Random Search in the Presence of Markets: A Note

By

Meenakshi Rajeev[1]

Institute for Social and Economic Change

Nagarbhavi, Bangalore-72, India

And

Dipankar Dasgupta

Visiting Professor

Department of Economics and Finance

City University of Hong Kong

Hong Kong

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Abstract

This paper examines in a dynamic game-theoretic framework, the role of social institution of money and markets in facilitating exchange. It reveals how, depending on the level of transaction costs associated with a market setup (synonymously, trading posts) appropriate monetary trade emerges, which like a hub and spoke network (Starr and Stinchcombe, 1999) makes some markets non-functioning. However, despite the obvious advantages of a market setup in reducing search costs, pure random search for a complementary trading partner prevails in many economies, especially, in many de-

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veloping economies. This paper models this feature of *developing economies* by introducing differences in transaction costs across agents and shows why sustainable equilibria might exist exhibiting random search for certain commodities even in the presence of established markets.

Key Words: Trading post, marketless trade, steady state Nash equilibrium.

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1   Introduction

Money is an essential intermediary in the process of exchange. Even in a competitive set-up where aggregate demand equals aggregate supply for each commodity, lack of a double coincidence of wants at the individual level can create serious problems in attaining ones desired bundle of goods (Ostroy and Starr (1994)). The earliest recognition of the problem came in Menger (1892) and later Hicks (1967) posed the problem in the context of a Walrasian economy. In recent times the celebrated papers of Ostroy and Starr (1990, 1994) and Kiyotaki and Wright (1989) examined the question of which commodity can be the best choice as a medium of exchange in terms of transactions costs like time or storage cost[2].

In addition to money, which facilitates the process of exchange, the social institution of markets also smoothens the transaction process by acting as a meeting ground for the buyers and sellers. Recognizing the role of markets

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[2]There are several recent models that theoretically formulates monetary exchange, for example see Wright, Schindler, Rupert (2001), Wright, Corbae and Temzilides(2003), Starr (2003), Dasgupta and Rajeev (1997).

in the process of exchange , a number of authors theoretically examined the coexistence of money and markets as facilitators of exchange (see Starr(2002), Howitt (2002), Rajeev (1997, 1999)). However, notwithstanding the obvious advantages of a market set-up where buyers and sellers of a particular good can meet separately (as against a pure random search for a matching trading partner)[3] marketless trade is commonly observed in **developing economies**. For example, at any traffic signal stop, one is surrounded by vendors peddling their fares. Hawkers of various commodities, ranging from perishable goods like fish or vegetables to non-food items, call out in residential areas in search for a potential trading partner. In other words, though a market set-up can reduce the search cost to a great extent, random search persists even for the commodities having established markets.Literature is generally silent about why one may observe such a phenomenon.

In the present paper we make an attempt to explain the simultaneous existence of monetary trade through pure random search and market specialization as a solution of a dynamic game problem. The problem is posed in a simple three-good economy characterized by complete absence of mutual coincidence of wants amongst the agents[4] with flexible trading rules. More precisely, trade can take place either in a trading post set-up (equivalently in markets to exchange different pairs of goods) or, through pure random search (marketless setup). Though it is appropriate to incorporate flexible price regimes (see Kiyotaki 'and Wright (1993)), we abstain from doing so in this exercise as the purpose here is to show that even with pre-determined market clearing prices, monetary trade can be indispensable and marketless

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[3]To be called synonymously as marketless trade.

[4]See Kiyotaki and Wright (1989).

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trade can coexist in equilibrium with specialized markets, even though the latter seems to reduce transaction costs.

Equilibrium here is a steady state Nash equilibrium involving the optimal trading strategies of agents engaged in a process of exchange. The paper recognizes the fact that in addition to time or storage costs, there may be additional resources necessary for one to be engaged in trade through a market set-up. These may include taxes payable, rentals or power charges and so on. The kind of economy we consider here has different types of agents who are not only characterized by their excess demands and supplies but also by their resource constraints. For example, one group of agents may be constrained by financial resources in participating in a market setup, while the other group may have inherited (or otherwise endowed with) wealth, which may be used as collateral for acquiring capital necessary to participate in a trading post or equivalently, market setup. This is captured in the model by incorporating a cost for participating in a market setup, measured in terms of instantaneous disutility, which is higher for the resource-constrained agents. This leads to the possibility that the people who are more constrained with respect to financial resources (than with respect to time) may prefer to go in for random search. However, this need not imply that a meaningful steady state Nash equilibrium will necessarily exist. We here derive the parametric restrictions under which such an equilibrium exists and look at the feasibility issue. The important question that arises is why such an equilibrium is not common in a developed economy. This is possibly due to the fact that even a resource-constrained agent (in a developed nation) has sufficient funds or receives adequate state support to be able to participate in a market set-up. In other words, not just relative deprivation, absolute level of poverty seems

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to matter. It is interesting to note however that the equilibrium we derive is Pareto non comparable to the one characterized by complete marketised trade.

Given this background the next section describes the basic framework under consideration. Section 3 looks at the possible equilibrium strategies under two different trading arrangements. A concluding section follows thereafter. An appendix provides the technical details. Furthermore, in appendix we discuss a model under a complete marketised setup.

2 Frame-work

To begin with we consider a competitive economy in a state of equilibrium, like in the case of Kiyotaki and Wright (1989) but modified appropriately to suit our objectives. The economy consists of three types of agents (type 1, 2 and 3), each specialised in consumption and production.

Every type consists of a equal number of agents producing one unit of a specific good. There are three indivisible commodities, viz., good1, 2 and 3. Type k agents derive utility from the consumption of good k only and produces k^*. In our model we assume $1^* = 2, 2^* = 3, 3^* = 1$. As soon as a type k agent acquires good k he consumes it and produces one unit of k^*. Each good can be stored at a cost, but an agent's capacity to store is restricted to one unit only. Let b_{kc} denote the cost (in terms of instantaneous disutility) to the type k agents of storing good c. It is assumed that $0 < b_{k1} < b_{k2} < b_{k3}$, for all k. For a type k agent, let u_k denote the instantaneous utility from consumption of good k net of disutility of producing k^* and $\beta_\epsilon(0,1)$ the

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common discount factor. An economy with these features is denoted by E.

For the economy E we consider two types of trading arrangements viz. the marketless arrangement and the trading post set-up. In a marketless arrangement (Kiyotaki and Wright (1989), Aiyagari and Wallace (1991)), the agents meet each other randomly in pairs (irrespective of the goods they want to trade) and exchange of goods takes place when it is mutually agreeable.

On the other hand, in a trading post set-up there exists three different markets to deal with good 1 against 2, good 1 against good 3 and good 2 against good 3. By the (c, c') trading post we refer to the market where good c is exchanged against good c'. Agents wishing to trade good c against c' visit the (c, c') trading post where buyers and sellers (of c against c') can identify each other and meet and trade. It appears therefore, that the trading post set-up would be able to avoid meetings between the agents who are unlikely to benefit from trade. However, though there is a saving of time cost in a trading post economy, one needs to incur additional costs (above the storage costs) for the setting up and maintenance of a market system. More precisely, let $\gamma_{cc'}$ be the per period cost to be incurred by an agent trading in the (c, c') trading post to run the market (it includes eg. tax payable, electricity charges etc.)[5].

We would consider a specific relation amongst the costs to be incurred in a trading post set-up.

Relation I $\qquad b_{k2} + \gamma_{12} < b_{k1} + \gamma_{31} < b_{k3} + \gamma_{23}, \gamma_{12} \leq \gamma_{31} \leq \gamma_{23}$

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[5]Here we have made these costs market specific. Similar exercise can be carried out if one makes these costs agent specific or alternatively dependent on the good one wants to sell in that market.

All other possible relationships can be considered and dealt with similarly. It is assumed that the utility u_k is large enough compared to the costs (measured in terms of instantaneous disutility) so as not to induce any agent to drop out of the market economy. (This may be ensured through the following sufficient condition

$$u_k - \frac{b_{kk^*} + \gamma_{ck^*}}{1 - \beta} \geq -\frac{b_{kc} + \gamma_{cc'}}{1 - \beta}, \ \forall \ c \ \text{and} \ c'$$

In a set-up with trading posts a type k agent has two (pure) strategies: either to go for direct barter i.e., to exchange k again k^* directly or to go for indirect trade by exchanging k^* against some good c and then c against k. In the next section we would examine the possible equilibrium strategies for such a scenario. The concept of equilibrium used is the steady state Nash equilibrium[6].

2.1 Mix of trading post set-up and marketless arrangement

The basic question we ask here is: If the option of trading in a marketless set-up is available together with a trading post set-up, will trading without some markets be preferred to trading through them, resulting thereby in the coexistence of marketless trading and exchange through a network of trading posts ? Under some restrictions on the parameter values, the answer to this question is in the affirmative.

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[6]A steady state Nash equilibrium is a set of trading strategies S_k one for each type k, together with a steady state distribution $\bar{p}$, which gives the proportion of type k agents with good c, that satisfies
(i) each individual k chooses S_k to maximize his expected utility, given the best strategies of others and the distribution $\bar{p}$;
(ii) given $S_k, \bar{p}$ is the resulting steady state distribution.

To show this identification of the strategy set is a necessary prerequisite. For this let us define an ordered triplet $\tau_k(i, j, h)$ such that

$$
\begin{aligned}
i &= 0 \text{ if } k \text{ goes for direct barter} \\
&= 1, \text{ otherwise.} \\
j &= 0, \text{ if } k \text{ trades first in a trading post} \\
&= 1, \text{ otherwise.} \\
h &= 0, \text{ if } k \text{ trades in the second step in a trading post} \\
&= 1, \text{ otherwise.}
\end{aligned}
$$

In terms of this notation we plan to represent different possible strategies in the following manner.

$S_{k1} : \tau_k = (0, 0, 0)$ i.e., type k agents go for direct barter through a trading post.

$S_{k2} : \tau_k = (1, 0, 0)$ i.e., type k agents go for indirect trade through trading posts.

$S_{k3} : \tau_k = (0, 1, 1)$ i.e., type k agents go for direct barter through marketless trade.

$S_{k4} : \tau_k = (1, 1, 1)$ i.e., type k agents go for indirect trade through marketless set-up.

$S_{k5} : \tau_k = (1, 0, 1)$ i.e., type k agents go for indirect trade by trading first in a market and then in a marketless set-up.

$S_{k6} : \tau_k = (1, 1, 0)$ i.e., type k agents go for indirect by trading in a marketless set-up and then through a trading post.

Given the possible strategies, we have the following result :

Proposition 1 *Under Relation I, the strategy profile (S_{11}, S_{26}, S_{33}) consti-*

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tutes a set of steady state Nash Equilibrium strategies if the following conditions hold

$$p_1^{12} u_1 - \gamma_{12} > 0 \ and \ p_{31}\{\beta u_2 - (b_{21} + \gamma_{12})\} \geq b_{23} - b_{21}$$

where, p_1^{12} is the steady state probability of meeting an (type 2) agent with good 1 in the (1, 2) market by a type 1 agent and p_{31} is the probability of meeting a type 3 agent with good 1 in the marketless set-up.

Proof: See Appendix.

Under the above strategy profile (S_{11}, S_{26}, S_{33}), the type 1 agents would go for direct barter in the (1, 2) trading post and type 3 agents would opt for direct trade in a marketless set-up. It is the type 2 agents who would act as the intermediaries by exchanging good 3 against good 1 in a marketless set-up and then buy good 2 for good 1 in the (1, 2) market.Thus the good with the lowest storage cost emerges as a medium of exchange.

Next to look at the welfare levels (or equivalently steady state utility levels) under the two types of trading arrangements viz., complete marketisation and the one considered in proposition1, we define the welfare derived by a type k agent as:

$$WF_k = (1 - \beta) \sum_c p_{kc} V_{kc}$$

where, p_{kc} is the proportion of type k agents with good c in the steady state and V_{kc} is the utility derived by a type k agent by acquiring good c.

The complete marketisation case is the one where the agents need to trade only through designated trading posts viz., (1,2) , (2,3) or (3,1). Under complete marketisation, one can show that type 1 and 3 agents going for

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direct barter and type 2 agents opting to act as an intermediary (by selling good 3 in (1,3) market and then buying good 2 in (1, 2) market) forms a steady state Nash equilibrium.This case being straight forward is discussed in the appendix (see Proposition A.1).

The equilibrium under complete marketisation discussed above, however, will be Pareto non-comparable with the one of Proposition 1. This is because type 1 agents are going to be worse off in this new equilibrium as their complementary trading partners (i.e., the type 2 agents) are now going through a more time consuming trading process, whereas the type 3 agents would be better off if the running cost of the market revelent for them, i.e., γ_{31}, is sufficiently high. Thus we have

Proposition 2 *The equilibrium derived in Proposition 1 is Pareto non-comparable with that of an equilibrium with complete marketisation if the running costs of some markets (viz., (1, 3) and (2, 3)) are sufficiently high. However, if $\gamma_{cc'}$'s are sufficiently small, in particular $\gamma_{cc'} \to 0$ for all (c, c') and the welfare levels are positive, the equilibrium under complete marketization is welfarewise Pareto superior to the equilibrium derived in Proposition 1.*

Proposition 2 establishes our intuition that the utility of trades through monetized markets cannot be dominated by monetized trade in the absence of markets.

2.2 Random search in a trading post set-up

In the set-up considered in Section 2.1, a particular pair of goods is either traded through·markets or through marketless trade. However, in a developing country we often observe a mix of both types of trading arrangements for the *same pair of goods*. This can happen due to the fact that the cost of trading in a particular market (measured in terms of instantaneous disutility here) differs for different agents within the same type. More precisely, "time" is relatively less expensive than the cost of establishing and maintaining a trading post for a subset of agents. We next try to incorporate this feature in our model.

Let us assume that for the half of the type 2 and 3 agents time is relatively less expensive[7]. We call these agents "time surplus-resource constrained" agents or "RC" in short. Similarly, the other half i.e., the time constrained agents will be called "NRC" in short. An RC type k agent will be denoted by $RC(k)$ etc. Let $\gamma_{cc'}^{k}$ be the per period cost to be incurred by a type k agent in the (c, c') market to trade. Given the above specification.

$$\gamma_{cc'}^{RC(k)} > \gamma_{cc'}^{NRC(k)}, k = 2, 3$$

We then consider the following set of strategies:

for a type 1 agent $\tau_1 = (0, 0, 0)$

for a type 2, RC agent $\tau_2^{RC} = (1, 1, 0)$

for a type 2, NRC agent $\tau_2^{NRC} = (1, 0, 0)$

for a type 3, RC agent $\tau_3^{RC} = (0, 1, 0)$

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[7]Other specifications can be dealt with similarly.

for a type 3, NRC agent $\tau_3^{NRC} = (0, 0, 0)$

Proposition 3 *The strategy profile given above forms a steady state Nash equilibrium under a feasible set of parametric restrictions.*

Proof: See Appendix.

Under the above strategy profile type 1 agents would go for direct barter in the (1, 2) trading post.

Type 2 non-resource-constrained (NRC) agents would go for indirect trade in a trading post set up. As before the good with the lowest storage cost will become a medium of exchange.

The resource constrained (RC) type 2 agents would sell their produced goods in a marketless set-up and purchase their consumption good in a trading post. In fact this is what we usually observe in a developing economy.

Non resource constrained type 3 agents would opt for direct barter in a trading post set-up and the resource constrained ones would trade in a marketless arrangement.

As in the case of Proposition 2 , the above equilibrium (of Proposition 3) will also be non comparable with that of an equilibrium characterised by complete marketisation and also that of Proposition 2.

3 Conclusion :

This paper looks into the possibility of trade through a trading post set-up together with a marketless trading arrangement. In this context, interesting

steady state Nash equilibria are derived and the steady state utility levels are compared. It is seen that even in the presence of a well established market setup, pure random search for trading partner can prevail as a meaningful equilibrium. This corroborates what one observes in a developing economy and brings out through feasible equilibria why such unique feature is observed in the less developed economies. One interesting extension of the work can be introduction of fiat money.

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Appendix

Proposition A.1 *For Relation I, in a completely marketised set-up the strategies under which type 1 and 3 agents opt for direct barter through their respective trading posts and type 2 agents go for indirect trade, form a set of steady state Nash equilibrium strategies under the following sufficient condition:*

$$\gamma_{13} - \gamma_{12} < \frac{\beta}{2} u_3$$

Proof of Proposition A.1 :

Let V_k^D and V_k^I denote respectively the (expected, discounted, lifetime) utility derived by a type k agent by going through direct and indirect trades respectively. We want to show $V_1^D \geq V_1^I, V_2^D \leq V_1^I$ and $V_3^D \geq V_3^I$. Let p_{kc} denote the probability of finding a type k agent with good c.

Let us first consider a type 1 agent. As soon as he decides to go through direct barter he has to pay the costs $b_{12} + \gamma_{12}$. Next period he would meet a type 2 agent with good 1 with probability p_{21} and attain net utility u_1 and the entire process starts again. With probability $(1 - p_{21})$ he has the option of choosing V_1^D or V_1^I whichever is larger.

$$V_1^D = -(b_{12} + \gamma_{12}) + \beta[p_{21}(u_1 + \max(V_1^D, V_1^I)] + (1 - p_{21}) \max(V_1^D, V_1^I)]$$

When he decides to go through indirect trade, he would visit (2, 3) market and for entire life time would not meet any complementary trading partner given $V_2^D \leq V_2^I$ and $V_3^D \geq V_3^I$. Hence

$$V_1^I = -\frac{b_{12} + \gamma_{23}}{1 - \beta} \Rightarrow V_1^D > V_1^I$$

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Similarly, one can show that V_2^I and V_3^D are also optimal strategies (under the condition on the parameters stated above). It can be easily checked that in the steady state, $p_{21} = \frac{1}{2}$.

Next to look at the welfare levels under the two types of trading arrangements viz., complete marketisation and completely marketless trade (Kiyotaki and Wright, 1989), we define the welfare derived by a type k agent as:

$$WF_k = (1 - \beta) \sum_c p_{kc} V_{kc}$$

where, p_{kc} is the proportion of type k agents with good c in the steady state and V_{kc} is the utility derived by a type k agent by acquiring good c.

If we were to compare the steady state welfare levels of the equilibrium of Proposition A.1 with that of the corresponding equilibrium in a marketless economy (see Kiyotaki and Wright (1989)) we arrive at the following result :

Proposition A.2 *For the economy E defined above the welfare of every agent is higher under the fundamental strategies in a trading post set-up, (that is, where the good with the lowest storage cost becomes the medium of exchange) as compared to that of the marketless trading arrangement if the following conditions hold:*

$$\frac{\beta u_1}{3} > \gamma_{12}, \frac{\beta u_3}{3} > \gamma_{13}, \frac{\beta u_2}{3} > \frac{1}{2}(\gamma_{12} + \gamma_{13})$$

Proof of Proposition A.2 :

For a marketless set-up one can derive the welfare levels of a type k agent, WF_k as (see Kiyotaki and Wright (1989))

$$WF_1 = \frac{\beta u_1}{6} - b_{12}, \quad WF_2 = \frac{\beta u_2}{6} - \frac{1}{2}(b_{21} + b_{23}), \quad WF_3 = \frac{\beta u_3}{6} - b_{31}$$

For a trading post set-up the corresponding welfare levels are

$$\overline{WF_1} = (1 - \beta)V_1^D = -(b_{12} + \gamma_{12}) + \frac{\beta}{2}u_1 \text{ and } \overline{WF_3} = -(b_{31} + \gamma_{31}) + \frac{\beta}{2}u_3$$

For a type 2 agent let V_{21} and V_{23} denote respectively the indirect utilities of acquiring good 1 (by visiting the (1, 3) trading post) and of acquiring good 3 (by visiting (1, 2) trading post i.e. by acquiring good 2 and then producing good 3). We have $p_{21} = p_{23} = \frac{1}{2}$.

$$
\begin{aligned}
V_{23} &= -(b_{23} + \gamma_{13}) + \beta V_{21}, \\
V_{21} &= -(b_{21} + \gamma_{12}) + \beta(u_2 + V_{23}) \\
\overline{WF_2} &= (1 - \beta)\left(\frac{1}{2}V_{21} + \frac{1}{2}V_{23}\right) = \frac{\beta u_2}{2} - \frac{(b_{23} + \gamma_{13}) + (b_{21} + \gamma_{12})}{2}
\end{aligned}
$$

Comparing $\overline{WF_k}$ with WF_k we get the result.

Proof of Proposition 1 :

Let U_{ki} denote the (expected, discounted, lifetime) utility derived by a type k agent by adopting the strategy $S_{ki}(i = 1, 2, \ldots, 6)$.

$$
\begin{aligned}
U_{11} = -(b_{12} + \gamma_{12}) \quad &+ \quad \beta[p_1^{12}(u_1 + \max(U_{ki}, i = 1, 2, \ldots, 6) \\
&+ \quad (1 - p_1^{12})\max(U_{ki}, i = 1, 2, \ldots, 6)]
\end{aligned}
$$

where p_1^{12} is the probability of meeting a trader with good 1 in the (1, 2) trading post.

$U_{12} = -\frac{b_{12} + \gamma_{23}}{1 - \beta}, \quad U_{13} = -\frac{b_{12}}{1 - \beta}$

$$
\begin{aligned}
U_{14} = -b_{12} \quad &+ \quad \beta[p'_{23}\{-b_{13} + p'_{31}(u_1 + \max(U_{ki}, i = 1, 2, \ldots, 6)) + (1 - p'_{31})v_{13}\} \\
&+ \quad (1 - p'_{23})\max(U_{ki}, i = 1, 2, \ldots, 6)]
\end{aligned}
$$

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where p'_{23} is the steady probability of meeting a type 2 agent with good 3 in a marketless set-up when a type 1 agent adopts S_{14}, p'_{31} is the steady state probability of meeting a type 3 agent with good 1 in a marketless set-up when a type 1 agent adopts S_{14}, v_{13} is the indirect utility of acquiring good 3 by a type 1 agent.

$$U_{15} = -\frac{b_{12} + \gamma_{23}}{1 - \beta}$$

$$U_{16} = -b_{12} + \beta[p''_{23}(-\frac{b_{13} + \gamma_{13}}{1 - \beta}) + (1 - p''_{23})\max(U_{ki}, i = 1, 2, \ldots, 6)]$$

p''_{23} is the probability of meeting a type 2 agent with good 3 when the type 1 agents opt for S_{16}.

Now given the optimal strategy of type 2 agents, in the steady state $p'_{23} = 0$. Thus, U_{11} will be optimal if $-(b_{12} + \gamma_{12}) + \beta p_1^{12} u_1 > -b_{12} \Rightarrow \beta p_1^{12} u_1 - \gamma_{12} > 0$.

Proceeding in a parallel fashion it can be shown that U_{26} would be optimal if $p_{31}\{\beta u_2 - (b_{21} + \gamma_{12})\} \geq b_{23} - b_{21}$. Similarly optimality of U_{33} can be shown.

Steady State Probability Distributions :

Let $\frac{N_1}{N}$ and $\frac{N_2}{N}$ be the steady state proportions of the type 2 agents in a marketless arrangement and in a trading post set-up respectively. Thus, in the steady state we get :

$$p_1^{12} = \frac{N_2}{N} \text{ and } p_{23} = \text{ probability of meeting a type 2 agent with good 3}$$
$$\text{in the marketless arrangement (by a type 3 agent)}$$
$$= \frac{N_1}{N + N_1}$$

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p_{31} : probability of meeting a type 3 agent with good 1 in the marketless arrangement (by a type 2 agent) $\Rightarrow p_{31} = \frac{N}{N+N_1}$.

Also in the steady state $N_1 p_{31} = N_2$. Using these relations we get $p_{31} = \frac{\sqrt{5}-1}{2}$,

$$p_1^{12} = \frac{2\sqrt{5}-4}{\sqrt{5}-1}, \quad p_{23} = \frac{3-\sqrt{5}}{2}$$

Proof of Proposition 2 :

Let $\bar{\bar{W}}_k$ be the welfare derived by a type k agent under the equilibrium strategy of Proposition 1.
Then

$$
\begin{aligned}
\bar{\bar{W}}_1 &\simeq -(b_{12} + \gamma_2) + (.38197)\beta U_1 \\
&< -(b_{12} + \gamma_{12} + \frac{1}{2}\beta u_1 = \overline{WF_1}, \quad p_1^{12} = \frac{2\sqrt{5}-4}{\sqrt{5}-1} \simeq .38197 \\
\Rightarrow \bar{\bar{W}}_3 &\simeq -(b_{31}) + \beta(.38197)u_3 \\
&= -b_{31} + \frac{1}{2}\beta u_3 - .11903\beta u_3 \\
\Rightarrow \bar{\bar{W}}_3 &< \bar{W}_3 \text{ if } .11903\beta u_3 - \gamma_{31} > 0
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{\bar{W}}_2 &= \frac{N_1}{N}V_{21} + \frac{N_2}{N}V_{23} \\
V_{21} &= \frac{-[b_{23} + \beta p_{31}(b_{21} + \gamma_{12})] + \beta^2 p_{31} u_3}{1 - = \beta^2 p_{31}} \\
V_{23} &= \frac{-(\beta b_{23} + b_{21} + \gamma_{12}) + \beta u_2}{1 - \beta^2 p_{31}}
\end{aligned}
$$

It can be checked in a straight forward manner that if $\gamma_{cc'} \to 0, \forall(c,c')$ and $\bar{\bar{W}}_k > 0, \forall k$ then $\bar{\bar{W}}_k < \overline{WF_k}, \ \forall k$.

Proof of Proposition 3 :

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We recall that a type k agent has 6 possible strategies delineated above by $S_{k1}, S_{k2}, S_{k3}, \ldots, S_{k6}$.

Let us consider each type of agents separately.

Type 1:

Given the optimal strategies of others, for a type 1 agent S_{k2}, S_{k4} and S_{k6} would lead to incurring storage costs indefinitely without any possibility of attaining the consumption good. Assuming that U_1 is reasonably high we will concentrate only on S_{k1}, S_{k3} and S_{k5}.[8]

Let q_{ij} and $\bar{q}_{ij}$ are the probabilities of meeting an agent with good i who is ready to accept good j in a marketless and trading post set-up respectively. Let U_{ki} denote the expected, discounted, lifetime utility derived by a type k agent by adopting strategy $S_{ki}, i = 1, 2, \ldots, 6$.

Let us now compute the expected discounted life time utilities

$$U_{11} = -(b_{12} + \gamma_{12}) + \beta(\bar{q}_{21}(u_1) + \max(1)) + (1 - \bar{q}_{21})\max(1))$$

where, $\max(k) = \max\{U_{ks}, s = 1 \ldots 6\}$, s is index for the strategies $\bar{q}_{21}$ is the probability of meeting a complementary trading partner in the (1, 2) market

$$U_{14} = -b_{12} + \beta\{q_{32}(-b_{13}) + \beta q_{31}(U_1 + \max(1)) + \beta(1 - q_{31})U_{13})\} + \beta(1 - q_{32})\max(1)$$

where q_{32} is the probability of meeting an agent with good 3 who is ready to accept good 2 U_{13} indirect utility of acquiring good 3

$$U_{16} = -b_{12} + \beta[q_{32}(-b_{13} - \gamma_{13} + \beta\bar{q}_{13}(U_1 \max(1)) + \beta(1 - q_{13})\bar{U}_{13} + (1 - q_{32})\max(1)].$$

---

[8]This is an assumption we make for all types.

We have, steady state value of

$$\bar{q}_{13} = 1.$$

Since $1/2$ of the type 2 agents (NRC) are the complementary trading partner in a marketless set up for a type 1 agent

$$q_{32} = \frac{1/2}{1/2 + 1/2 + 1} = \frac{1}{4}.$$

Probability of meeting a type 3 (RC) agent in a marketless set-up gives

$$q_{31} = \frac{1/2}{1/2 + 1/2 + 1} = \frac{1}{4}.$$

Since $\frac{1}{4}$ of the type 2 NRC agents and $\frac{3-\sqrt{5}}{2}$ proportion (see (A.2) below) of the type 2 RC agents would visit (1,2) trading partner in the steady state. Therefore,

$$q_{21} = \frac{1}{4} + \frac{3 - \sqrt{5}}{2} > \frac{1}{2}.$$

Thus if $\gamma_{12} \to 0 \Rightarrow U_{11} > U_{1i}, i = 2, \ldots, 6$. Hence one gets a condition γ_{12} such that U_{11} as the optimal choice. Let us now concentrate on the type 2-NRC agents.

Only possibility for this type to attain its consumption good is by opting for strategy 3 or 5 i.e., an agent has to go through indirect trade in the trading post set-up. Or alternatively, to go through indirect trade by first trading in a marketless set-up and then in a trading post set-up

$$
\begin{aligned}
U_{23}^{NRC} &= -(c_{23} + \gamma_{31}^{NRC}) + \beta(v_{21}^{NRC}) \\
&= -(c_{23} + \gamma_{31}^{NRC}) + \beta\{\beta(-c_{21} - \gamma_{21}^{NRC}) + u_2 + \max(2)\} \\
&= -(c^{23} + \gamma_{31}^{NRC}) - \beta^2(c_{21} + \gamma_{21}^{NRC}) + u_2 + \max(2NRC)
\end{aligned}
$$

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$$
\begin{aligned}
U_{25}^{NRC} &= -(c_{23} + \beta\{q_{13}^{2NRC} U_{21}^{2NRC} + (1 - q_{13}^{2NRC})\max(2NRC)\}) \\
&= -c_{23} + \beta[q_{13}^{2NRC}\beta((-c_{21} - \gamma_{12}^{NRC}) + u_1 + \max(2NRC)) \\
&\quad + (1 - q_{13}^{2NRC})\max(2NRC)] \\
&= -c_{23} + q_{13}^{2NRC}\beta^2(-(c_{21} + \gamma_{21}^{2NRC}) + u_1 + \max(2NRC)) \\
&\quad + (1 - q_{13}^{2NRC}\max(2NRC)).
\end{aligned}
$$

Therefore under the condition

$$
-\gamma_{31}^{2NRC} - (1 - q_{13}^{2NRC})\beta^2(c_{21} + \gamma_{12}^{2NRC}) + \beta^2(1 - q_{13}^{2NRC})u_1 > 0
$$

would give $U_{33}^{2NRC} > U_{35}^{2NRC}$. Similarly one can compute the feasibility condition for the type 2 RC agents under which they would opt for strategy 6 viz., indirect trade though marketless trading arrangement and then trading in a trading post set-up. According to our notation $\bar{q}_{12}$ is the proportion of type 2 agents in the $(1, 2)$ trading post. In the steady state following condition will be satisfied:

$$
\left\{(1 - \bar{q}_{12})\frac{x}{2}\right\} \cdot -\frac{\frac{x}{2}}{\frac{x}{2} + (1 - \bar{q}_{12})\frac{x}{2}} = \bar{q}_{12} \cdot \frac{x}{2} \qquad (A.1)
$$

Since $\bar{q}_{12}$ proportion of agents would be in the trading post, $(1 - \bar{q}_{12})$ proportion of agents will be in the marketless set-up. All the type 3 RC agents ($\frac{x}{2}$ of them) will be their complementary trading partner.

Hence probability of meeting a complementary trading parter is

$$
\frac{\frac{x}{2}}{\frac{x}{2} + (1 - \bar{q}_{12})\frac{x}{2}} = \frac{1}{2 - \bar{q}_{12}}.
$$

Solving (A.1)

$$
\bar{q}_{12} = \frac{3 - \sqrt{5}}{2}. \qquad (A.2)
$$

Let us now move on to the type 3NRC agents. Two strategies viz., 1 and 3 are relevant in this context

$$U_{31}^{NRC} = -(b_{31} + \gamma_{31}^{NRC}) + \beta(\bar{q}_{31}^{3NRC}(u_1 + \max(3NRC))$$
$$+ (1 - \bar{q}_{31}^{3NRC})\max(3NRC))$$
$$U_{33}^{NRC} = -b_{31} + \beta(q_{31}^{3NRC}(u_1 + \max(3NRC)) + (1 - q_{31}^{3NRC})\max(3NRC)).$$

It can be easily checked that

$$\bar{q}_{31}^{3NRC} = \frac{1}{2}.$$

In a marketless set-up a type 3NRC agent has to meet a type 2 RC agent with good 3. Therefore,

$$q_{31}^{3NRC} = \frac{\frac{\sqrt{5}-1}{2}}{1 + \frac{\sqrt{5}-1}{2}} < \frac{1}{2}.$$

Hence we have a feasibility a condition on γ_{31}^{NRC} which would make strategy 1 optimal for a type 3 NRC agent.

Let us now find the proportion of type 2 RC agents who are in a marketless set-up. Let α proportion of type 2 agents go to the (1, 2) trading partner each period.

Let N be the total number of agents in each type.

In the marketless set-up there would be $\frac{N}{2}$ type 3 RC agents and $(1 - \alpha)\frac{N}{2}$ type 2 agents. Thus we need the following condition to be satisfied

$$\left(\frac{N}{2}(1 - \alpha)\right) \cdot \frac{\frac{N}{2}}{\frac{N}{2}(1 - \alpha) + \frac{N}{2}} = \alpha \frac{N}{2}$$
$$\Rightarrow (1 - \alpha) \times \frac{1}{(1 - \alpha) + 1} = \alpha$$
$$\Rightarrow \alpha^2 - 3\alpha + 1 = 0 \Rightarrow \alpha = \frac{3 - \sqrt{5}}{2}. \qquad (A.3)$$

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For the type 3 RC agents also exactly 2 strategies are available as that of type 3 NRC viz., 1 and 3.

Calculations of steady state utilities are also similar. However if, all type 3 agents opt for strategy 1, then corresponding problem of meeting a trading partner $\bar{q}_{31}^{3NRC+3RC}$ decreases to $\frac{1}{4}$.

Further more given the fact that γ_{31}^{3RC} is higher, direct barter through marketless set up (strategy 3) becomes the optimal strategy.