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ABSTRACT

Some social scientists argue that the widespread of cooperation within societies is the fact that modern economics theory fails to capture. Fukuyama, for example, argues that economics fails to take into account cultural factors affecting individual behaviors (Fukuyama 1995). Trust, as he argued, is culturally embedded in societies and social virtue promoting prosperity in some nations. Yet, studies on cooperative behaviors by game theorists lay theoretical groundwork arguing that cooperation can be sustained within self-interested individuals. This paper is, mainly, an attempt to review cooperative behavior under the random matching game, particularly a seminal work by Kandori. This paper will discuss basic properties of the model in detail.

Keywords: the random matching game, cooperative behavior, the repeated game, game theory.

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1. Introduction

Adam Smith’s assertion in the Wealth of Nation that market will function well even when individuals are self-interest is considered that any cooperative behavior of individuals would be at odd with selfishness individuals. Some social scientists argue that the widespread of cooperation within societies is the fact that modern economics theory fails to capture. Fukuyama, for example, argues that economics fails to take into account cultural factors affecting individual behaviors (Fukuyama 1995). Trust, as he argued, is culturally embedded in societies and social virtue promoting prosperity in some nations.

Yet, studies on cooperative behaviors by game theorists lay theoretical groundwork arguing that cooperation can be sustained within self-interested individuals. What more surprising in these studies is that the practice of cooperative behaviors takes place in some circumstances even when agents only concern with their own welfares. This finding contradicts with a view saying that cooperative behaviors are not compatible with self-interested individual framework. As long as individuals concern about their future payoff and are patient enough, the theory of repeated games predicts that cooperation can be achieved.

The theory of repeated games, moreover, points out that the threat of retaliation because of pursuing non-cooperative actions (cheating, for example) encourages individuals to take cooperative action. Hence cooperative outcome can be provided in equilibrium. However, many economic activities happen as agents change their partner overtime. In this circumstance, the threat of retaliation could be avoided while agents would do defective behaviors. Therefore the standard theory of repeated games analysis cannot be used to grasp the possibility of cooperation when agents meet different partners over periods.

Yet some studies on the random-matching model offers interesting findings suggesting that under certain environments cooperation still can be provided in equilibrium even an agent’s partners change. Kandori (1992) considerably extends the theory of repeated games into the matching games and argues that if cooperation outcome is supported by the equilibrium in the repeated games, the same outcome can be achieved in the matching random game.

This paper is, mainly, an attempt to review the development of this theory. A seminal work worth discussing for is a Kandori's work (Kandori 1992). Kandori basically proposes that under some particular circumstances, cooperation can be sustained in the equilibrium. This paper will discuss basic properties of the model in detail. One section, moreover, is devoted to discuss any refinement of Kandori propositions. Regarding this part, credit should be attributed to Ellison's work whose comes up with more robust properties, particularly regarding cooperation under the contagious equilibrium. Another section discusses applications of this theory done by Dal Bo (2007) and Greif (1993).

2. Cooperation under The extended repeated Game:

A seminal work on cooperation under the matching game may be at good attributed to Kandori (Kandori 1992). Kandori extends the theory of repeated games applied to the matching game (Kandori 1992). He, moreover, proposes the theory of self-enforcing agreements where social pressure or reputation affects the outcome of transaction among agents. His study, allows a situation where an agent may change their partners over time. Even under weak condition, i.e. when an agent's past history is not publicly observed by other players, any outcome can be supported in equilibrium. Two results can be inferred from his propositions. First they asserts that a community is able to support cooperation among members even when each agent only observes experiences with his partner but not what his partners have done to others. Second, they show that a community can process certain information about each agent labels which result in mutual benefit among them.

In his seminal paper (Kandori 1992), he defines a social norm as a desirable behavior within societies and an agent deviating from this behavior gets a sanction by others- community's sanction. Hence, a social norm, moreover, would work effectively in making all agents cooperate if it provides proper incentive to the players. Therefore punishment upon the deviators is not only necessity but a player who fails to punish is in turn to be punished.

Yet the further problem arises: as an agent changes his opponents over time, such punishments or retaliations are almost impossible to be done. A community needs certain information regarding an agent's past history so that any deviating behavior of an agent can be traced and any sanction can be imposed upon. In other words, social norms would work if a community can processes information about each agent's behaviors and *only* punish the deviators. Therefore Kandori evaluates three environments in which any outcome can be supported in equilibrium: first is a situation when an agent's past history is publicly observable, second is a situation when information about an agent's history is absent and third is a situation when an agent's history is decentralized.

a. General Assumptions

Before formalizing the model, we discuss the general assumptions. As a community requires information about an agent's past history, this knowledge is assumed to be attached in individuals by dark spot i.e. a deviating agent is darkly spotted by a community. A social norm is assumed to entail that an agent should cooperate only and if only he encounters an unspotted partner. Now consider there is two unspotted agents matched each other and let say one of them is likely to face spotted agents in the future. If punishment were costly enough to carry on, then each player would not have incentive to cooperate.

Other assumptions can be summarized as: first a label consisting agent's past information is attached to each agent. Second agent's and his opponents' labels are observed first before any transaction take place. The third is current actions and label of an agent and his opponents affect their future labels.

b. Formal model for the repeated matching game

As the model essentially an extension version of repeated games to matching model, the basic formalization of the extended repeated games is discussed in this section. Suppose there are N player divided equally in size N_k (k=1,2). Each agent is matched and the game is repeated infinitely. An agent's payoff, moreover, is discounted by $\delta \in (0,1)$. Under this game, the match follows uniform random matching model with probability (Kandori 1992).

Prob{μ(i,t)=j}=1/n for all $i \in N_1$ and $j \in N_2$ and for all t

μ(i,t)=j is the probability of player i is matched with player j.

Furthermore we define the payoff function as $g : A \to R^2$ (A=A_1 X A_2 where A_k is set of actions for type-k players, k=1,2). And we set minimax point $M_1 \in$ A for type-1 player as

$$M_1^1 \in \arg\max_{a_1 \in A_1} g_1(a_1, M_2^1)$$

$$M_2^1 \in \arg\min_{a_2 \in A_2} (\max_{a_1 \in A_1} g_1(a_1, a_2))$$

Moreover, mutual minimaxing poit ($M_1^2\ M_2^1$) is rewritten as m = (m_1,m_2) and normalized to zero (g_1 (M_1)=g_2 (M_2)=0). These properties basically argue that when an agent and his opponent are the same i.e. both of them are the same type; both will play action mutually beneficial. Yet, as one encounters another type of opponent, both will play minimax action. Any feasible payoff is assumed convex and the element of the set of payoff function (V).

c. Folk Theorem under perfect information

A remarkable generalization of Kandori's model is that any such outcome supported in the two-player repeated games can also be provided in the matching game, though the rule of matching games is arbitrary. In this section we will discuss the proofs of this generalization taken from Kandori in detail. Generalization can be made: if there is any feasible payoff, v, supported by sub game perfect equilibrium in the repeated games, such feasible payoff, $v \in V$, also can be figured out in the matching game.

Based on Kandori's work, there are several propositions (Kandori 1992), namely, :

Proposition 1: *if any feasible payoff v sustained in the equilibrium in the repeated game for some δ would also be supported by the matching game for the same the same δ with arbitrary population size and matching rule,* $v \in V$.

Intuition: this proposition essentially suggests that any discounted factor, δ, provided in the two-player repeated game could be applied in the matching game for any population and matching rule. The intuition behind is that under public observability environment each agent's past actions are observed by *all* players. Although an agent changes his opponents over periods, the potential opponents already observed his past action toward other players. Any cheat behavior is known by the public and as such, an agent has an incentive to cooperate as if he faced the same partner in every period.

Proof Suppose the player in the matching game start playing the equilibrium path of the two-player repeated game. If the type-l of player deviates (l=1,2), then all type-l's are punished by all of other type in the similar way to two-player repeated game since each player faces the same sequence of action profiles and retains information about the other's past action profiles.

Proposition 2. *Any feasible payoff point $v \in V$ supported by an equilibrium in the random matching game exists only cheaters are punished, if $\delta \in (\delta^*,1)$, where δ^* can be chosen independently of the population size.*

Intuition: the second proposition argues that as each agent's action profiles are publicly informed by all players, a community (the players) can recognize the deviators and take sanctions immediately upon them. Additional restriction in this proposition, moreover, is that simultaneous punishment is implausible so that only the latest deviator is being punished. Therefore whoever punishes the deviators is not an issue as community sanctions function in the same way as personal retaliation in the two-player repeated games.

Proof. Suppose in the equilibrium all players play the action profile a* (a* $\in$ A). Any deviation by an agent turns him and his partner to play minimax strategy, m, for T period and then back play a* afterward. If the deviator defects during the punishment period (T period) then the punishment is carried on again for T-period. Yet if another player deviates while the last deviator is punished, then the punishment is replaced to the latter deviator and the former deviator is forgiven.

Formal model

The formal model is discussed below. Let *V* is the payoff function. Suppose x = g(m)-the payoff of guilty player during punishment and *v* is the payoff after the punishment. Then we may define the average payoff of guilty player as

V=(1-δ^T)x+ $\delta^T v$

$\delta^T \in$ (0,1). If a guilty player deviates when he is punished, his payoff at most is 0+ δVk (recall that g(m) is normalized to zero when both player play minimax strategy) which is less than original payoff V.

Moreover, the average payoff of an innocent player will be

Π=(1-δ^T)x_k+ $\delta^T v_k$+ (δ-δ^T)[(1-1/n)v_k+(1/n)x_k]

If he deviates he earns at most

$\Pi^{'} = (1-\delta)v_k^* + \delta V_{k,}$ where $v_k^* = \max_{a\in A} g_k(a)$

It is very clear that as $\Pi - \Pi^{'} \rightarrow (1-\delta^T)(1-1/n)(v_k - x_k) > 0$, and action profiles are publicly observed, any deviation is not profitable.

Within these two propositions, the generalization of Kandori's model finds that any cooperative outcome supported in the repeated games can be figured out in the matching game with arbitrary discounted factor, δ, and the matching rule as long as the action profiles of each agent are publicly known by all players.

d. Cooperation in the absence of information processing

The foregoing discussion evaluates an environment where each agent's past actions are publicly observed and it is shown that folk theorem properties can be applied in the random matching game. In this section, we will discuss outcome where such information is very limited. In the absence of information processing, the game can be considered as prisoner's dilemma with the strategic form as below:

Table 1. Prisoner's dilemma

	C	D
C	1 , 1	-*l* , 1+g
D	1+g , -*l*	0 , 0

Let say that there are *n* pairs player matched randomly and population size M=2n. Where *g* is gain from defection and *l* is loss of being cheated. It is assumed that each player only observes the history of action profiles of the stage game in which he has played. As the game assumes that any direct communication among players does not exist, each player has no information about what has gone on the community. Therefore it is very obvious that folk theorem equilibria assuming perfect information cannot be implemented in this game.

However, an agent has belief that what has happened in his stage game also happens in a community. Based on his belief, an agent will choose action maximizing his payoff. In other words, if an agent was cheated then he believes that a community also has been cheated. Hence in the next period, he starts deviating by playing D. Therefore outcome (D,D) by all players exists through long periods (not immediately because there is missing information processing such that punishment cannot be taken instantly) and becomes equilibrium or what so-called contagious equilibrium. The contagious equilibrium can at best be imagined as trust attached to the community as a whole and not to each individual. It is called contagious because any single defection done by a member will end the whole community trust. Moreover, another player finding dishonest behavior start cheating all of his partners and finally, defection spreads within a community like epidemic.

To formalize the game, let assume that there are two types of player. A player becomes type c if his opponents have not deviated in his past history and otherwise he turns to be type-d. In other words, once he is cheated, he becomes type-d. Theorems by Kandori (Kandori 1992)

Theorem 1: *The contagious strategy s* is an action supported by a sequential equilibrium strategy for any given gain of defection (g) and population size M if δ and loss (l) are sufficiently large*.

Intuition: as δ is large enough it is better for a player to pursue equilibrium path-the player is patient enough to cooperate. This argument can be simply explained like this: when a player defects, he starts the contagious process in which decreases his future payoff. As δ is large enough (close to 1), the loss of future payoff will be higher than the one-period gain of cheating. This encourages an agent to play C.

However trade-off occurs as contagious process has started. An agent may still play C in hope to slow down contagious process and enjoy the higher future payoff, although he realizes that playing C may be costly but as δ is close to unity, it may outweigh the cost. Yet, if the cost, in terms of loss (*l*) is large enough, once type-c player meet type-d player, it is the player's best interest to play D and follow contagious effect.

Formal model

The formal model is discussed here. Suppose X_t is the number of type *d* players at time *t*. As the contagious process takes place through periods, Kandori introduces the probability of this transition by matrix *M* X *M*. Let define A as probability matrix a_{ij} ($A = (a_{ij})$) where $a_{ij} = \text{prob}(X_{t+1}=j|X_t=i)$-the probability that *j* becomes type d player (at time t+1) given there is type-d player before (at time t). Moreover define $B(b_{ij})$ where $b_{ij} = \text{prob}(X_{t+1}=j|X_t=i)$ and one of the d-types deviates to play c at time t. Under the contagious process, type-d player may also deviate by playing C so that the contagious effect is delayed. The probability of delayed contagious effect can be defined as matrix H where H = B- A.

Hence, the probability of d-type player faces a c-type can be written by a column vector

$$\rho = \frac{1}{M-1}(M-1, M-2,...,1,0)^T$$

Since M is total number of players, then M - 1 means that at least one player is type-c player. Thus if conditions described below hold, we can find that contagious strategy constitutes a sequential equilibrium. These two conditions are[1]:

$$\frac{1}{1+g} \geq (1-\delta)e_i(I-\delta A)^{-1}\rho \qquad (1)$$

$$\frac{\left(\frac{M-k}{M-1}\right)g + \left(\frac{k-1}{M-1}\right)l}{1+g} \geq \delta e_k H(I-\delta A)^{-1}\rho \qquad (2)$$

Equation 1 basically is drawn from one-shot deviation property. It is argue that one-shot deviation property is unprofitable if

$$\frac{1}{(1-\delta)} \geq \sum_{t=0}^{\infty} \delta^T e_i A^t \rho(1+g)$$

Left hand side is the expected payoff when the player does not deviate while the right hand side is the payoff when the player defects forever. *e*i*A*t is the probability of meeting c-type player at time t given that he defected at time 0.

[1] For detail discussion please see Kandori (1992)

As l and δ is large enough and $\lim_{\delta \to 1}(I-\delta A)^{-1}\rho < \infty$ we can find the payoff stream in the form of matrix as:

$$\sum_{t=0}^{\infty} \delta^{T} A^{t} \rho = (I-\delta A)^{-1}\rho$$

Putting this payoff stream into the one-shot deviation property equation and rearranging the equation, we can find the condition 1.

Meanwhile equation 2 comes up from

$$\sum_{t=0}^{\infty} \delta^{t} e_{k} A^{t} \rho (1+g) \geq \left(\frac{M-k}{M-1}\right) - \left(\frac{k-1}{M-1}\right) l + \delta \sum_{t=0}^{\infty} \delta^{t} e_{k} BA^{t} \rho (1+g)$$

The left hand side is the payoff stream if a player plays d forever and the right hand side is the payoff if he plays C today and D forever. (M-k)/(M-1) is the probability if he meets c-type player and k-1/M-1 is the probability of encountering d-type player. Rearranging the equation above, we may figure out the inequality on equation 2.

What equation 1 and 2 tell us is that there are any value of g, l and δ supporting contagious equilibrium as these values fit these two inequalities above. In the absence of information processing, a community does not have much power to enforce players for not cheating compared to perfect information situation. The reason is because the punishment upon deviator is delayed. With this environment, moreover, and as M (population) is large, cooperation can hardly be achieved. It is because information processing about players' past history takes time. The absence of information processing, moreover, also allows innocent players of being punished. Contagious equilibrium, in the end, may lead a society into collapse, i.e. distrust spreads among players.

e. Cooperation in Local Information processing

If cooperation outcome is hardly supported in the absence of information processing setting, would the cooperation be provided in the equilibrium if we allow little information spread among the players? Here Kandori shows that under decentralized information i.e. an agent could observe his opponents' action profiles through the attached label and take action afterward. It is assumed that the information processing itself is exogenous. As information is decentralized, players' best actions do not require the information in the community level. Label carried by each agent is enough as a record of agent's past actions.

Kandori, moreover, assumes certain definitions drawn from Okuno-Fujiwara and Postlewaite (1989). Let every period in which an agent encounters his opponents consist of single state (z_i where i is player-type (i = 1,2). Hence the next states of an agent are determined by the current state. Therefore information of action in each state is attached in the label and each player observes the labels to take an action in the current state. This is what decentralized information means. This information is enough for each agent to carry on the best action, although additional information is available. In the equilibrium, moreover, any payoff ,$v \in V$ should be globally stable. Otherwise the stability of social norms is in question and any such outcome is hardly found in the equilibrium.

Kandori discusses situation of one-sided incentive problem i.e. when only an agent's label is under concern of his opponents. For example is borrower-lender case. Lender e.g. banks have an incentive to know borrowers' loan histories. Kandori generalizes that cooperation can be supported in the equilibrium for some δ*. Yet, what more interesting discussion is when both players have incentive problem (two-sided incentive problem). Further Kandori assumes payoff of action profiles that are

$$g_1(m_1, r_2) > g_1(m) \geqq g_1(r_1, m_2) \qquad (1)$$

$$g_2(r_1, m_2) > g_2(m) \geqq g_2(m_1, r_2) \;\; r \in A$$

This assumption implies that if a guilty player meets another guilty player, they will minimax each other. If a guilty player meets an innocent player, the innocent player will minimaxs the guilty, but not in another way around. In this case, a guilty player repents (r) by taking action r which is less harmful for the opponent (strict inequality) but more costly for himself (weak inequality).

Theorem 2 *With local information processing and under the assumptions above, every point $v \in V$ is supported by the equilibrium if $\delta \in (\delta^*,1)$ for some δ^*, which is independent of the matching rule and the population size. This equilibrium is stable and globally optimal. Furthermore, only three actions are prescribed in equilibrium (a*,m, and r).*

Proof. Suppose there are two-type of players and finite sets which are defined as $Z_1 = Z_2 = Z = \{0,1,\ldots T\}$. $z = 0$ indicates that the player is innocent, otherwise he is guilty. T states are the number of punishment period for a guilty player. In the equilibrium, there are only three plausible actions, the action a* ($a^* \in A$) supporting the payoff v, the minimax action (m), and the action repent (r). If two innocent players are matched each other, they will choose action a*. When two guilty players encounter each other, they play minimax strategy. As an innocent player meets a guilty player, the former will choose minimax strategy while the guilty one will take repent action as defined in the assumption above. Formally we can write stategy profile $z \in Z \times Z$,

$$\sigma(z) = \begin{cases} a^* & if \;\; z = (0,0) \\ (m_1, r_2) & if \;\; z_1 = 0, z_2 \neq 0 \\ (r_1, m_2) & if \;\; z_1 \neq 0, z_2 = 0 \\ m & if \;\; z_1, z_2 \neq 0 \end{cases} \qquad (2)$$

We can say that it is unprofitable for type-1 players to deviate from prescribed strategies when this following condition is satisfied

$$(1-\delta^T)g_1(r_1,m_2)+\delta^T v_1 \geqq 0 \qquad (3)$$

$$(1-\delta^T)\min\{g_1(m_1,r_2),v_1\}+\delta^T v_1 \geqq (1-\delta^T)v_1^* + \delta[(1-\delta^T)g_1(m)+\delta^T v_1] \qquad (4)$$

Inequality 3 basically satisfies condition that type-1 player takes the prescribed actions as he is guilty. Suppose a player guilty, he can earn payoff at least

$$x(1)+\delta x(2)+\ldots+\delta_{T-1}x(T)+(\delta T+\delta T+1+\ldots)v_1 \qquad (5)$$

The magnitude of x(t) depends on an agent's partners i.e. g_1(m) if he encounters a guilty player or g_1 (r_1,m_2) if he encounters an innocent player. After *T* period, every player can earn payoff v_1 as long as he does deviate during the punishment period and all players adhere to the equilibrium path. This intuition comes up since at time *T* period ahead, punishment is vanished and each player is matched with an innocent player. Yet if he deviates, his payoff at most

$$0+\delta x(2)+\ldots+\delta Tx(T)+\delta\, g_1(r_1,m_2)+(\delta T+1+\delta T+2+\ldots)v_1 \qquad (6)$$

From these inequalities in 5 and 6, it is very obvious that the payoff on 5 is higher than the payoff 6. Therefore it is not profitable for a guilty player to deviate.

Inequality 4 argues that it is better for an innocent player not deviating. The left hand side is the least payoff of innocent player if he does not deviate; meanwhile the right hand side is the payoff he can earn at most if he deviates. Kandori, moreover, shows that this equilibrium is stable as at *T* period and after, all agents will turn back playing a*.

3. General conclusion on Kandori's model

Kandori gives insight that any outcome provided in the repeated games can be generalized, under some circumstances, in the matching game. By allowing several degree of information flow among players, Kandori shows that cooperation can be supported even under weak condition i.e. when information is decentralized. However, in the absence of information processing, cooperation outcome is hardly provided in equilibrium. Yet it does not mean that such equilibrium does not exist. The contagious equilibrium can replace the equilibrium in the standard folk theorem. Even though the final outcome is defection (D,D), it does not happen immediately. The contagious equilibrium requires that defection spread among players and takes time due to the limited information.

This generalization, moreover, is a remarkable result showing that even self-interested individuals who are concerned with their future payoff can achieve cooperation. Regarding the contagious equilibrium, however, some studies relax the restricted assumptions made by Kandori and find that cooperation can still be achieved although information is not perfectly transferred among players. Ellison (1994) comes up with another model and shows that sufficiently small amount of noise does not discourage agents to cooperate.

4. Cooperation in the contagious equilibrium: Ellison model

The previous discussion argued that as the information processing is absence, the only possible equilibrium in this environment is the contagious equilibrium. However, this equilibrium puts cooperation outcome into very weak condition and social norms, basically, become unstable. Another issue about this equilibrium is that the consequences of few mistakes done, perhaps, accidentally by an agent situate social setting at stake. This shows that any outcome supported by the contagious equilibrium becomes inefficient as any accidental single-deviation break cooperation down and destroy the social norms (Ellison 1994).

Ellison, moreover, utilizes the contagious equilibrium properties and finds that even single-deviation takes place, under particular situations, cooperation is still supported by the contagious equilibrium. The main difference result between Kandori's and Ellison's model can be attributed into two reasons. First, instead of using information processing, Ellison employs public randomizations. In other word, in Ellison's model, the action profiles of an agent are based on public random variable, *q*. Second, the severity level of the punishment (the duration of punishment) adjusted through public randomization variable is not an issue in Kandori's model since any single deviation leads to breakdown in cooperation. Contrary to Kandori, Ellison argues that any single deviation in the contagious equilibrium does not necessarily lead to the cooperation breakdown as long as the future punishment determined by the level of public randomization is severe enough. As the average duration of punishment can be defined as 1/(1-q(δ)), the higher public randomization (i.e. higher q), the longer the average duration of punishment.

The game in Ellison model follows the strategic form game as in Kandori's (the game is the same as in the table 1 above, page 5). Yet, Ellison assumes that there are two stages (Ellison 1994).

Stage I Play C in period t

If the outcome is (C,C), the matching game rules that both matched player (i and j) play C in stage 1 in period t+1. Other outcome depends on the level of public randomization, if $q_{t+1} \geq q(\delta)$ both play following stage I, if $q_{t+1} \leq q(\delta)$ both play following stage II.

Stage II Play D in period t

If $q_{t+1} \geq q(\delta)$ both play following stage I in the period t+1, if $q_{t+1} \leq q(\delta)$ both play following stage II in the period t+1.

Ellison argues that in the random matching model above with public randomization and the form of game is prisoner's dilemma with the number of players is at least 4, any strategy profile, s*(δ), is supported by the sequential equilibrium for some $\underline{\delta} < 1$ and $\underline{\delta}$ [0,1).

To proof this assertion, suppose *k* is the number of player playing in the stage II and the continuation payoff function is defined as *f*(k, δ, q) from period t. If none plays in the stage II, the continuation payoff of i's player is 1. In other words, all players will cooperate.

Therefore deviation in the stage I is not profitable as the inequality below hold

$$(1-\delta)g \leq \delta q(\delta)(1-f(2,\delta,q(\delta))) \qquad (1)$$

Intuition as δ is large enough, potential gain from single deviation in stage I is less attractive in encouraging player i to deviate although he potentially encounter the deviators in the future. In other words, the average expected continuation payoff outweighs the potential gain of one-period cheating.

Moreover, deviating in the stage II is also unprofitable if

$$(1-\delta)g \geq \delta q(\delta)E_j(f(j,\delta,q(\delta))-f(j+1,\delta,q(\delta))) \quad (2)$$

Intuition essentially inequality in 2 says that the small level of randomization (i.e. the low duration of punishment) is sufficiently enough to discourage player *i* to deviate by playing C in the stage II, given his belief about the number of player playing D in the stage II.

Ellison proves that the continuation payoff is convex in k[2]. This property brings significant result regarding the contagious equilibrium. Based on convexity property, Ellison shows that

$$g = [\underline{\delta}\,(1-f(2,\delta,1))]/(1-\underline{\delta}) \text{ where } q(\delta)=1 \quad (3)$$

$$g = [\delta q(\delta)(f(1,\delta,q(\delta))-f(2,\delta,q(\delta)))]/(1-\delta) \quad (4)$$

Intuition the equation 3 and 4 are derived from inequalities 1 and 2. Equation 3 says that when public randomization is close to unity, there is any discounted payoff $\underline{\delta}$ which makes player *i* indifferent between playing C and D in the stage I. In equation 4, the player *i* is indifferent between playing D in period t and playing C in period t then deviating and by playing D in t+1 in the stage I.

These equations imply that it is plausible that player *i* will be indifferent between deviating in *t* period and cooperating in *all* future periods. In other words, single deviation under the contagious equilibrium, with specific conditions, *does not lead to* cooperation breakdown i.e. an

[2] for detail discussion about convexity properties of the continuation function please see Ellison (Ellison 1994)

agent still cooperate after the deviation has started. If 3 and 4 hold, cooperation can be supported in the contagious equilibrium. Ellison, moreover, extends the model by imposing noise into the model and the basic result does not change, namely, cooperation can be supported by the equilibrium. Even in the absence of public randomization, under very restricted circumstances, cooperation can be sustained by the equilibrium.

Ellison model regarding the contagious equilibrium shows more robust results. Assumptions made in his model capture social reality that even when a community lacks information processing mechanism we still find that cooperation is common. Moreover, a concern about defection which may spread over within the community is sufficient enough to encourage cooperation among players.

5. Applications

Dal Bó relaxes some basic assumptions of Kandori's model (Dal Bó 2007). He abandons assumptions in Kandori model that any outcome sustained in the relationship between two agents can also be provided in two groups. His study mostly relies on the properties of Kandori's model. However he applies the situation into the social sphere, for example inequality in the caste system. One striking finding is that inequality in the caste system might be supported by social norms even though all members within the caste are basically equal.

Another interesting study about social norms in the context of this game is Greif's research on the early Maghribi traders (Greif 1993). The Maghribi traders in 11^{th}-century trading used overseas groups to gather information regarding their trading partners. This agency was formed under a coalition, which basically is similar to contractual relations. These relations, moreover, allowed traders to encourage self-commitment and the coalition helped in coordinating actions of the members. Therefore retaliation upon the cheating traders does not require personal retaliation, the coalition would punish the cheaters and work in the same way as personal retaliation.

6. Conclusion

Game theory has long recognized that cooperation can be sustained in equilibrium as the future threat of retaliation can be taken. However, economic activities show that an agent may change his partners over time and this becomes a problem to encourage self-enforcement and commitment. Related to this context, the extension of the repeated games into the matching game gives insight how such cooperative behavior can be sustained although an agent is matched randomly every period. Kandori's model should get credit in this research.

Ellison, moreover, offers more robust result particularly regarding the contagious equilibrium. In contrast with Kandori, Ellison finds that the contagious equilibrium is powerful enough to encourage agents to cooperate. So single deviation does not alter individual to cooperate immediately and cooperation is still sustained. Some questions remain unanswered. In Kandori model, if information processing among agents becomes vital, then how much is the cost and who would share the cost? The formalization of these issues gives avenue for further research.

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