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可轉換證券在創投融資的角色

The Role of Convertible Securities in Venture Capital Financing

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摘 要

在本文，我們研究可轉換證券在新創事業融資所扮演的角色，這時候創業家比創投家擁有更多訊息。我們證明，在一個設計良好的契約之下，證券的換股比例可做為訊息傳遞機制，用來克服訊息不對稱的問題。如果報酬的變動夠大，創業者將發現可藉由可轉換證券的換股比例來披露部分訊息，也就是說，將會出現「分離均衡」。由於「混合均衡」無法披露私有訊息，容易引發誘因問題，「分離均衡」則具有避免上述誘因問題的優勢。再者，我們證明在投資及換股之間的決策時差將會使創投者獲利，此額外報酬稱為「時間價值」。此外，我們研究了「技術股」的影響，創業家不用投入任何資金即可獲得股份，我們將這種融資方式和可轉換證券做比較，並解釋為何可轉換證券已成為新創事業最廣泛使用的融資工具。

關鍵詞：創投者、創業者、可轉換證券、訊息傳遞

Abstract

In this paper we study the role of convertible securities in the financing of start-up enterprises when the entrepreneurs are better informed than the venture capitalists (VCs). We demonstrate that for a well-designed contract the conversion ratio of the securities can be used as a signaling device to overcome the problem of information asymmetry. If the variability of the return is sufficiently large, the entrepreneurs will find it desirable to rely on convertible securities with the conversion ratio revealing part of his information, that is, a "separating equilibrium" will arise. Such an equilibrium has the advantage of avoiding the incentive constraints that appear in the other "pooling equilibrium", in which the privately held information is not revealed. We show that the time-lag of decisions between investment and conversion will also benefit the VCs, with the extra return as the "time value". In addition, we study the impact of introducing "technical shares" with which the entrepreneurs are awarded equity shares without investment outlays. We compare the different financing devices with convertible securities and explain why convertible securities have become the most commonly used financial instrument for start-up enterprises.

Key words: Venture capitalists, Entrepreneurs, Convertible securities, Signaling

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I. Introduction

As a new investment project is initiated, it is often difficult to obtain funding through traditional channels since the entrepreneur (hereafter as EN) may own little physical asset to be used as collaterals for loans. The other major constraint for an EN at the startup stage is that the project may not generate sufficient income to pay for the interest expenses. Therefore, we observe the emergence of venture capitalists (hereafter as VCs) to provide the necessary financing of start-up enterprises by using an intricate system of financial instruments, often beyond the normal form of debt or equity. It is also apparent that serious information asymmetry exists between EN and VC before entering any contractual arrangement. Furthermore, the empirical evidence shows that a particular form of convertible securities has become the most important and popular form of investment instrument in venture capital financing[1]. These phenomenon need to be explained by a coherent theory.

In this paper we focus on the contracting stage and explore how convertible securities can be designed to mitigate the problem of ex ante information asymmetry. In particular, we demonstrate that convertible securities can serve as a signaling device from the EN to VC and facilitate the financing of start-up enterprises. Before an EN signs a contract with a VC, the exists ex ante information asymmetry with a problem of adverse selection. The conversion ratio of convertible securities is shown in this paper to serve the role as choices for signaling. When the privately held information indicates that the investment plan is promising, the EN can propose a lower conversion ratio. Or on the contrary, if the privately held information reveals that the investment is risky, the EN can allow for a higher conversion ratio. In the contracting stage, the VC can perceive the EN's private information by evaluating the conversion ratios offered. We will demonstrate that a ”separating equilibrium” exists and overcomes the problem of information asymmetry in the contracting stage of start-up enterprises.

For the theoretical works on VC financing, they are either restrained to the use of conventional equity shares or bonds[2], or they are concerned with the use of convertible securities in reinforcing the incentives of EN after the investment contracts have been signed [3]. But for start-up enterprises the phenomenon of ex ante

1 For example, Kaplan and Strömberg (2003) investigated 213 financing rounds of VC enterprises during the period of December 1986 to August 1999, and found that convertible preferred stock was used in 204 of them.

2 Many other studies focus on the use of equity shares or bonds in VC financing. Admati and Pfleider (1994) show that without an inside investor such as VC ”the choice of securities is unlikely to reveal all private information”. Hellmann (1998) demonstrates that in order to overcome the moral hazard problem VC has to have more control right than EN, including the firing of EN from the manager position. In Kirilenko (2001), VC is shown to need more controlling power than his share fraction to deal with incentive issues according to the extent of information asymmetry.

3 Information asymmetry is present in various stages of VC financing. After the fund has been invested, VC can still be unsure about the actions adopted by EN. This kind of moral hazard problem has been studied by Casamatta (2003), Cornelli and Yosha (2003) and Schmidt (2003). It is shown that convertible securities can be used to reinforce the work incentive as in Casamatta (2003), to reduce the effort for window dressing in Cornelli and Yosha (2003), or to allocate the cash-flow rights as a function of the EN's effort in Schmidt (2003).

information asymmetry is pervasive. There is an apparent difficulty for VC to collect enough information of the project proposed by EN. As Kaplan and Strömberg (2001) point out, VC spend a lot of time and efforts to evaluate investment plans, which indicates that ex ante information asymmetry is a critical issue requiring a more careful examination.

There are also important works about the use of financial instruments as a signaling device [4]. However, they are concerned neither with use of convertible securities nor with VC financing. In this paper we focus on the adverse selection problem faced by EN and VC before reaching an investment agreement. We find reasons for them to prefer the use of convertible securities when the investment risk is sufficiently high and is not publicly known to VC, who has to examine the terms of financial arrangements offered by EN. In order to fully analyze the strategic interactions between EN and VC, we adopt a dynamic framework, making it distinct from the works of Leland and Pyle (1977) or Ross (1977) in which the discussion is limited to the unilateral decisions by the managers or EN. Similar to the analysis of Cho and Kreps (1987) and Laffont and Maskin (1990), in our model the concept of Perfect Bayesian Equilibrium (PBE) is applied to analyze the dynamic signaling game in which EN first proposes a financial arrangement and VC then decides on whether to accept it. We study how convertible securities, in particular the conversion ratios, can be used to reveal some private information of EN so as to achieve an equilibrium for both EN and VC.

Furthermore, we also examine other alternative financial arrangements, including the use of ”technical shares” when EN obtains certain amount of equity shares without investment outlays. Since EN owns the technology for developing the product, an assessment of the contribution of the special technology in terms of certain amount of equity shares is awarded to or demanded by the EN at the contracting stage, with payoffs realized when shares are marketed later[5]. When the EN believes that the investment outcome will be more valuable, he might ask for more technical shares, or vice versa. Given the asymmetric information faced by the VC, the amount of technical shares demanded by the EN can become a source of getting certain information about the investment project. However, VC can also reject an unreasonable demand by the EN. The final equilibrium is determined through strategic interactions of the two parties. Its outcome is then compared with that of using convertible securities.

4 Based on the seminal work of Spence (1973), Leland and Pyle (1977) demonstrate that the manager's investment intention is a signal about the quality of the project. Furthermore, Ross (1977) shows that the amount of borrowing can be considered as a signal about the corporation type. Myers and Majluf (1984) present a model in which capital structure can be used to signal the quality of the corporations, with good corporations issuing bonds and bad ones issuing stocks. Stein (1992) extends Myers and Majluf (1984) to demonstrate that convertible bonds can serve as a middle ground choice between bonds and stocks, for the financing of corporate investment.

5 Leland and Pyle (1977) show that the fraction of managers' equity shares can become a signal to outside investors. In their model the managers obtain the shares through investing their own money, not through the way of ”technical shares” as discussed here. The use of technical shares has also been observed for VC financing.

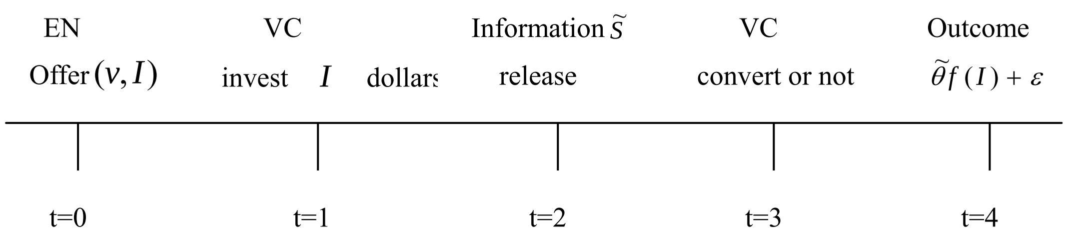
This paper is organized as follows. The basic model will be introduced in Section 2. In Section 3 we will study the conversion decision, the conditions for the existence of ”separating equilibrium” with convertible securities and the properties of such equilibrium. In Section 4 we analyze the properties of ”pooling equilibrium”. The entrepreneur’s favorite equilibrium is then characterized by the extent of exogenous uncertainty in Section 5. In Section 6 we compare the relative advantages of convertible securities and ”technical shares” and discuss why the start-up enterprises may want to adopt convertible securities as the financing instrument. Section 7 concludes this paper.

II. Basic Model

We consider a model with a continuum of identical risk-neutral VCs [6]. EN has a investment plan requiring capital input I with gross return $\tilde{\theta} f(I)+\varepsilon$, where $f(I)=I^{\alpha}$ and $0<\alpha<1$, ε is a random variable with standard normal distribution $N(0,\sigma^2)$, and θ is a random variable, independent of ε, that takes on the values θ_1 and θ_2 with probabilities μ and $(1-\mu)$, respectively. The variable $\tilde{\theta}$ is realized at the time 0 ($\theta_2 < \theta_1$) and is known to entrepreneur but not to all VCs.

The risk-neutral EN has no capital and will ask VCs for the required funds. Initially, EN who has private information will propose a conversion ratio v to VCs. Then, VC will decide on whether to invest I dollars. If one VC agrees to invest in this plan , this VC and EN will sign a contract (v,I) which allows VC to decide whether to exercise its conversion right to get equity v (with the total amount of equity

Figure 1. Timeline



normalized to one), or remain to be a debt holder who will receive dI dollars in the end, where d is the exogenously given gross rate of debt return.

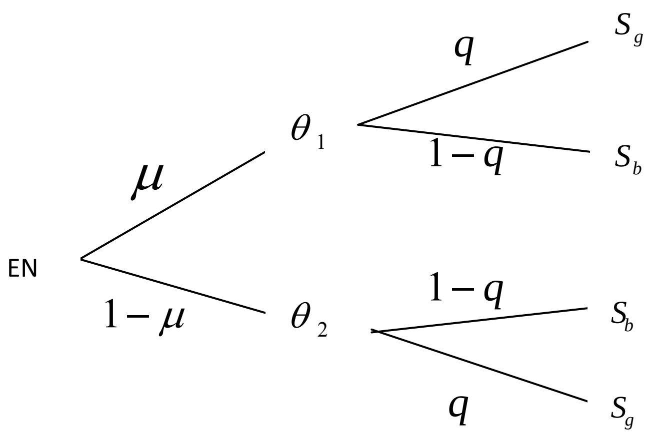
The sequence of dynamic interactions is characterized in Figure 1. At time 0, EN knows the true value of $\tilde{\theta}$, but VCs do not. After EN knows $\tilde{\theta}$, he offers a contract (v,I).

At time 1, VC will evaluate the investment plan and decide on whether to invest I. After more information is revealed about the profitability of the investment plan in period 2, VC decides to be an equity holder (equity share v) or debtholder (debt

6 In general, VC will invest in many venture projects, so as to reduce risk from any single project.

$D = dI$)[7] at time 3. Finally, in period 4, the gross return will be divided between EN and VC according to the contract (v, I) and the conversion decision made at time 3.

Figure 2. The belief and



The VC has the prior belief μ , as the likelihood of getting a final payoff θ_1, $0 \le \mu \le 10$. At t = 2, the information revelation $\tilde{S}$ arrives with some degree of inaccuracy. For simplicity, we assume that the public information is described by a binomial distribution: the information $\tilde{S}$ can either be S_g with probability q , $0.5 < q \le 1$, or S_b with probability $1-q$ if this is a good project: Prob $\{S_g \mid \theta = \theta_1\} = q$, Prob $\{S_b \mid \theta = \theta_2\} = 1-q$. Similarly, a probability q is assigned to the bad signal S_b if this is a bad project $(\theta = \theta_2)$.

After observing signal S, the VC updates his belief based on Bayes' rule and make the conversion decisions. The special case of $q = 1$ means that private information is completely revealed at t = 2. The belief and signal structure are described in Figure 2. At t = 2, based on the proposed contract (v, I) and the outcome of information revelation $\tilde{S}$, VC revises his beliefs regarding the EN's private information $\tilde{\theta}$ in the Bayesian way. The (ex ante) posterior probability μ_j of obtaining a good return θ_1 for VC can be calculated when Sg or Sb is observed.

7 In this model, we discuss the conversion ratio as a signal for private information revelation. To simplify, we can regard I as face value and (d−1) as the interest rate if the convertible security is a convertible bond, and regard D − I as dividends if it is a convertible preferred stock .In addition, we suppose the market interest rate is zero.

$$\begin{cases} \mu_g = \Pr ob(\theta_1 \mid v, I, S_g) = \dfrac{\mu q}{\mu q + (1-\mu)(1-q)} \\ \mu_b = \Pr ob(\theta_1 \mid v, I, S_b) = \dfrac{\mu(1-q)}{\mu(1-q) + (1-\mu)q} \end{cases} \tag{1}$$

EN's strategy is a mapping $v : \theta_i \to [0,1]$. $v(\theta_i)$ describes a conversion ratio, on the basis of EN's private information θ_i, $i = 1,2$. Suppose the debt return d and investment I are constant. *VC's strategy at time 1* is a mapping $I : v \to \{0, I\}$, that represents the investment decisions of VC for each conversion ratio v. After the information $\tilde{S}$ is released, *VC's strategy at time 3* is a mapping $C : (v, I, \tilde{S}) \to \{0, C\}$, that represents the conversion decisions of VC for each conversion ratio v, the capital input I, and information $\tilde{S}$.

Conditional beliefs for VC before he make the investment decision are represented by a mapping that associates to each conversion ratio v a probability function $h(. \mid v)$ on $\{S_g, S_b\}$, where $h(. \mid v)$ is the probability that the VCs attaches to information revelation $\tilde{S}$ given conversion ratio v. In the pooling equilibrium to be discuss below, the VC chooses to convert once the signal S_g is observed, and not to convert if the signal S_b is observed.

The conditional belief for VC before he make the conversion decision is $g(. \mid v, I, S)$, the probability that the VCs attaches to the value θ given conversion ratio v, capital input I, and information revelation S. We assume that EN will expend the management cost $c(I)$, where $c(I) = cI^{\beta}$ with $0 < c < 1$ and $\beta > 1$, if VC decide to invest I dollars.

Separating Perfect Bayesian Equilibrium

A *perfect Bayesian equilibrium* in our model is a pair of strategies $[v(.), I(.), C(. \mid v, I, S)]$ and two families of conditional beliefs $h(. \mid v)$ and $g(. \mid v, I, S)$ such that

(1) for all v in the range of $v(.)$, $h(. \mid v)$ is the conditional probability of $\tilde{S}$ and, for given v, I and $\tilde{S}$, $g(. \mid v, I, \tilde{S})$ is the conditional probability of $\tilde{\theta}$ obtained by updating in the Bayesian fashion.

(2) for all v, d, S, I and $h(S \mid v)$, VC convert $C(. \mid v, I, S) = C$ if

$$E\{v_1[(\theta_1 f(I_1) + \varepsilon)g(\theta_1 \mid v_1, I_1, S_j) + (\theta_2 f(I_1) + \varepsilon\}g(\theta_2 \mid v_1, I_1, S_j)]\} \geq dI_1$$

and $E\{v_2[(\theta_1 f(I_2) + \varepsilon)g(\theta_1 \mid v_2, I_2, S_j) + (\theta_2 f(I_2) + \varepsilon\}g(\theta_2 \mid v_2, I_2, S_j)]\} \geq dI_2$

and VC does not convert $C(. \mid v, I, S) = 0$ if

$$E\{v_1[(\theta_1 f(I_1)+\varepsilon)g(\theta_1 \mid v_1, I_1, S_j)+(\theta_2 f(I_1)+\varepsilon\}g(\theta_2 \mid v_1, I_1, S_j)]\} < dI_1$$

and

$$E\{v_2[(\theta_1 f(I_2)+\varepsilon)g(\theta_1 \mid v_2, I_2, S_j)+(\theta_2 f(I_2)+\varepsilon\}g(\theta_2 \mid v_2, I_2, S_j)]\} < dI_2$$

where $j = g, b$.

(3) for all v,

$$I(v_1) \in \operatorname{argmax}_I E\{v_1[(\theta_1 f(I)+\varepsilon)h(S_j \mid v_1)] - I\}$$

and

$$I(v_2) \in \operatorname{argmax}_I E\{v_2[(\theta_2 f(I)+\varepsilon)h(S_j \mid v_2)] - I\}$$

where $j = g, b$.

(4) for θ_1 and θ_2,

$$v(\theta_1) \in \operatorname{argmax}_{v_1} E\{(1-v_1)[\theta_1 f(I_1)+\varepsilon] - c(I_1)\}$$

$$v(\theta_2) \in \arg\max_{v_2} E\{(1-v_2)[\theta_2 f(I_2)+\varepsilon] - c(I_2)\}$$

Condition (1) stipulates that VC has rational expectations; Condition (2) is the required condition of VC'conversion decisions. Condition (3) and (4) are the requirements that EN and VC be optimizing respectively.

III. Separating Perfect Bayesian Equilibrium

There are many empirical evidences indicating that convertible securities are adopted widely in venture capital financing. In general, convertible security is a kind of portfolio of debt and conversion right which allow investor to make conversion decision in future. In this section, we will study the properties of convertible security financing and compare the characteristics of multiple equilibria. We start by showing the existence of a separating perfect Bayesian equilibrium.

Depending on EN's private information is θ_1, θ_2 at time 0, $\theta_1 > \theta_2$, he will propose a conversion ratio $v_1 = v(\theta_1)$ or $v_2 = v(\theta_2)$, respectively. And then VC can decide whether to invest I_1 or I_2 after receiving the signal v_1 or v_2 at time 1. When EN proposes a conversion ratio v, he has to take VC's willingness of conversion into consideration.

When EN proposes a conversion ratio v_i and VC decides to invest I_i dollars, the project will go on to the next stage. Information $\widetilde{S}$ is released to the public at time 2. Then, VC decides whether to convert debt into equity or not at time 3. We will estiblish the existence of a separating equilibrium with the following beliefs. If EN proposes v_1, VC invests I_1 dollars, and the information revelation is S_j, $j = g,b$, VC believes that the final return θ_1 and θ_2 with probabilities $g(\theta_1 \mid v_1, I_1, S_j) = 1$ and $g(\theta_2 \mid v_1, I_1, S_j) = 0$, respectively. For the same reasons, he believes that the final return θ_1 and θ_2 come with probabilities $g(\theta_1 \mid v_2, I_2, S_j) = 0$ and $g(\theta_2 \mid v_2, I_2, S_j) = 1$, if EN propose v_2, VC invest I_2 dollars, and the information revelation is S_j, $j = g,b$.

Given that EN proposes a conversion ratio v_i and VC offers capital input I_i and receives debt return dI_i, VC's private information θ_1 or θ_2 is revealed completely. The necessary conditions that VC will exercise the conversion right if the following conversion constraints are satisfied

$$E[v_i(\theta_i I_i^{\alpha} + \varepsilon)] \geq dI_i, \quad \text{where } i = 1,2 \qquad (2)$$

When EN proposes a conversion ratio v_1 and debt return d at time 0, VCs knows the true type of this project is θ_1, and believes that $h(S_g \mid v_1) = 1$ and $h(S_b \mid v_1) = 0$. In contrary, when EN proposes a conversion ration v_2 and debt return d at time 0, VCs knows the true type of this project is θ_2, and believes that $h(S_g \mid v_2) = 0$ and $h(S_b \mid v_2) = 1$. VC's expected return ER^{is} is

$$ER^{is}(I_i, \theta_i) = E[v_i(\theta_i I_i^{\alpha} + \varepsilon)] - I_i, \quad \text{for } i = 1,2.$$

So we find the first order condition

$$v_i \theta_i \alpha I_i^{\alpha-1} - 1 = 0, \quad \text{for } i = 1,2.$$

and the second order condition is also satisfied. The optimal choice of capital input is

$$I^*(v_i) = (\alpha v_i \theta_i)^{\frac{1}{1-\alpha}}, \quad \text{where } i = 1,2. \tag{3}$$

Back to time 0. Given VC's conversion decision constraints and investment decision $I^*(v_i)$, EN's expected profit EN^{is} is

$$EN^{is} = E[(1-v_i)(\theta_i I_i^{*\alpha}(v_i) + \varepsilon) - c(I^*(v_i))], \quad \text{for } i = 1,2. \tag{4}$$

From the first order condition, we can find the optimal choice of the conversion ratio as

$$\alpha - v_i^* = \beta c (\alpha v_i^*)^{\frac{\beta-\alpha}{1-\alpha}} \theta_i^{\frac{\beta-1}{1-\alpha}} \tag{5}$$

The second order condition is also satisfied. We are certain that there exists an interior solution v_i^* and $\alpha > v_i^*$. Then, the optimal conversion ratio $v_i^*(\theta_i)$ and capital inputs $I_i^*(\theta_i) = I_i(v_i^*(\theta_i))$ constitute a separating perfect Bayesian equilibrium. Hence, we have proved the following proposition.

Proposition 1. *There exists a separating PBE* $[v_1^*, (I_1^*, (C,C))]$ *and* $[v_2^*, (I_2^*, (C,C))]$, *in which* $v_1^* < v_2^*$, *in venture capital financing.*

In this separating equilibrium, EN's expected profit is $EN^{is}(v_i^*)$, $i = 1,2$. If EN's private nformation is θ_1, he proposes a conversion ratio v_1^*, VC will choose to invest I_1^* dollars and exercise the conversion right no matter what the information revelation is S_g or S_b. If EN's private information is θ_2, he proposes a conversion ratio v_2^*, VC will invest I_2^* dollars and exercise the conversion right no matter where the information revelation is S_g or S_b. There exists at least one such separating perfect

Bayesian equilibrium.

From the maximization conditions of v_1^* and v_2^*, we can check that the single crossing condition is satisfied. Furthermore, the properties of a separating PBE can be also demonstrated in the following proposition.

Proposition 2. *For the separating PBE, the endogenously chosen conversion ratio* v *will decrease with exogenously given* θ *and the capital input* I *will increase with* θ*, if* $\alpha > v$*. That is, the conversion ratio* v *and the capital input* I *will change in opposite directions.*

Proof: By equation (5), we have

$$\ln(\alpha - v) = \ln\beta + \ln c + \frac{\beta-\alpha}{1-\alpha}\ln(\alpha v) + \frac{\beta-1}{1-\alpha}\ln\theta$$

Differentiating this equation with respect to θ, we find

$$[\frac{-1}{\alpha-v} - \frac{\beta-\alpha}{1-\alpha}\frac{1}{v}]dv = \frac{\beta-1}{1-\alpha}\frac{1}{\theta}d\theta$$

The following comparative statics can be derived

$$\frac{dv}{d\theta} = \frac{\dfrac{\beta-1}{1-\alpha}\dfrac{1}{\theta}}{\dfrac{-1}{\alpha-v} - \dfrac{\beta-\alpha}{1-\alpha}\dfrac{1}{v}} \tag{6}$$

If $\alpha > v$, we can show that $\frac{dv}{d\theta} < 0$.

Also, by equation (3),

$$\ln I(v) = \frac{1}{1-\alpha}\ln(\alpha v\theta).$$

Differentiating this equation with respect to θ, and applying equation (6), we find

$$\frac{1-\alpha}{I}\frac{dI}{d\theta} = \frac{1}{v}\frac{dv}{d\theta} + \frac{1}{\theta} = \frac{\dfrac{\alpha}{\alpha-v}}{\dfrac{v}{\alpha-v} + \dfrac{\beta-\alpha}{1-\alpha}}\frac{1}{\theta}.$$

It is assumed that $\alpha < 1$ and $\beta > 1$. Therefore, $\frac{dI}{d\theta} > 0$ if $\alpha > v$. *Q.E.D*

In this separating PBE, the monotonic properties of $\frac{dv}{d\theta} < 0$ and $\frac{dI}{d\theta} > 0$ are the basis for the conversion ratio v to perform as a signaling mechanism. The management cost plays the role of ”indirect” signaling cost, which then results in the truth-telling condition. EN, who knows the project is of a bad type, doesn’t have the incentive to deviate, that is , to propose a smaller converting ratio and raise a larger amount of capital input, which would cause higher management cost. Therefore, as his private information θ turns out to be better, EN will offer a smaller conversion ratio v. Then VC is willing to provide a larger amount of capital input I in the separating PBE.

IV. Pooling Perfect Bayesian Equilibrium

In this section, we will study whether a pooling equilibrium may exist and explore its properties, if it exists. In such an equilibrium, EN with private information θ_1, θ_2 will propose a single conversion ratio $v^p = v^0(\theta_1, \theta_2)$. Then VC decides to invest $I^0 = I(v^0(\theta_1, \theta_2))$. We can demonstrate the existence of a pooling equilibrium with the peoperty that VC will decide to convert only after receiving a good signal. We also find conditions on exogenous variables for the existence of pooling equilibrium. In comparison with the results of last section, the pooling equilibrium appears to be a good description of VC financing when exogenous θ's are sufficiently close.

Proposition 3. *For* θ_1 *sufficiently near* θ_2 *, there exists a pooling PBE* $[v^0, (I^0, (C, 0))]$ *in which* $v^0 = v(\theta_1) = v(\theta_2) = v^0(\theta_1, \theta_2)$.

Proof: Suppose that the probability of VC receiving a good information S_g at time 2 is λ, where $\lambda = \mu q + (1-\mu)(1-q)$, and the probability of VC receiving a bad information S_b is $1-\lambda$. At time 3, VC decides whether to convert debt into equity or not. Given that EN proposes a conversion ratio v, VC decides to invest I dollars and information revelation is S_g, we can find VC's posterior belief for the realization of θ_1 is μ_g. With the same logic, VC's posterior belief for θ_2 is μ_b can also be written as in equation (1):

$$\begin{cases} \mu_g = \Pr ob(\theta_1 \mid v, I, S_g) = \dfrac{\mu q}{\mu q + (1-\mu)(1-q)} > \mu \\ \mu_b = \Pr ob(\theta_1 \mid v, I, S_b) = \dfrac{\mu(1-q)}{\mu(1-q) + (1-\mu)q} < \mu \end{cases} \tag{1}$$

Equation (1) means that VC become more optimistic (pessimistic) when the signal $S_g(S_b)$ has been observed. At time 4, VC will exercise the conversion right in the state of information revelation S_g, if the expected return of becoming a equity holder is greater than or equal to that of debt. In contrast,VC will not exercise the conversion right with signal S_b if the following equation is satisfied:

$$\begin{cases} E\{v[\mu_g(\theta_1+\varepsilon)I^\alpha+(1-\mu_g)(\theta_2+\varepsilon)I^\alpha]\}\geq dI \\ E\{v[\mu_b(\theta_1+\varepsilon)I^\alpha+(1-\mu_b)(\theta_2+\varepsilon)I^\alpha]\}\leq dI \end{cases} \tag{7}$$

We can derive the boundary of *convertible price* $\dfrac{d}{v}$

$$[\mu_b\theta_1+(1-\mu_b)\theta_2]I^\alpha\leq\frac{dI}{v}\leq[\mu_g\theta_1+(1-\mu_g)\theta_2]I^\alpha \tag{8}$$

When EN propose a conversion ration v at time 0, and VC invest I dollars at time 1, given the posterior probability distribution of information revelation $\widetilde{S}$, VC's expected profit ER^p is

$$\begin{aligned} ER^p &= \mu\{E[qv(\theta_1 I^\alpha+\varepsilon)+(1-q)dI-I]\} \\ &\quad+(1-\mu)\{E[(1-q)v(\theta_2 I^\alpha+\varepsilon)+qdI-I]\} \end{aligned}$$

By the first order condition, the optimal choice of capital input is

$$I^0(v,\theta_1,\theta_2)=[\frac{\alpha v\lambda}{1-(1-\lambda)d}\overline{\theta}_g]^{\frac{1}{1-\alpha}}$$

where $\overline{\theta}_g=\mu_g\theta_1+(1-\mu_g)\theta_2$. Then, EN's expected profit EN^p is

$$\begin{aligned} EN^p &= \mu\{E[q(1-v)(\theta_1 I^{0\alpha}+\varepsilon)+(1-q)(\theta_1 I^{0\alpha}+\varepsilon-dI^0)-cI^{0\beta}]\} \\ &\quad+(1-\mu)\{E[(1-q)(1-v)(\theta_2 I^{0\alpha}+\varepsilon)+q(\theta_2 I^{0\alpha}+\varepsilon-dI^0)-cI^{0\beta}]\} \end{aligned} \tag{9}$$

From the first order condition, we find the choice of the optimal conversion ratio $v^0(\theta_1,\theta_2)$ as

$$(\alpha\overline{\theta}-v\lambda\overline{\theta})-\frac{(1-\alpha)-(1-2\alpha)(1-\lambda)d}{1-(1-\lambda)d}v\lambda\overline{\theta}=c\beta[\frac{\alpha v\lambda}{1-(1-\lambda)d}\overline{\theta}]^{\frac{\beta-\alpha}{1-\alpha}}$$

where $\overline{\theta}=\mu\theta_1+(1-\mu)\theta_2$, and the optimal capital input is $I(v^0(\theta_1,\theta_2))=I^0(v^0(\theta_1,\theta_2),\theta_1,\theta_2)$. Then, we can check the existence of the pooling equilibrium. Define $I(v_i)=\hat{I}(v(\theta_i),\theta_i)$, $i=1,2.$, Taking $v(\theta_1)=v(\theta_2)=v^0(\theta_1,\theta_2)$,

$I(v^0(\theta_1,\theta_2)) = I^0(v^0(\theta_1,\theta_2),\theta_1,\theta_2) = I^0$ [8], $prob(\theta_1 \mid v^0(\theta_1,\theta_2)) = \mu$ and $prob(\theta_2 \mid v^0(\theta_1,\theta_2)) = 1-\mu$, we also construct the equilibrium with out-of equilibrium conversion ratios. For any conversion ratio $v \neq v^0(\theta_1,\theta_2)$, let $prob(\theta_1 \mid v) = 1$. We show that, given this beliefs, the entrepreneur will never set such a conversion ratio if θ_1 and θ_2 are closed enough.

Suppose first that $\tilde{\theta} = \theta_1$. If $\theta_2 = \theta_1$, then the entrepreneur will not gain by setting $v \neq v^0(\theta_1,\theta_2)$ since $I^0(\theta_1,\theta_2) = I^*(\theta_1)$. Thus it suffices to show that, at $\theta_2 = \theta_1$, the entrepreneur's net payoff $EN^p(\theta_1) = q(1-v^0)\theta_1 I^{0\alpha} + (1-q)(\theta_1 I^{0\alpha} - dI^0) - cI^{0\beta}$ is decreasing in θ_2, since

$$\begin{aligned}
&q(1-v^0)\theta_1 I^{0\alpha} + (1-q)(\theta_1 I^{0\alpha} - dI^0) - cI^{0\beta} \\
&= (1-v^0)\theta_1 I^{0\alpha} - cI^{0\beta} + (1-q)[v^0\theta_1 I^{0\alpha} - dI^0] \\
&> (1-v^*(\theta_1))\theta_1 I^{*\alpha}(\theta_1) - cI^{*\beta}(\theta_1) \geq (1-v)\theta_1 \hat{I}^{\alpha}(v,\theta_1) - c\hat{I}^{\beta}(v,\theta_1), \quad \text{for all } v,
\end{aligned}$$

where θ_2 is near θ_1. At $\theta_2 = \theta_1$, the maximization problem in equation (9) becomes

$$EN^p(\theta_2 = \theta_1) = EN^p(\theta_1) + (1-\mu)(2q-1)[v^0\theta_1 I^{0\alpha} - dI^0],$$

with the first-order condition

$$\frac{dEN^p(\theta_2 = \theta_1)}{dv} = 0.$$

Now

$$\begin{aligned}
&\frac{dEN^p(\theta_1)}{d\theta_2} \\
&= \frac{d}{d\theta_2}\{EN^p(\theta_2 = \theta_1) - (1-\mu)(2q-1)[v_0\theta_1 I^{0\alpha} - dI^0]\} \\
&= \frac{dEN^p(\theta_2 = \theta_1)}{d\theta_2} v_2^0 - (1-\mu)(2q-1)[\theta_1 I^{0\alpha} + v_0\theta_1 I^{0\alpha} - d]I^0 v_2^0 \\
&= -(1-\mu)(2q-1)[\theta_1 I^{0\alpha} + v_0\theta_1 \alpha I^{0(\alpha-1)} - d]I^0 v_2^0 \qquad (10)
\end{aligned}$$

Since $q > 0.5, I^0 < 0$, $v_2^0 < 0$, equation (10) is negative at $\theta_2 = \theta_1$.

Assume next that $\tilde{\theta} = \theta_2$. In this case, if the entrepreneur failed to set $v = v^0(\theta_1,\theta_2)$, his best alternative is to choose $v = v^1(\theta_1,\theta_2)$ solve the maximization problem

$$EN^p(v^0,\theta_2) = q(1-v^0)\theta_2 \hat{I}^{\alpha}(v,\theta_1) + (1-q)[\theta_1 \hat{I}^{\alpha}(v,\theta_1) - d\hat{I}(v,\theta_1)] - c\hat{I}^{\beta}(v,\theta_1)$$

Since $g(\theta_1 \mid v) = 1$ for all $v \neq v^0(\theta_1,\theta_2)$. Thus it suffices to show that his gain from choosing v^0 rather than v^1 is decreasing in θ_2 at $\theta_2 = \theta_1$, that is,

$$\frac{d}{d\theta_2}\{EN^p(v^0,\theta_2) - EN^p(v_1,\theta_2)\} < 0 \qquad (11)$$

from the same application of the envelope theorem that we used in the previous paragraph. So equation (11) holds, as required. *Q.E.D*

[8] We know that $v_2^0 = \frac{dv^0(\theta_1,\theta_2)}{d\theta_2} < 0$ and $I_1^0 = \frac{dI^0(v^0(\theta_1,\theta_2),\theta_1,\theta_2)}{dv^0} < 0$

Proposition 4. *Given* θ_2*, there exists* $\overline{\theta}_1$ *such that there is no pooling equilibrium for* $\theta_1 > \overline{\theta}_1$.

Proof: Suppose that there exists a pooling equilibrium in which $v(\theta_i) = v$ and $I(v(\theta_i)) = I$, $i = 1,2$. If $0 < v < \alpha$, by the first-order condition of VC's expected profit ER^p

$$\frac{dER^p}{dI} = \alpha\lambda\mu[\mu q\theta_1 + (1-\mu)(1-q)\theta_2]I^{\alpha-1} + (1-\lambda)d - 1 \le 0 \qquad (12)$$

As θ_1 tends to infinity, so does I. Thus eventually the left-hand side of equation (12) exceeds 0, a contradiction of equation (12). Hence, for v near 0 and I near infinite, we can find a $\tilde{\theta} = \theta_1$ with the entrepreneur's payoff $EN^p = \theta_1 I^\alpha - (1-q)dI < 0$. But with $I^*(\theta_1)$, the entrepreneur can obtain a positive payoff by setting $v = v^*(\theta_1)$, a contradiction. *Q.E.D*

V. The Entrepreneur's Favorite Equilibrium

There exists a continuum of "semiseparating" equilibria (where either $v(\theta_1)$ or $v(\theta_2)$ is a random variable) between complete pooling and complete separating if a pooling equilibrium exists. We suppose that the entrepreneur can influence VCs' beliefs and ensure a favorable equilibrium. Hence, we assume that the entrepreneur can predict his ex ante Best PBE (BPBE) (see Laffont and Maskin (1990)). We use a numerical example to explain the EN's favorite equilibrium, a formal proof follows.

Example 1.

Suppose that the two states of nature θ_1 and θ_2 occur with probability $\mu = 0.25$ and $(1-\mu) = 0.75$, and we also take the following numerical values $\alpha = 0.5$, $\beta = 2$, $c = 0.25$, $q = 0.8$. We calculate EN' expected profit in the three sets of return (A) $(\theta_1 = 1.2, \theta_2 = 1, d = 1.1)$ and (B) $(\theta_1 = 5.0, \theta_2 = 1.0, d = 1.1)$ (Note that the gross rate of debt return d is a exogenous variable, but it is constrained by equation (8).), the numerical results are shown in *Table 1*. Compare the sets of return (A) and (B), they are consistent results coincide with the propositions in this papers:

(1) We can not only find that there exists a separating PBE and a pooling PBE respectively, but also derive the properties that, if θ_1 increases from 1.2 to 5.0, the optimal conversion ratio v_1 decrease from 0.489447 to 0.4 and the optimal capital input I_1 will increase from 0.086241 to 1.0. the characteristic of Proposition 2 is verified.

(2) Given θ_2 = 1.0, EN's expected profit in pooling PBE is greater than that in separating PBE if θ_1 = 1.2, that is, $EN_1^P = 0.215615 > EN_1^S = 0.17806$; however,

EN's expected profit in separating PBE is greater than that in pooling PBE if θ_1 = 5.0, that is, $EN_1^S = 2.75 > EN_1^P = 2.12185$. EN will offer the separating PBE if θ_1 is sufficiently greater than θ_2, which is the same as the description of proposition 4.

(3) By the ex ante point of view. In the set of return (A), EN's expected profit in pooling BPBE is $EN^P = 0.25EN_1^P + 0.75EN_2^P = 0.202478$, EN's expected profit in separating BPBE is $EN^S = 0.25EN_1^S + 0.75EN_2^S = 0.17805475$, $EN^P > EN^S$. If θ_1 (= 1.2) and θ_2 (= 1.0) is sufficiently close, the ex ante BPBE is the pooling equilibrium. Proposition 5 will verify this finding; Similarly, in the set of return (B), EN's expected profit in pooling BPBE is $EN^P = 0.25EN_1^P + 0.75EN_2^P = 0.69817$, EN's expected profit in separating BPBE is $EN^S = 0.25EN_1^S + 0.75EN_2^S = 0.82103975$, $EN^S > EN^P$. If θ_1(= 5.0) is sufficiently greater than θ_2 (= 1.0), the ex ante BPBE is the separating equilibrium. A formal proof is provided in Proposition 6.

Table 1. A numerical result

(θ_1, θ_2, d)	Separating PBE	Pooling PBE
(A)	$v_1 = 0.4899447$, $v_2 = 0.492532$	$v_0 = 0.472958$, $v_0 = 0.472958$
(1.2,1.0,1.1)	$I_1 = 0.086241$, $I_2 = 0.086241$	$I_0 = 0.1047193$, $I_0 = 0.1047193$
	$EN_1^S = 0.17806$, $EN_2^S = 0.178053$	$EN_1^P = 0.215615$, $EN_2^P = 0.198099$
(B)	$v_1 = 0.4$, $v_2 = 0.492532$	$v_0 = 0.28526$, $v_0 = 0.28526$
(5.0,1.0,1.1)	$I_1 = 1.0$, $I_2 = 0.0606469$	$I_0 = 0.331847$, $I_0 = 0.331847$
	$EN_1^S = 2.75$, $EN_2^S = 0.178053$	$EN_1^P = 2.12185$, $EN_2^P = 0.22361$

We next turn to find that the EN expects to gain from concealing his private information if θ_1 and θ_2 are not too far apart.

Proposition 5. *The BPBE is the pooling equilibrium that solves equation* (9) *if* θ_1 *and* θ_2 *are sufficiently close* .

Proof: Figure 3(a) illustrate the situation which EN prefer a separating equilibrium. From proposition 3, equation (9) defines a pooling equilibrium that exists for θ_2 near enough θ_1 . We need compare it only with the EN's favorite separating equilibrium. For clarification, the separating PBE solves the program

$$\text{Max} \quad \mu EN^{1S}(v_1) + (1-\mu)EN^{2S}(v_2) \tag{13}$$

subject to

$$EN^{1S}(v_1) \geq EN^{1P}(v_2) \tag{14}$$

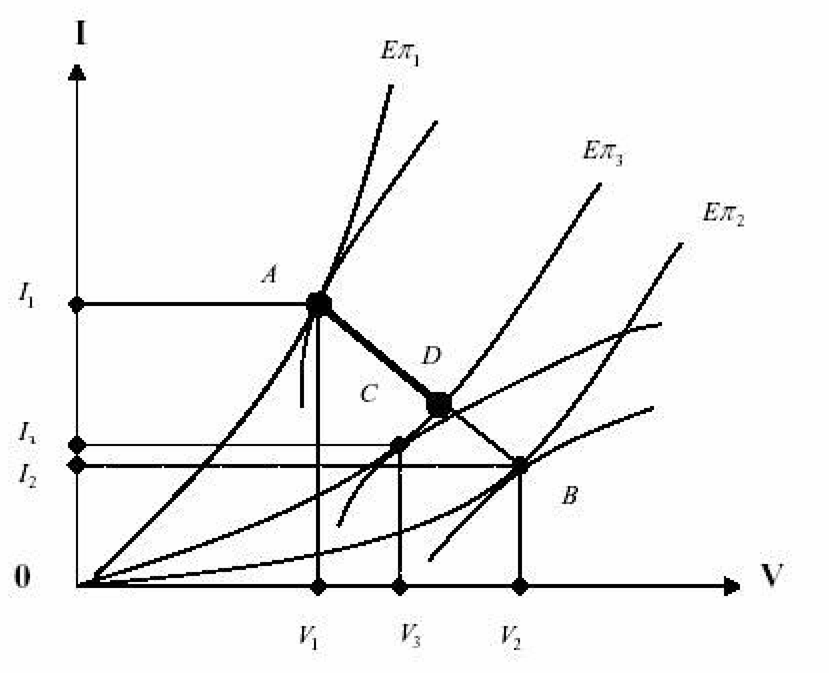
Clearly, the solution $[v_1(\theta_1), v_2(\theta_2)]$ to equation (13) and (14) satisfies $v(\theta_1) = v^*(\theta_1)$. We first show that, for θ_2 near θ_1, $v(\theta_2) \neq v^*(\theta_2)$. Applying the envelope theorem, we have

$$\frac{d}{d\theta_2}[(1-v^*(\theta_2))\theta_1 I^\alpha(v^*(\theta_2),\theta_2) - cI^\beta(v^*(\theta_2),\theta_2)]\,|_{\theta_2=\theta_1}$$

$$= -\theta_2 I^\alpha(v^*(\theta_2),\theta_2)[\frac{dv^*(\theta_2)}{d\theta_2} + \frac{dI(v^*(\theta_2),\theta_2)}{d\theta_2}]$$

which is negative because $\frac{dv^*(\theta_2)}{d\theta_2} + \frac{dI(v^*(\theta_2),\theta_2)}{d\theta_2} > 0$. Thus if $v(\theta_2) = v^*(\theta_2)$, equation (14) is contradicted. Thus $v(\theta_2) \neq v^*(\theta_2)$.

Figure 3(a) EN prefer a separating equilibrium



Now for θ_2 near θ_1, $v(\theta_2) = v^*(\theta_2)$ violates equation (14) but $v(\theta_2)$ is near $v^*(\theta_2)$. Hence, $v(\theta_2)$ must satisfy equation (14) with equality. But equation (14) is violated for all v_2 between $v^*(\theta_2)$ and $v^*(\theta_1)$, and, from proposition 2, $v(\theta_2) \geq v^*(\theta_1)$. Hence $v(\theta_2)$ is the smallest conversion ratio less than $v^*(\theta_2)$ such that equation (14) holds with equality.

The derivative of EN's expected profit in the separating equilibrium with respect to θ_2 is

$$\frac{d}{d\theta_2}\{\mu[(1-v_1)\theta_1 I^\alpha(v(\theta_1),\theta_1) - cI^\beta(v(\theta_1),\theta_1)]$$
$$+ (1-\mu)[(1-v_2)\theta_2 I^\alpha(v(\theta_2),\theta_2) - cI^\beta(v(\theta_2),\theta_2)]\}$$
$$= (1-\mu)\frac{d}{d\theta_2}[(1-v_2)\theta_2 I^\alpha(v(\theta_2),\theta_2) - cI^\beta(v(\theta_2),\theta_2)]\} \qquad (15)$$

The right-hand side of equation (15) can be rewritten as

$$(1-\mu)\{(1-v_2)\theta_2 I^\alpha + [-\theta_2 I^\alpha \frac{dv_2}{d\theta_2} + (1-v_2)\alpha I^{\alpha-1}\frac{dI}{d\theta_2} - c\beta I^{\beta-1}\frac{dI}{d\theta_2}]\} \qquad (16)$$

Because equation (14) is binding,

$$\frac{d}{d\theta_2}[(1-v_2)\theta_2 I^{\alpha}(v(\theta_2),\theta_2) - cI^{\beta}(v(\theta_2),\theta_2)]\} = 0 \tag{17}$$

Hence

$$-\theta_1 I^{\alpha}\frac{dv_2}{d\theta_2} + [(1-v_2)\theta_2 \alpha I^{\alpha-1} - c\beta I^{\beta-1}]\frac{dI}{d\theta_2} = 0 \tag{18}$$

But at $\theta_2 = \theta_1$, the left-hand side of equation (18) and the expression in braces in equation (16) are the same. Therefore, from equation (15), (16), and (18),

$$\begin{aligned}&\frac{d}{d\theta_2}\{\mu[(1-v_1)\theta_1 I^{\alpha}(v(\theta_1),\theta_1) - cI^{\beta}(v(\theta_1),\theta_1)]\\ &\quad + (1-\mu)[(1-v_2)\theta_2 I^{\alpha}(v(\theta_2),\theta_2) - cI^{\beta}(v(\theta_2),\theta_2)]\}\\ &\quad = (1-\mu)(1-v_2)I^{\alpha}\end{aligned} \tag{19}$$

Applying the envelop theorem to the solution $v^0(\theta_1,\theta_2)$ to equation (9), we find that the derivative of the EN's expected profit with respect to θ_2 is

$$\frac{dEN^P(v^0,I^0)}{d\theta_2} = (1-\mu)(1-v^0)I^{0\alpha} - q(1-v^0)I^{0\alpha} - \frac{dI^0}{d\theta_2} \tag{20}$$

Because $\frac{dI^0}{d\theta_2} > 0$, the right-hand side of equation (20) is less than that of equation (19). Hence, because the pooling and separating equilibrium generate the same expected value when $\theta_2 = \theta_1$, the former yields the EN a higher profit for θ_2 near θ_1. Q.E.D

In addition, Figure 3(b) illustrate the situation when EN prefers a pooling equilibrium. When EN's expected profit $\mu E\pi_1 + (1-\mu)E\pi_2$ is located on $B'D'$, he always prefer pooling equilibrium to separating equilibrium.

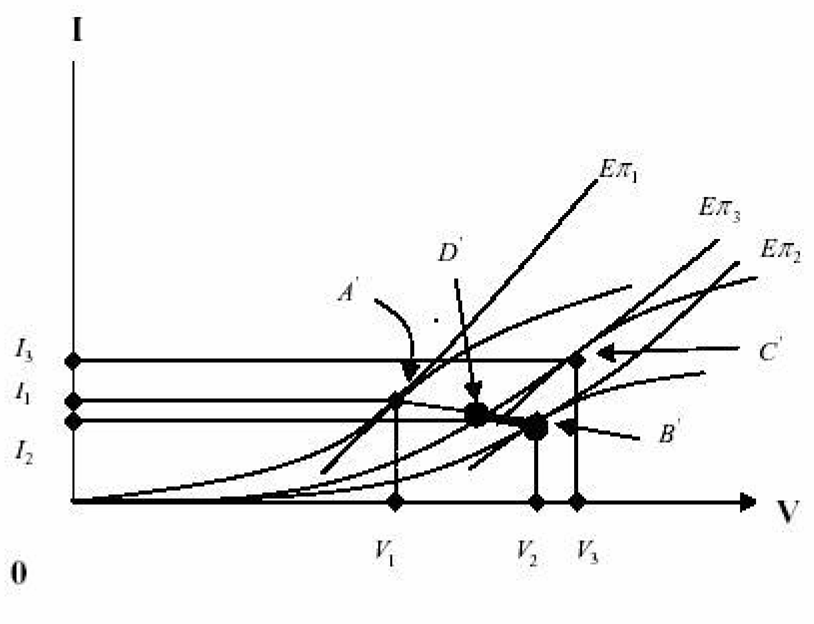
Proposition 6. *The BPBE is the separating equilibrium if* θ_1 *is sufficiently greater than* θ_2.

Proof: Given the BPBE is either completely separating or pooling. From proposition 4, a pooling equilibrium doesn't exist if θ_1 is sufficiently large relative to θ_2. Therefore, the BPBE must be the best separating equilibrium. Q.E.D

VI. Alternative Financing Arrangements

In this section, we will introduce technical share as an alternative financing mechanism. We consider a three-period model, where EN has a business plan with gross rate of return $\widetilde{\theta}+\widetilde{\varepsilon}$, $\widetilde{\varepsilon}$ is a random variable with standard normal distribution $N(0,\sigma^2)$ and $\widetilde{\theta}$ is a random variable and independent of $\widetilde{\varepsilon}$, which depends on the values θ_1 and θ_2 ($\theta_1>\theta_2$) with probabilities μ_1 and μ_2. The variable $\widetilde{\theta}$ is realized at the time 0, it is known to EN but not to all VCs.

Figure 3(b) EN prefer a pooling equilibrium



The risk-neutral EN has no capital and will ask the VCs for the required funds. EN with private information $\widetilde{\theta}$ asks a technical share $(1-v_t)$ to VCs who wish to invest I dollars in this project. After observing $(1-v_t)$, VC decides to invest I dollars [9]. The total share will be divided into two parts: VC gets v_t and EN gets $(1-v_t)$.

EN's strategy is a mapping $v_t:\theta_i\to\Re$, $\Re\in[0,1]$, and that assigns EN technical share $(1-v_t)$ on the basis of EN's private information $\widetilde{\theta}$. Because we assume $\widetilde{\theta}$ is a random variable, and $v_t(\theta)$ can be a random function. VC's strategy is a mapping $I:v_t\to\Re$, that represents the capital amount I that VC want to invest, and EN expend management cost $c(I)=cI^{\beta}$, $0<c<1$ and $\beta>1$.

We adapt the same assumption as in section 3 and section 4. First, consider the case of a separating equilibrium, At time 0, EN will propose a share ratio v_t to VC, $v_{t1}=v_t(\theta_1)$, $v_{t2}=v_t(\theta_2)$ if his private information is θ_1, θ_2 respectively. At time

9 Assume total share number is one, investment I divided equity v represents VC's valuation of stock .

1, VC can invest I_{t1} and I_{t2} dollars after receive the signals v_{t1}, v_{t2}.

By backward induction, at time 1, VC's expected profit ER_{t1}^{S} is

$$ER_{t1}^{S} = E[v_{ti}(\theta_i I_i^{\alpha} + \varepsilon) - I_i], \quad \text{for } i = 1,2 .$$

Hence, the first order condition of ERs is

$$v_{ti}\theta_i \alpha I_i^{\alpha-1} - 1 = 0, \quad \text{for } i = 1,2 .$$

and the second order condition is satisfied. We can find an optimal capital input

$$I^*(v_{ti}) = (\alpha v_{ti}\theta_i)^{\frac{1}{1-\alpha}}, \quad \text{for } i = 1,2 .$$

Back to t = 0, given VC's conversion decision constrains and investment constrains, when EN offer conversion ratio v_{ti}, consider EN's expected profit EN^{S} is

$$EN^{S} = E\{(1 - v_{ti})[\theta_i I^{*\alpha}(v_{ti}) + \varepsilon] - cI_i^{\beta}\} .$$

By the first order condition of EN^{S}, we can find the optimal path of the share ratio to VC is

$$\alpha - v_{ti}^{*} = \beta c(\alpha v_{ti}^{*})^{\frac{\beta-\alpha}{1-\alpha}} \theta_i^{\frac{\beta-1}{1-\alpha}}, \text{ for } i = 1,2 .$$

In a separating PBE, we can find a optimal conversion ratio $v_{ti}^{*}(\theta_i)$ and $I_i^{*}(\theta_i) == I_i(v_{ti}^{*}(\theta_i))$, $i = 1,2$. Compare to equation (3) and equation (5), the result is the same as the convertible securities financing.

Second, we consider the case of a pooling equilibrium. At time 0, EN will propose a share ratio v_{t3} to VC, $v_{t3} = v_t(\theta_1, \theta_2)$, if his private information is θ_1 and θ_2. At time 1, VC can invest I_{t3} dollars after receive the signals v_{t3}.

By backward induction, at time 1, VC's expected profit ER_t^{P} is

$$ER_t^{P} = E\{\mu[v_{t3}(\theta_1 I_3^{\alpha} + \varepsilon) - I_3] + (1-\mu)[v_{t3}(\theta_2 I_3^{\alpha} + \varepsilon) - I_3]\}$$

By the first order condition of ER_t^{P}, we derived VC's optimal path of capital input

$$I_3^{*}(\bar{\theta}) = (\alpha v_{t3}\bar{\theta})^{\frac{1}{1-\alpha}} \tag{21}$$

Where $\bar{\theta} = \mu\theta_1 + (1-\mu)\theta_2$. Then, consider EN's optimal decision, EN's expected profit EN_t^{P} is

$$\begin{aligned} EN_t^{P} &= \mu\{E[(1 - v_{t3})(\theta_1 I_3^{*\alpha}(\bar{\theta}) + \varepsilon)] - cI_3^{*\beta}\} \\ &\quad + (1-\mu)\{E[(1 - v_{t3})(\theta_2 I_3^{*\alpha}(\bar{\theta}) + \varepsilon)] - cI_3^{*\beta}\} \end{aligned} \tag{22}$$

From the first order condition of EN_t^{P}, the optimal choice of share to VC is

$$\alpha - v_{t3}^{*} = \beta c(\alpha v_{t3}^{*})^{\frac{\beta-\alpha}{1-\alpha}} \theta^{\frac{\beta-1}{1-\alpha}}$$

and the optimal capital input is $I_{t3}^{*}(\bar{\theta}) = I_3(v_{t3}^{*}(\bar{\theta}))$. We can find that if there exists a pooling equilibrium, the technical share is $(1 - v_{t3}^{*})$, VC will acquire equity shares v_{t3}^{*} and invest I_{t3}^{*} dollars.

In separating PBE under convertible securities financing, VC will always convert no matter the signal is S_g or S_b. By the proofs in section 2 and this section, EN is indifferenct between equity financing and convertible securities financing. In pooling PBE, we find that EN will get a higher expected profit with convertible securities financing.

Example 2.

In this numerical example, we have the same parameters as *table 1*. By Proposition 4 (or the result of *Table 1*), a pooling PBE exists in the set of return (A). We find that equity financing lead to a higher share ($v_{t3} = 0.4899447 > v_0 = 0.472958$) which VC holds, and a lower capital input ($I_{t3} = 0.0660283 < I_0 = 0.1047193$). Specially, EN's expected profit under equity financing are always less than that under convertible securities financing, that is, $EN_{t1}^P = 0.15634 < EN_1^P = 0.215615$, $EN_{t2}^P = 0.130102 < EN_2^P = 0.198099$. This result may explain why convertible securities have become the most commonly used financing instrument for EN. Hence, we can derive the following proposition:

Table 2. Pooling PBE in equity financing and convertible securities financing

(θ_1, θ_2, d)	Equity financing	Convertible securities financing
(A) (1.2,1.0,1.1)	$v_{t3} = 0.4899447$ $I_{t3} = 0.0660283$ $EN_{t1}^P = 0.15634$, $EN_{t2}^P = 0.130102$	$v_0 = 0.472958$ $I_0 = 0.1047193$ $EN_1^P = 0.215615$, $EN_2^P = 0.198099$
(B) (5.0,1.0,1.1)	$v_{t3} = 0.4$ $I_{t3} = 0.16$ $EN_{t1}^P = 1.1936$, $EN_{t2}^P = 0.2336$	$v_0 = 0.28526$ $I_0 = 0.331847$ $EN_1^P = 2.12185$, $EN_2^P = 0.22361$

Proposition 7. *EN prefers convertible securities financing to equity financing (technical share) in pooling equilibrium; however, EN is indifferent between equity financing and convertible securities financing in separating equilibrium .*

VII. Conclusion

In this paper we demonstrate the existence of separating equilibrium when convertible securities are used as the financing instrument. Convertible securities then serve the function of a signaling device to overcome ex ante informational asymmetry. We also show that pooling equilibrium exists when exogenous uncertainty is sufficiently small. With such pooling equilibrium, convertible securities also bring to the VC an extra return called "time value". Since there is a time lag between investment and conversion decisions, the VC benefits from waiting for more information before actual conversion. This factor also contributes to the popularity of convertible securities in VC financing.

We also analyze EN's preference for alternative equilibria in our framework when the VC cannot distinguish between good and bad projects, that is, EN's choice between a separating and a pooling equilibrium. It is shown that EN's preference depends on the variability of the return being sufficiently large or small. Furthermore, we compare the relative advantages and disadvantages of alternative financing schemes. We show that EN prefers convertible securities financing to technical share financing in pooling equilibrium. Hence the financing arrangement with convertible securities is shown to have many advantages as compared to other forms of arrangement for the start-up enterprises.

Some further issues about our model can also be explored. For example, we can study the possibilities of multiple equilibria. In fact, in the signaling game with continuous signals (conversion ratios), there exists typically an infinite number of separating, pooling and mixed equilibria. We can employ the equilibrium-domination based refinement criterion (see Cho and kreps (1987)) to focus on a unique separating equilibrium.

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