, when the government runs budget deficits, an increase in the target level for government bonds raises the growth rate of the low-growth economy, but reduces the growth rate of the high-growth economy. If the economy has a negative amount of asset holdings, the results are ambiguous.

JEL Classification:

Contents

1. Introduction 1

2. The Model 2

2.1 Households 2
2.2 The Firm 4
2.3 The Government 4

3. Model Equilibrium 4

4. Character of the Model's Steady States 8

5. Concluding Remarks 9

Appendix 11

References 12

1. INTRODUCTION

Productive government spending can play an important role in the development of economies, as shown by Barro (1990). A well organized police system, established courts, health facilities (such as sanitation), and other publicly provided services are indispensable for economic development. However, it is sometimes difficult for developing economies to finance these services because of difficulties collecting enough tax revenue to finance them. Therefore, developing economies usually must rely on the issue of government bonds to finance these costs.

Developing economies, however, are usually short of household savings. Therefore, they may need to introduce foreign capital into their economies through the sale of government bonds to foreign investors. If there are not adequate rules or discipline governing such borrowing, these countries might borrow beyond their ability to repay. In fact, many developing economies have faced the risk of default. In the 1980s, some countries in Latin America renegotiated their obligations to foreign lenders. In the late 1990s, East Asian countries and Russia were confronted with a currency crisis because of the flight of foreign capital.

In this paper, we examine the effects of introducing rules constraining government borrowing by using a continuous-time overlapping generations model of a small open economy. We assume that the government sets constraints on the amount of government bonds that may be issued by establishing an upper limit or target level for the ratio of government bonds to gross domestic product (GDP). In a closed economy, a decrease in this ratio implies a tighter government budget and thus a decrease in government spending, or an increase in tax revenue. In the small open economy model in this paper, however, the government can borrow from abroad. Therefore, the mechanism by which the government finances its expenditure in small open economies can be significantly different from that in closed economies.

We find constraints on government borrowing in the Maastricht Treaty (formerly the Treaty of the European Union [EU]) and in the Code for Fiscal Stability of the United Kingdom. The Maastricht criterion states that member countries of the EU and countries that hope to join the EU must maintain a government bonds to GDP ratio of less than 60%. The code in the United Kingdom states that the ratio must be kept under 30%.

There exists considerable research based on endogenous growth models with productive government spending and government bonds. Bruce and Turnovsky (1999) examined how a tax cut, or a tax cut with expenditure cuts, affects the government's long-run fiscal balance. Greiner and Semmler (1999, 2000) investigated how the financing methods of government affect the economy. Ghosh and Mourmouras (2004) considered that a benevolent government attempts to maximize the welfare of households and endogenously compared various fiscal rules under standard government budget constraints and the golden rule of public finance. The golden rule constrains government to use revenue raised by issuing bonds for public investment only. They compared optimal fiscal policy under three different regimes and showed that the golden rule can be an effective constraint. In contrast with these studies, Futagami, Iwaisako, and Ohdoi (2008) introduced an upper limit, or target level, for government debt relative to the size of the economy into an endogenous growth model with productive government spending. They showed that there were two equilibriums: one associated with high growth, the other associated with low growth. They also showed that government use of income taxes or bonds made the results significantly different. In particular, they showed that an increase in bonds outstanding reduced the economic growth rate at the high-growth steady state, and raised the growth rate at the low-growth steady state.

All the analyses cited in the preceding paragraph were limited to a closed economy, however. In this paper, we extend the analysis to a small open economy that has a positive amount of asset holdings. When the government is running a budget surplus with such an economy at the low-growth steady state, an increase in the upper limit, or target level, for government bonds reduces the economic growth rate. On the other hand, an increase in the bond upper limit raises the growth rate of an economy at a high-growth steady state with a budget surplus. [This result sharply contrasts with the results of Futagami Iwaisako, and Ohdoi (2008)]. However, when the government is running a budget deficit with the economy at the low-growth steady state and increases the target level for government bonds, the economic growth rate is raised. On the other hand, the growth rate is reduced when the bond target level is increased in an economy at a high-growth steady state with budget deficits. If the small open economy has a negative amount of asset holdings, the result becomes ambiguous.

According to conventional wisdom, an increase in government bonds absorbs private savings that would finance accumulation of private capital, thus reducing the level of capital stock or the growth rate of the economy (Modigliani 1965). However, when government expenditure promotes the productivity of private capital, an increase in productive government expenditure through issuing bonds can promote capital accumulation or raise the economic growth rate.

The remainder of this paper is presented in four sections. Section 2 describes the construction of the model of a small open economy. Section 3 considers the equilibrium of that model. The character of model equilibrium is examined in Section 4. Section 5 contains concluding remarks.

2. THE MODEL

We consider a small open economy with overlapping generations. It is assumed that the single final good and capital are freely traded. However, individuals cannot migrate across the country border. A continuous-time overlapping generations model without intergenerational altruism is used, based on Blanchard (1985). Each individual faces an instantaneous probability of death, which is denoted by *p*. We assume this probability is constant through an individual life. At every instant of time, new cohort is born. In order to simplify the analysis, it is assumed that the population of this economy is constant over time. Therefore, the birth rate becomes the same as the death rate, that is, *p*. We assume that the population size is unity.

2.1 Households

We first consider a representative household. Let *c*(*s*, *v*) be consumption at time *v* of generation *s*, where generation *s* implies households born at time *s*. The expected utility of generation *s* at time *t* is given by:

$$EU_t = E_t \int_t^z [\ln c(s,v)] e^{-\rho(v-t)} dv = \int_t^\infty [\ln c(s,v)] e^{-(\rho+p)(v-t)} dv ,$$

where ρ is the rate of time preference.

We assume that there exists an actuarially fair insurance company, as assumed by Blanchard (1985). The company collects funds and invests them in firms or in the international financial market. The government imposes a tax on the returns. We assume that the tax takes the residence base form. According to residence-based taxation, the income of residents is taxed at a uniform rate regardless of its source country, while nonresidents' income is not taxed. Returns after tax are paid to the insured households still living. The contract offered by the insurance company redistributes income from people who

have died to those who are still living. An individual's premium is equal to his or her probability of death. The insured who are still living can receive the insurance premium *p* as well as the interest after tax, $(1-\tau)r$, when he or she is a lender. When the insured is a borrower, he or she can only receive the insurance premium *p* and pays the interest. Thus, the flow budget constraint of each consumer is

$$\frac{\partial a(s,t)}{\partial t} = \begin{cases} (1-\tau)[ra(s,t)+w_t]-c(s,t)+pa(s,t), & a(s,t)\geq 0, \\ ra(s,t)+(1-\tau)w_t-c(s,t)+pa(s,t), & a(s,t)<0, \end{cases} \quad (1)$$

where $a(s, t)$ is financial assets at time *t* of generation *s,* and *r* stands for the world interest rate that is constant over time due the assumption of a small country. The government imposes a tax on the wage income of the households, and the tax rate takes the same rate as that of the interest tax, that is, τ. As mentioned above, when $a(s, t)$ takes a negative value, the interest payment is not taxed. We assume that the insurance premium, *p,* is not taxed. We further assume that an individual supplies one unit of labor inelastically; that the wage rate, w_t, does not depend on the age of households; and that the newly born households are not endowed with any financial assets:

$$a(s,s)=0$$

The optimal conditions are given by:

$$\frac{\partial c(s,t)}{\partial t} = \begin{cases} (1-\tau)r-\rho, & a(s,t)\geq 0, \\ r-\rho, & a(s,t)<0, \end{cases} \quad (2)$$

$$\begin{aligned} &\lim_{T\to\infty} a(t,T)e^{-[(1-\tau)r+p](T-t)} = 0, \quad a(s,t)\geq 0, \\ &\lim_{T\to\infty} a(t,T)e^{-[r+p](T-t)} = 0, \quad a(s,t)<0, \end{aligned} \quad (3)$$

The first equation (2) is the Euler equation, and the second equation (3) is the transversality condition. Accumulation of financial assets is determined by (1), (2), and (3) for given *r* and $\{w_t\}_{t=0}^{\infty}$.

Integrating budget constraint (1) and making use of (1), (2), and (3) result in the following consumption function of generation *s* at time *t*:

$$c(s,t)=(\rho+p)[a(s,t)+h(t)],$$

where $h(t)$ is the human wealth at time *t*:

$$h(t) \equiv \begin{cases} \int_t^{\infty}(1-\tau)w_v \exp\left[-\int_t^v[(1-\tau)r+p]dz\right]dv, & a(s,t)\geq 0 \\ \int_t^{\infty}(1-\tau)w_v \exp\left[-\int_t^v[r+p]dz\right]dv, & a(s,t)<0 \end{cases}$$

Letting aggregate variable X_t, defined by $X_t \equiv x(0,t)e^{-pt}+\int_0^t x(s,t)pe^{-pt}ds$ for each variable $x(s, t)$, we find the aggregate consumption function:

$$C_t=(\rho+p)[A_t+H_t] \quad (4)$$

Time differentiation of (4) leads to the following consumption dynamics:

$$\dot{C}_t = \begin{cases} [(1-\tau)r-\rho]C_t-p(\rho+p), & a(s,t)\geq 0, \quad \forall s\in(0,t] \\ [(r-\rho]C_t-p(\rho+p), & a(s,t)<0, \quad \forall s\in(0,t] \end{cases} \quad (5)$$

2.2 The Firm

Following Barro (1990), the government supplies productive public services. The production function takes the following form:

$$Y_t = F(K_t, G_t L_t), \quad (6)$$

where Y, K, G, and L are output, private capital, productive public services, and labor input, respectively. This production function satisfies the standard neoclassical characters, especially the constant returns to scale in both inputs. Accordingly, we can transform this into the following intensive form:

$$Y_t = f\left(\frac{G_t L_t}{K_t}\right) K_t = f(x_t) K_t \quad (7)$$

where $x_t \equiv (G_t L_t)/K_t$. The first order conditions for profit maximization become as follows:

$$w_t = f'(x_t) G_t \equiv \omega G_t, \quad (8)$$

$$r = f(x_t) - f'(x_t) x_t, \quad (9)$$

where the world interest rate is constant because of the assumption of a small country. Thus, x_t becomes constant over time. We denote this constant value as x.

For later use, we calculate the following:

$$\frac{G_t}{Y_t} = \frac{G_t}{f(x) K_t} = \frac{x}{f(x)} \equiv g \quad (10)$$

2.3 The Government

The government finances its expenditure by two methods. One is by levying an income tax, and the other is by issuing bonds. Thus, the government's budget constraint is

$$\dot{B}_t = \begin{cases} rB_t + G_t - \tau(rA_t + w_t), & A_t \geq 0, \\ rB_t + G_t - \tau w_t, & A_t < 0, \end{cases} \quad (11)$$

where B_t stands for government bonds.

We assume that the government has an upper limit, or target level for government bonds outstanding. The government tries to maintain the ratio of bonds to GDP at a constant level. We assume that the government adjusts $b_t \equiv B_t/Y_t$ gradually so that it equals the bond target level in the long run, as follows:

$$\dot{b}_t = -\phi(b_t - \bar{b}), \quad \phi > 0 \quad (12)$$

where $\bar{b}$ and ϕ represent the target level of government bonds and the adjustment coefficient of the rule, respectively. Therefore, given this adjustment rule, the government can borrow from the abroad to balance its budget.

3. MODEL EQUILIBRIUM

We derive the equilibrium paths of the economy. In order to derive the dynamic system, we define the following variables: $c_t = C_t/Y_t$, $a_t \equiv A_t/Y_t$, and $\gamma = \dot{Y}_t / Y_t$. From (5), we obtain the following:

$$\dot{c}_t = \begin{cases} [(1-\tau)r - \rho - \gamma_t]c_t - p(\rho + p)a_t, & a_t \geq 0 \\ [r - \rho - \gamma_t]c_t - p(\rho + p)a_t, & a_t < 0 \end{cases} \quad (13)$$

By the definition of aggregate assets and (8), we obtain the following:

$$\dot{A}_t = \begin{cases} (1-\tau)[rA_t + f'(x)G_t] - C_t, & A_t \geq 0 \\ rA_t + (1-\tau)f'(x)G_t - C_t, & A_t < 0 \end{cases}$$

This can be rewritten as follows:

$$\dot{a}_t = \begin{cases} [(1-\tau)r - \gamma_t]a_t + (1-\tau)\omega g - c_t, & a_t \geq 0 \\ [r - \gamma_t]a_t + (1-\tau)\omega g - c_t, & a_t < 0 \end{cases} \quad (14)$$

From (11), we obtain

$$\dot{b}_t = \begin{cases} (r - \gamma_t)b_t - [\tau(ra_t + \omega g) - g], & a_t \geq 0 \\ (r - \gamma_t)b_t - [\tau\omega g - g], & a_t < 0 \end{cases} \quad (15)$$

Note that the second term of the right side of this equation means the primary balance per GDP.

Substituting (12) into (15), we can obtain

$$\gamma_t = \begin{cases} (r + \phi) - \dfrac{1}{b_t}[\phi\bar{b} + \tau(ra_t + \omega g) - g] \equiv \gamma(a_t, b_t; \tau), & a_t \geq 0 \\ (r + \phi) - \dfrac{1}{b_t}[\phi\bar{b} + \tau\omega g - g] \equiv \hat{\gamma}(b_t; \tau), & a_t < 0 \end{cases} \quad (16)$$

Because $b_t = \bar{b}$ in the long run, the following relationship holds:

$$\gamma^* = \begin{cases} r - \dfrac{1}{\bar{b}}[\tau(ra^* + \omega g) - g], & a_t \geq 0 \\ r - \dfrac{1}{\bar{b}}[\tau\omega g - g] & a_t < 0 \end{cases} \quad (17)$$

When the households are creditors, that is, $a_t \geq 0$, substituting (16) into (13) and (14) leads to the following dynamic system:

$$\dot{c}_t = [(1-\tau)r - \rho - \gamma(a_t, b_t; \tau)]c_t - p(\rho + p)a_t, \quad (18)$$

$$\dot{a}_t = [(1-\tau)r - \gamma(a_t, b_t; \tau)]a_t + (1-\tau)\omega g - c_t. \quad (19)$$

On the other hand, when the households are debtors, that is, $a_t < 0$, we obtain

$$\dot{c}_t = [r - \rho - \hat{\gamma}(b_t; \tau)]c_t - p(\rho + p)a_t, \quad (20)$$

$$\dot{a}_t = [r - \hat{\gamma}(b_t; \tau)]a_t + (1-\tau)\omega g - c_t. \quad (21)$$

When the households are creditors, that is, $a_t \geq 0$, (18), (19), and (12) constitute the dynamic system of the economy. On the other hand, when the households are debtors, that is, $a_t < 0$, (20), (21), and (12) constitute the dynamic system of the economy. Because (18) and (19) do not affect (12), it is enough to consider only (18) and (19). The same applies to (20) and (21).

We next examine the steady state of the economy where c_t, a_t, and b_t become constant over time. We use the phase diagram to do this. $\dot{c}_t = 0$ line is defined by the following:

$$c_t = \frac{p(\rho + p)\bar{b}a_t}{\tau(ra_t + \omega g) - g - (\rho + \tau r)\bar{b}}, \quad a_t \geq 0. \quad (22)$$

By differentiating (22) with respect to a_t, we obtain

$$\frac{dc_t}{da_t} = p(\rho + p)\bar{b}\frac{\tau\omega g - g - (\rho + \tau r)\bar{b}}{[(ra_t + \omega g) - g(\rho + \tau r)\bar{b}]^2}. \quad (23)$$

We can show that $\dot{c}_t\big|_{a_t \geq 0} = 0$ line is upward sloping (downward sloping) when the numerator of (23) takes a positive (negative) value, that is, $\tau\omega g - g > (<)(\rho + \tau r)\bar{b}$. Figure 1a shows an upward sloping $\dot{c}_t\big|_{a_t \geq 0} = 0$ line, and Figure 1b shows a downward sloping $\dot{c}_t\big|_{a_t \geq 0} = 0$ line, respectively. When the tax rate, τ, takes a sufficiently large value and the target level of government bonds, $\bar{b}$, is sufficiently small, $\dot{c}_t\big|_{a_t \geq 0} = 0$ line becomes upward sloping. If the government runs budget surpluses, the result as shown in Figure 1a tends to appear. If, on the other hand, the government runs budget deficits, $\dot{c}_t\big|_{a_t \geq 0} = 0$ line becomes downward sloping and the result as shown in Figure 1b tends to appear.

Figure 1a: Model Dynamics with Government Budget Surpluses

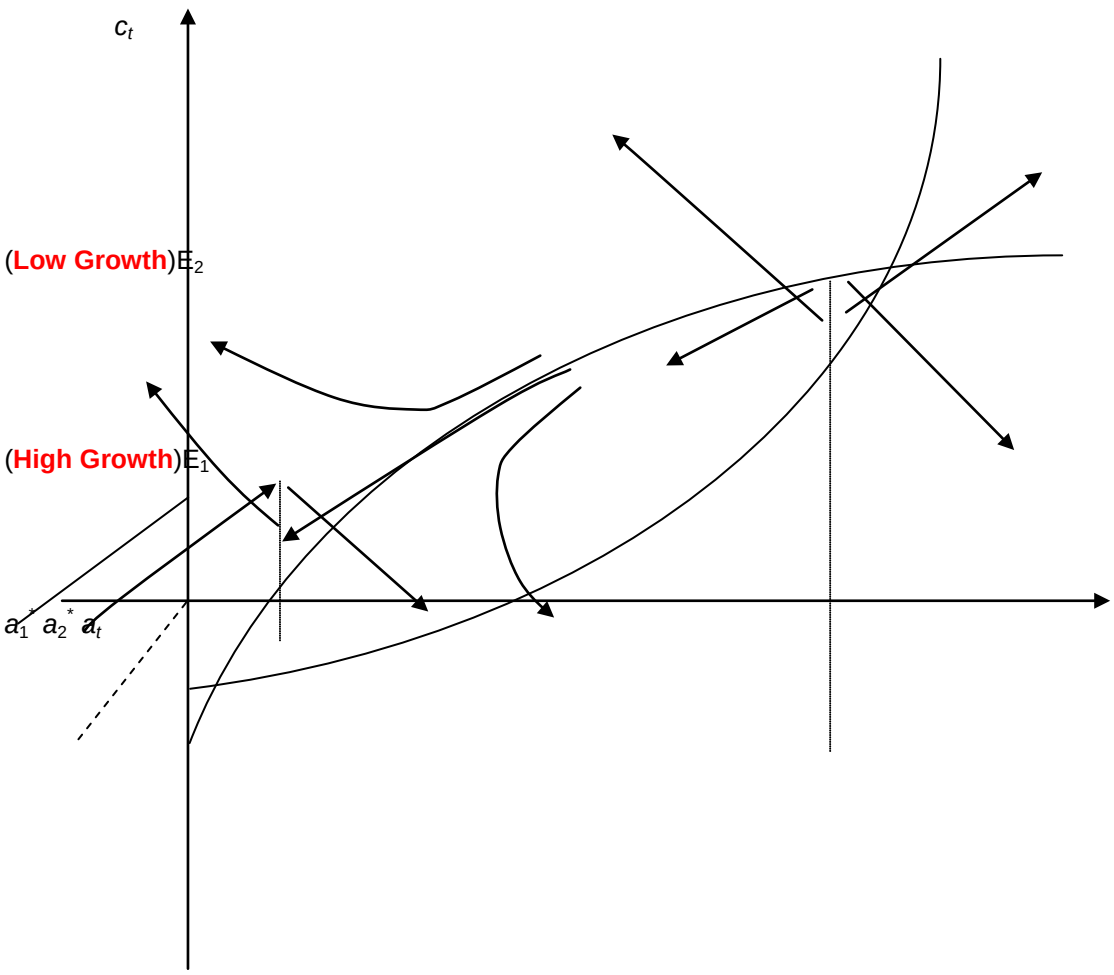
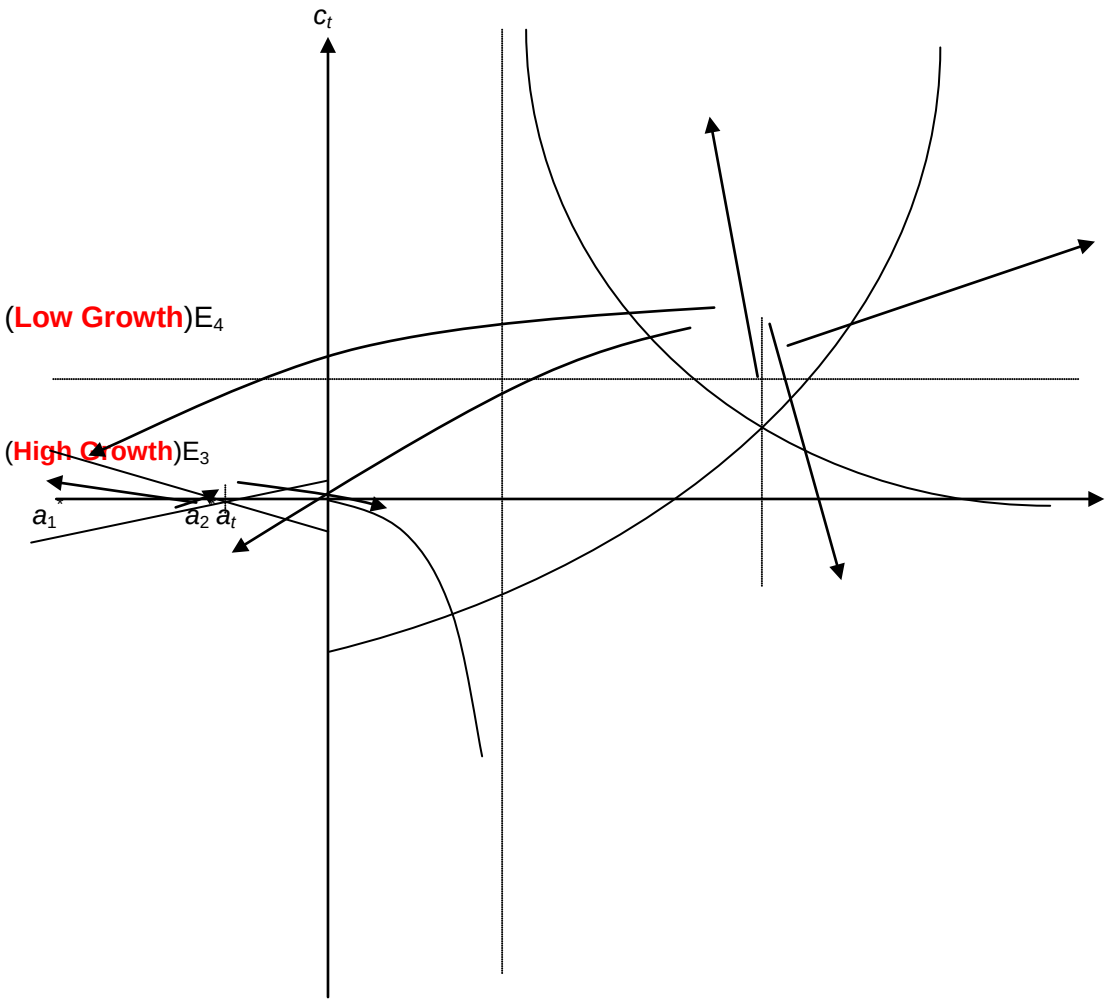


Figure 1b: Model Dynamics with Government Budget Deficits



When the households are debtors, that is, $a_t < 0$, $\dot{c}_t\big|_{a_t<0} = 0$ line becomes

$c_t = \dfrac{p(\rho+p)}{r-\rho-\hat{\gamma}(\bar{b};\tau)} a_t = \dfrac{p(\rho+p)}{\frac{1}{\bar{b}}[\tau\omega g - g]-\rho} a_t, \quad a_t < 0$. (24) When $\dot{c}_t\big|_{a_t\geq 0} = 0$ line becomes upward sloping, this implies that $\tau\omega g - g > (\rho+\tau r)\bar{b}$. Therefore, when the inequality, $r-\rho-\hat{\gamma}(\bar{b};\tau) > 0$ holds, $\dot{c}_t\big|_{a_t<0} = 0$ line also becomes upward sloping. We can therefore depict $\dot{c}_t\big|_{a_t<0} = 0$ line as shown in Figure 1a.

$\dot{a}_t\big|_{a_t\geq 0} = 0$ line is defined by the following

$$c_t = (1-\tau)\omega g + \frac{1}{\bar{b}} a_t[\tau(ra_t + \omega g) - (g + \tau r\bar{b})], \quad a_t \geq 0 \quad (25)$$

This is a convex parabola. When $a_t < 0$, $\dot{a}_t\big|_{a_t<0} = 0$ line is defined by the following:

$$c_t = (1-\tau)\omega g + \frac{1}{\bar{b}} a_t[\tau\omega g - g], \quad a_t < 0 \quad (26)$$

This is a straight line.

We focus on the cases where steady states exist. Because c_t and a_t are jump variables, all steady states have to be examined. There can exist two steady states. From (17), we can show that the economy that has a smaller amount of assets per GDP exhibits a higher growth rate. Thus, in Figure 1a (1b), E_1 (E_3) is the steady state with high growth, while E_2 (E_4) is the steady state with low growth.

4. CHARACTER OF THE MODEL'S STEADY STATES

Comparative statics analyses of the steady states are conducted to examine their character. When the economy has a positive amount of asset holdings per GDP, by using (18) and (19), we obtain the following:

$$\begin{pmatrix} (1-\tau)r-\rho-\gamma(a_i^*,\bar{b};\tau) & -\frac{\partial\gamma(a_i^*,\bar{b};\tau)}{\partial a_t}c_i^*-p(\rho+p) \\ -1 & (1-\tau)r-\gamma(a_i^*,\bar{b};\tau)-\frac{\partial\gamma(a_i^*,\bar{b};\tau)}{\partial a_t}a_i^* \end{pmatrix}\begin{pmatrix} dc_i^* \\ da_i^* \end{pmatrix}$$
$$=\begin{pmatrix} \frac{1}{\bar{b}^2}[\tau(ra_i^*+\omega g)-g]c_i^* \\ \frac{1}{\bar{b}^2}[\tau(ra_i^*+\omega g)-g]c_i^* \end{pmatrix}d\bar{b}+\begin{pmatrix} [r-\frac{1}{\bar{b}}(ra_i^*+\omega g)]a_i^* \\ [r-\frac{1}{\bar{b}}(ra_i^*+\omega g)]a_i^*+\omega g \end{pmatrix}d\tau, \quad i=1,2,4. \tag{27}$$

When the primary balance per GDP, $\tau(ra_i^*+\omega g)-g$, at the steady state takes a positive (negative) value, the coordinates of the vector that is multiplied by $d\bar{b}$ take positive (negative) values. We obtain the effects of a change in the target level of government bonds on the asset holdings when keeping the tax rate constant ($d\tau$ =0) as follows:

$$da_i^*=\frac{\frac{1}{\bar{b}^2}[\tau(ra_i^*+\omega g)-g]}{|J_i|}\begin{vmatrix} (1-\tau)r-\rho-\gamma(a_i^*,\bar{b};\tau) & c_i^* \\ -1 & a_i^* \end{vmatrix}d\bar{b}, \quad i=1,2,4. \tag{28}$$

Here, $|J_i|$ stands for the determinant of the coefficient matrix of (27). Because the steady state E_1 exhibits the saddle point stability, the determinant of the Jacobi matrix, $|J_1|$ takes a negative value. On the other hand, because the steady state E_2 and E_4 are sources, that is, unstable, the determinants of the Jacobi matrix, $|J_i|$ (i = 2, 4) , take positive values (see Appendix for these signs).

First, suppose that the primary balance takes a positive value at both steady states, that is, $\tau(ra_i^*+\omega g)>g$. Consumption must take positive values at the steady states, and the inequality $(1-\tau)r-\rho>\gamma(a_i^*,\bar{b};\tau)$ holds at both steady states. Then, the result of the comparative statics for the asset holdings at the steady state E_1 is opposite that at the steady state E_2. An increase in the target level of government bonds reduces the level of asset holdings at the steady state E_1. On the other hand, an increase in the target level of government bonds increases the level of asset holdings at the steady states E_2 and E_4.

We next consider how an increase in the target level of government bonds affects the growth rate of the economy. From (25), an increase in the target level of government bonds decreases the level of asset holdings at the steady state with high growth. By taking account of (17), we can show that this decrease raises the growth rate of the economy at the steady state E_1. Therefore, an increase in the target level of government bonds raises the growth rate of the high-growth economy. On the other hand, as stated before, we obtain the opposite result at the steady states E_2 and E_4. An increase in the target level of government bonds reduces the growth rate of the low-growth economy.

In their analysis of a closed economy, Futagami, Iwaisako, and Ohdoi (2008) showed that an increase in the target level of government bonds reduces the economic growth rate at the steady state with high growth. The opposite result obtains in the steady state with low growth. The present analysis of a small open economy sharply contrasts with their result. In a closed economy, a large amount of government bonds lowers private capital; in an open economy, however, government can borrow from abroad. Therefore, the mechanism by which the government finances its expenditure in open economies works in a different way from that in closed economies.

Second, suppose the government runs budget deficits at the both low-growth and high-growth steady states, that is, $\tau(ra_i^* + \omega g) < g$. As can be seen from (25), the results obtained are completely opposite and are similar to those obtained by Futagami, Iwaisako, and Ohdoi in this case. In their model, however, the government must make its primary balance positive in the long run.

When the economy has a negative amount of asset holdings per GDP, by using (20) and (21), we obtain the following:

$$\begin{pmatrix} r-\rho-\hat{\gamma}(\bar{b};\tau) & -p(\rho+p) \\ -1 & r-\hat{\gamma}(\bar{b};\tau) \end{pmatrix} \begin{pmatrix} dc_i^* \\ da_i^* \end{pmatrix} = \begin{pmatrix} \frac{1}{\bar{b}^2}[\tau\omega g - g]c_i^* \\ \frac{1}{\bar{b}^2}[\tau\omega g - g]a_i^* \end{pmatrix} d\bar{b} + \begin{pmatrix} -\frac{1}{\bar{b}}\omega g a_i^* \\ -\frac{1}{\bar{b}}\omega g a_i^* + \omega g \end{pmatrix} d\tau, \quad i = 3. \tag{29}$$

Consequently we can obtain

$$da_i^* = \frac{\frac{1}{\bar{b}^2}[\tau\omega g - g]}{|J_3|} \begin{vmatrix} r-\rho-\hat{\gamma}(\bar{b};\tau) & c_i^* \\ -1 & a_i^* \end{vmatrix} d\bar{b}, \quad i = 3. \tag{30}$$

Because the element of the first column and the first row of the determinant takes a negative value, the result becomes ambiguous in this case.

5. CONCLUDING REMARKS

A model of a small open economy with overlapping generations is constructed. The government supplies productive spending in the economy. A policy rule for government borrowing is introduced when the government establishes constraints on the amount of government bonds outstanding in the form of an upward limit, or target level for the ratio of government bonds to GDP.

We first show that there exist multiple steady states in the model small open economy. One is a steady state with high growth, the other a steady state with low growth. We next examine how changes in the target level for the ratio of government bonds to GDP affect the economic growth rates of the steady states. If the economy has a positive amount of asset holdings, we obtain the following results. When the government is running budget surpluses, an increase in the target level for government bonds reduces the growth rate of the low-growth economy, but raises the growth rate of the high-growth economy.

However, when the government is running budget deficits, an increase in the target level for government bonds raises the growth rate of the low-growth economy, but reduces the growth rate of the high-growth economy.

The results obtained in this paper have implications for some Asian economies. Economies with high growth include the People's Republic of China and India. On the other hand, economies with low growth include Japan. Changes in the upper limit, or target level for government bonds outstanding would have different implications for these two types of

economies. Let us consider the Japanese economy. The Japanese government is now running large deficits to cope with economic difficulties. An increase in the target level for government bonds could thus promote growth of the Japanese economy. However, the government cannot operate with budget deficits continuously. In the long run, it must run a budget surplus. Hence, a conclusion of this paper is that continued increases in the target level for the ratio of government bonds to GDP can reduce the growth of the Japanese economy.

If the small open economy has a negative amount of asset holdings, the model produces ambiguous results. While the economies of developed countries, such as United States, can have a negative amount of asset holdings, developing economies face various difficulties when they have a negative amount of asset holdings. Therefore, the analytical results in this case do not have any implications for developing economies.

APPENDIX

Liberalizing (18) and (19) around the steady state E_i (i=1,2,4), we obtain

$$\begin{pmatrix} \dot{c}_t \\ \dot{a}_t \end{pmatrix} = \begin{pmatrix} (1-\tau)r - \rho - \gamma(a_i^*, \bar{b}; \tau) & -\frac{\partial \gamma(a_i^*, \bar{b}; \tau)}{\partial a_t} c_i^* - p(\rho + p) \\ -1 & (1-\tau)r - \gamma(a_i^*, \bar{b}; \tau) - \frac{\partial \gamma(a_i^*, \bar{b}; \tau)}{\partial a_t} a_i^* \end{pmatrix} \begin{pmatrix} c_t - c_i^* \\ a_t - a_i^* \end{pmatrix} \quad i = 1,2,4.$$

Due to the shape of the curves depicted in Figures 1a and 1b, the sign of the determinant of the Jacobi matrix takes a negative value at the steady state E_1, and takes positive values at the steady states E_2 and E_4.

REFERENCES

Barro, R. J. 1990. Government Spending in a Simple Model of Endogenous Growth. *Journal of Political Economy* 98 (5) part 2: s103-s125.

Blanchard, O. 1985. Debt, Deficits, and Finite Horizons. *Journal of Political Economy* 93(2): 223–247.

Bruce, N., and Turnovsky, S. 1999. Budget Balance, Welfare, and the Growth Rate: "Dynamic Scoring" of the Long-run Government Budget. *Journal of Money, Credit and Banking.* 31 (2): 162-186.

Futagami, K., T. Iwaisako, and R. Ohdoi. 2008. Debt Policy Rule, Productive Government Spending, and Multiple Growth Paths. *Macroeconomic Dynamics* 12 (4): 445-462.

Ghosh, S., and I. A. Mourmouras. 2004. Endogenous Growth, Welfare, and Budgetary Regimes. *Journal of Macroeconomics* 26 (4): 624-635.

Greiner, A., and W. Semmler. 1999. An Endogenous Growth Model with Public Capital and Government Borrowing. *Annals of Operations Research* 88: 65-79.

———. 2000. Endogenous Growth, Government Debt and Budgetary Regimes. *Journal of Macroeconomics* 22 (3): 363-384.

Modigliani, F. 1961. Long-Run Implications of Alternative Fiscal Policies and the Burden of the National Debt. *Economic Journal* 71 (284): 730-755.