WP-2008-001

An Alternative Approach to Measure HDI

Hippu Salk Kristle Nathan, Srijit Mishra, and B. Sudhakara Reddy

Indira Gandhi Institute of Development Research, Mumbai
January 2008
http://www.igidr.ac.in/pdf/publication/WP-2008-001.pdf

An Alternative Approach to Measure HDI[1]

Hippu Salk Kristle Nathan, Srijit Mishra, and B. Sudhakara Reddy

Indira Gandhi Institute of Development Research (IGIDR)
General Arun Kumar Vaidya Marg
Goregaon (E), Mumbai- 400065, INDIA
Emails: hnathan@igidr.ac.in, srijit@igidr.ac.in, and sreddy@igidr.ac.in

Abstract

The popularly known Human Development Index (HDI) is obtained through linear averaging (LA) of indices in three dimensions - health, education and standard of living. LA method assumes perfect substitutability among the indices. We question its appropriateness and propose an alternative measure, which is the inverse of the Euclidian distance from the ideal. Following Zeleny (1974), we refer to this, as the Displaced Ideal (DI) method. Through an axiomatic characterization, the paper shows that the advantages in the DI method are the following. Uniform, as against skewed, development is rewarded. Through an ideal path, it signals a future course of action. These signify that a given increment in any one dimension, with other dimensions remaining constant, has a greater significance for the index at a lower level than at a higher level. In other words, stagnancy in the dimension that has a lower value is more serious than stagnancy in other dimensions. Finally, an empirical illustration has been done by taking the statistics in Human Development Report 2006. We strongly propose that the DI method be considered over the LA method in the construction of HDI.

Key words: Displaced ideal, Euclidian distance, Ideal point, Linear averaging, Uniform development

JEL Codes: D63, I31, O15

[1] This paper is the culmination of a challenge thrown to the first author by the second author while discussing the notion of displaced ideal and its usage in a different context that is being developed in consultation with the third author. The current work forms a part of the first author's Ph.D. thesis. Earlier versions were presented by the first author and also received the best paper prize in two conferences: the Student's Colloquium of Development Research (SCODER) held at IGIDR, Mumbai, 28-29 September 2007 and in the Consortium of Student's in Management Research (COSMAR) held at Indian Institute of Science, Bangalore, 12-13 December 2007. This paper will also be presented at the 44th Annual Conference of the Indian Econometric Society, University of Hyderabad, Hyderabad, 3-5 January 2008. The authors blame each other for caveats, if any.

An Alternative Approach to Measure HDI

Hippu Salk Kristle Nathan, Srijit Mishra, and B. Sudhakara Reddy

1. Introduction

The larger human development paradigm stresses on human beings as *ends* in themselves and not so much as *means* of development.[2] Further, the ends can be in multiple domains. It is in this context that Mahbub ul Haq, the founder of *Human Development Reports*,[3] considers one-dimensionality as the most serious drawback of the income-based measures. This led to the birth of the Human Development Index (HDI), see Haq (2003b).

The calculation of HDI involves three dimensions – health, education and the ability to achieve a decent standard of living. Health, h, is measured by life expectancy at birth; education, e, is measured in terms of weighted average of adult literacy rate and enrolment ratio;[4] and standard of living, y, is measured through logarithm of income.[5] All the three dimensions are normalized such that $0 \leq h,e,y \leq 1$.[6] The composite HDI for each country is calculated by linear averaging (LA) of the above three dimensions.[7] We denote this as HDI^{LA}.

Acknowledging the importance of HDI and without going into the rationale of choosing the particular three indices or the way these three individual indices are measured, scaled, weighed and normalized this paper suggests a change in the way the HDI is constructed. The objective of the paper is to critically evaluate the appropriateness of the LA technique, as against an alternative proposed which is the inverse of the normalized Euclidian

---

[2] For discussions on this see Haq (2003a) and Sen (2003a, 2003b and 1999) among others.
[3] The human development report is being published annually since 1990 and serves as a cornerstone in terms of philosophy as well as an approach of the United Nations Development Programme.
[4] Adult literacy rate is given 2/3rd weight and enrolment ratio is given 1/3rd weight.
[5] Logarithm of income represents diminishing returns to an increase in income.
[6] The normalization used: Index=(actual-minimum)/(maximum-minimum).
[7] The measurement of HDI has its share of critiques (Raworth and Stewart 2003) which in a sense helped in the refinement of the measure over time (Jahan 2003) and leading to construction of related indices to capture various dimensions of deprivation (Anand and Sen 2003a and 2003b). It also contributed to policy discourse (Fukuda-Parr, Raworth and Shiva Kumar (2003).

distance from the ideal.[8] Following Zeleny (1974), we refer to this as the displaced ideal (DI) method and we denote this as HDI^{DI}.[9]

An axiomatic characterization of HDI is done in section 2. The LA and DI methods are discussed and their axiomatic comparison has been done in section 3. In section 4, there is an empirical exercise using data given in HDR 2006. The ranking of the countries obtained by applying DI method is compared with those obtained from the conventional LA method.

2. Axiomatic Characterization of HDI

This section presents a number of intuitive properties that a measure of HDI should satisfy.

Normalization (Axiom N): A measure of HDI should have a minimum and a maximum, HDI $\in$ (0,1). At its minimum, HDI=0 indicates no development in all the three dimensions (h=0, e=0, y= 0); and at its maximum, HDI=1 indicates complete attainment in all the dimensions (h=1, e=1, y= 1). Alternatively, in a three-dimensional Cartesian space, the two positions refer to the origin, O, and ideal, I, respectively.

Anonymity (Axiom A): A measure of HDI should be indifferent to swapping of values across dimensions. With two countries j and k, this would mean that $\text{HDI}_j=\text{HDI}_k$ if values are interchanged across two dimensions (say, health and education such that $h_j=e_k$ and $h_k=e_j$) and remains the same in the third dimension of income, $y_j=y_k$.

Monotonicity (Axiom M): A measure of HDI should be greater (lower) if the index value in one dimension is greater (lower) with indices value remaining constant in all other dimension. With two countries j and k, this would mean that indices value remaining same in two dimensions (*say* health and education such that $h_j=h_k$ and $e_j=e_k$) and different in the dimension of income, $y_j \neq y_k$, then $\text{HDI}_j \gtreqless \text{HDI}_k$ iff $y_j \gtreqless y_k$.

8 The distance from the ideal would be a deprivation indicator. For a discussion on measurement of deprivation adjusted for group disparities see Subramaniam and Majumdar (2002).

9 The two HDI measures discussed here also turn out to be special cases of a class of HDI measures based on the Minkowski Distance Function. This is being discussed in a companion paper.

Proximity (Axiom P): A measure of HDI should be such that greater (lower) value indicates that it is closer to (farther from) the ideal. For two countries j and k with Euclidian distance from ideal indicated by d_j and d_k respectively then $\mathrm{HDI}_j \gtreqless \mathrm{HDI}_k$ iff $d_j \lesseqgtr d_k$.

Uniformity (Axiom U): A measure of HDI should be such that for a given mean of indices value, μ, a greater (lower) dispersion across dimensions, σ, should indicate a lower (greater) value. For two countries j and k, if $\mu_j=\mu_k$ and $\sigma_j \gtreqless \sigma_k$ then $\mathrm{HDI}_j \lesseqgtr \mathrm{HDI}_k$. This is in line with the notion that human development should be balanced or uniform in all dimensions (Sen, 1999).

Signaling (Axiom S): A measure of HDI should indicate a unique optimal path to reach a higher value. There exists one and only one distance $d^*=\min(d_m)$; m=all possible paths. This supports the view that an indicator should not only convey to us about the present state of affairs, but also have a futuristic role. It should be reactive and proactive so that it can help in devising a future course of action (Department of Environmental Affairs and Tourism (DEAT), 2001). A corollary of this, which is also relevant from the perspective of uniform development, is that an improvement in a dimension that has lower value is more important than an equivalent improvement in a dimension that has higher value. In other words, stagnancy in a dimension that has lower value is more serious.

3 Linear Averaging versus Displaced Ideal

3.1 Linear Averaging – thinking in one-dimension

The LA method applied to any set of parameters has an underlying assumption that the parameters are perfectly substitutable. The perfect substitutability assumption means that a differential improvement (or increment) in one indicator at any value can be substituted or neutralized by an equal differential decline (or decrement) in another indicator at any other value. This assumption is understandable when used in the case of same parameters like finding the average height of students in a class, or when similar items like pulses and cereal are added to obtain per capita availability of total food grains. Thus, LA essentially makes the thinking one dimensional, wherein same or similar parameters, which in principle are perfectly substitutable, are added linearly and averaged out. By using LA in the construction of HDI, it is assumed that health, education, and income are perfectly substitutable. Mathematically,

$$\mathrm{HDI}^{\mathrm{LA}}=1/3(h+e+y). \tag{1}$$

In the three dimensional space, one will have triangular inclined iso-HDI$^{\mathrm{LA}}$ planes indicating common HDI$^{\mathrm{LA}}$, the corresponding locus in two dimension will be 45^0 inclined lines. For presentation convenience and without loss of generality, the iso-HDI$^{\mathrm{LA}}$ plot for a two-dimensional space has been given in Figure 1.

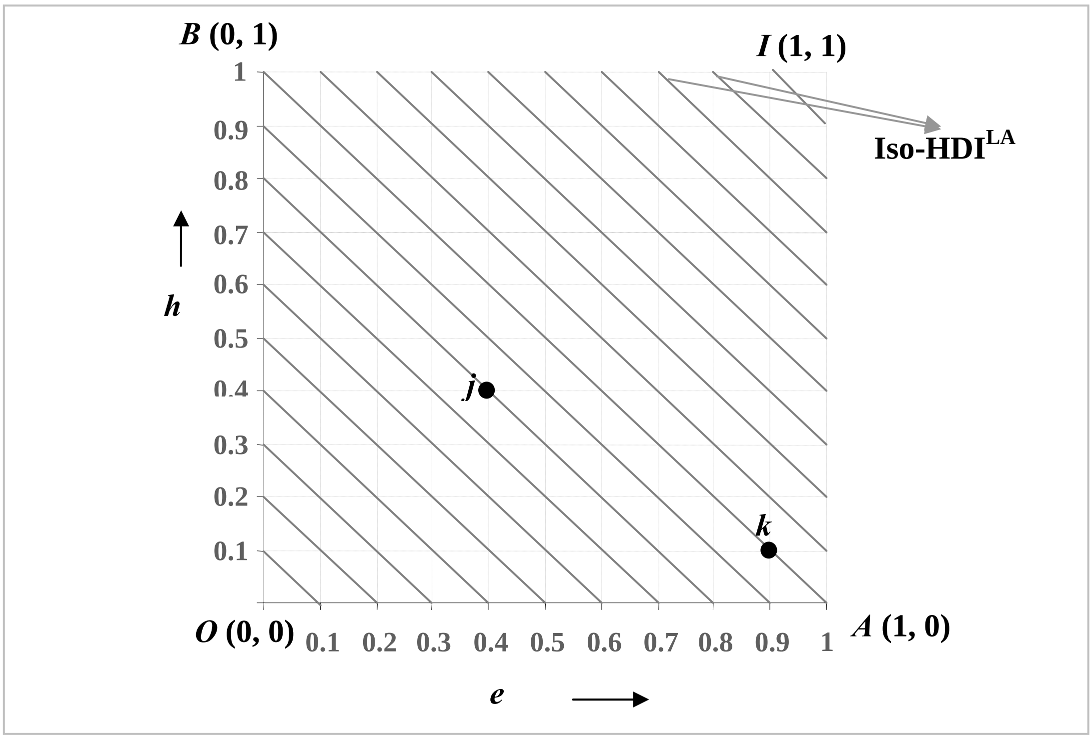

Figure 1: Iso-HDI$^{\mathrm{LA}}$ in a two-dimensional space

Figure 1 shows HDI space *OAIB* with origin, *O* (0, 0), presenting education, *e*, and health, *h*, at their minimum, and ideal, *I* (1, 1) where both the indicators are at their maximum. Any random country will occupy a point in the space *OAIB*. The locus of the points having same HDI^{LA} measure is indicated through the 45^0 inclined (or backward hatched) iso-HDI^{LA} lines. It is apparent that *j* (0.4,0.4) is lower than *k* (0.9,0.1) in terms of HDI^{LA}.

3.2 *Displaced Ideal*

The DI method is based the on the concept that the better system should have less distance from ideal (Zeleny, 1974). In a three-dimensional HDI space the ideal, *I*, denotes full attainment on all the three dimensions, (h=1, e=1, y=1). Inverse of the normalized Euclidian distance from the ideal gives

$$HDI^{DI} = 1-(\sqrt{((1-h)^2+(1-e)^2+(1-y)^2)}/\sqrt{3}) \quad (2)$$

where $\sqrt{((1-h)^2+(1-e)^2+(1-y)^2)}$ is the Euclidian distance from the ideal, dividing with $\sqrt{3}$ normalizes it in the three-dimensional space and then subtracting the normalized distance from unity gives the inverse. Thus, for country *j* the lower the distance from ideal, d_j, the higher is HDI^{DI}_j

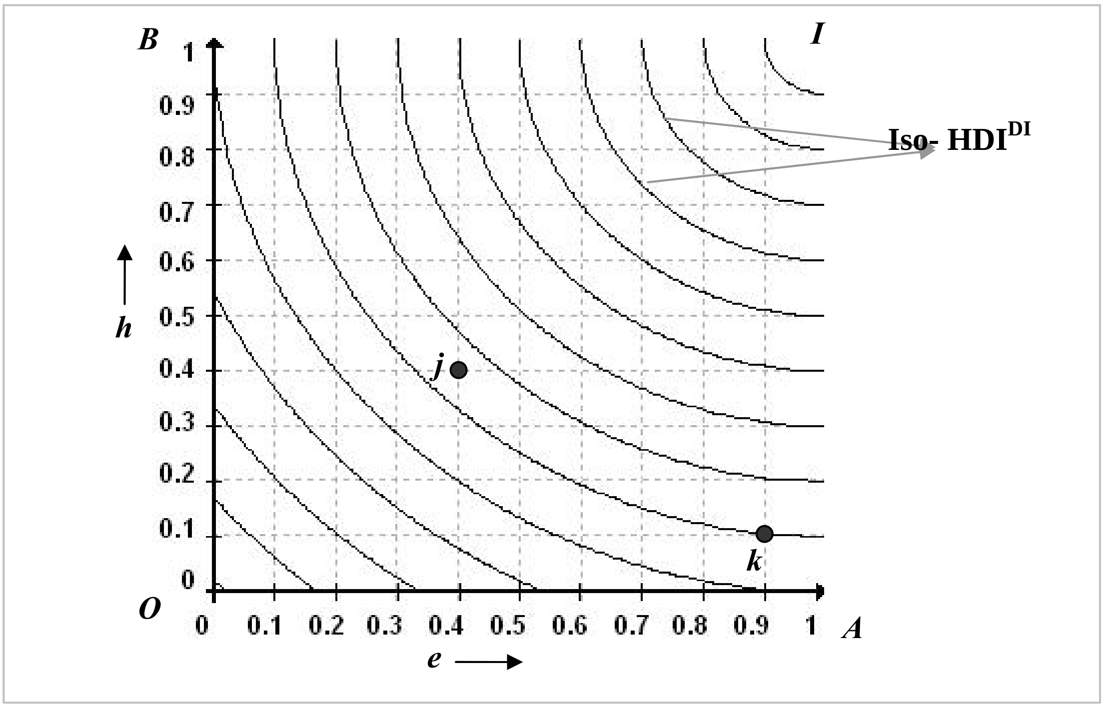

Figure 2: Iso-HDI^{DI} in a two-dimensional space

In the three-dimensional space, one will have concentric quarter spheres with centre being ideal as iso-HDI^{DI} planes indicating common HDI^{DI}, the corresponding locus in two dimension will be concentric quarter circles. For presentation convenience and without loss of generality, the iso-HDI^{DI} plot for a two-dimensional space has been given in Figure 2. The HDI space presenting the two dimensions of education and health and the two points j and k representing two countries are the same as in Figure 1. The difference being that the hierarchy between j and k has changed. $HDI^{DI}_j > HDI^{DI}_k$.

4. Axiomatic Comparison between LA and DI methods

The axioms of Normalization, Anonymity, Monotonocity, Proximity, Uniformity, and Signaling are collectively referred to with the acronym of NAMPUS. Both the LA and DI methods of calculating HDI satisfy the axioms of Normalization, Anonymity and Monotonicity. The axioms of Proximity, Uniformity and Signaling are satisfied by DI method alone. Let us elaborate.

Normalization: In both the methods, the countries are bounded by the minimum, $HDI^{LA}=HDI^{DI}=0$ at the origin, O (h=0,e=0,y=0); and the maximum, $HDI^{LA}=HDI^{DI}=1$ at the ideal I (h=1,e=1,y= 1).

Anonymity: Both satisfy this. If values are swapped across dimensions then this does not alter the value of HDI. For two countries j and k if values across the domains of health and education are interchanged, $h_j=e_k$ and $h_k=e_j$, and income is the same, $y_j=y_k$, then $HDI^{LA}_j=HDI^{LA}_k$ and $HDI^{DI}_j=HDI^{DI}_k$. Graphical explanation in a two-dimensional space is given in Figure 3.

Monotonicity: This is also satisfied for both. For two countries j and k if the value in one dimension is higher for one with the other dimensions being the same, $h_j>h_k$, $e_j=e_k$ and

$y_j=y_k$, then $HDI^{LA}_j > HDI^{LA}_k$ and $HDI^{DI}_j > HDI^{DI}_k$. Graphical explanation in a two-dimensional space is given in Figure 4.

LA:

$h_j + e_j = h_k + e_k$

hence, $HDI^{LA}_j = HDI^{LA}_k$

DI:

$d_j = \sqrt{((1-h_j)^2+(1-e_j)^2)}$

$d_k = \sqrt{((1-h_k)^2+(1-e_k)^2)}$

hence, $d_j = d_k$

$HDI^{DI}_j = HDI^{DI}_k$

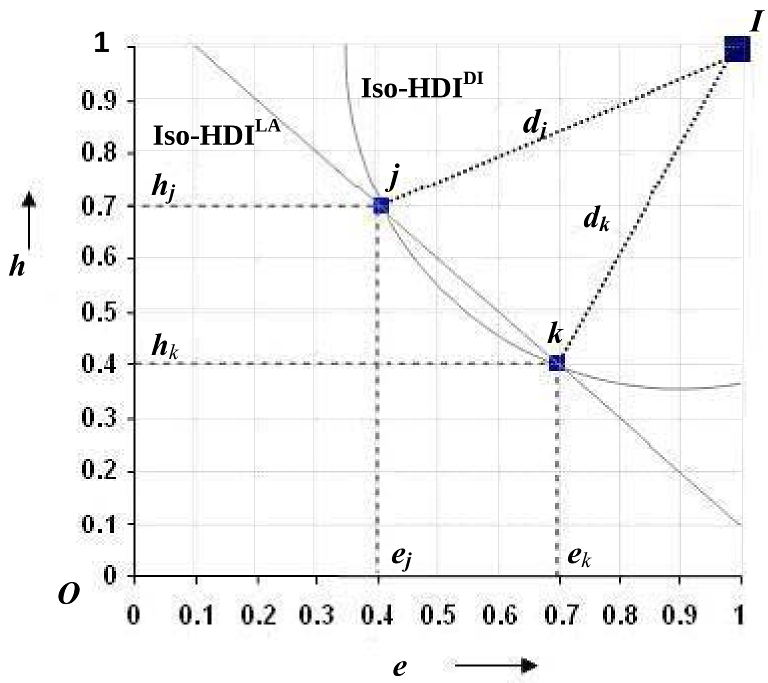

Figure 3: Anonymity axiom applied to LA and DI

If $h_i > h_k$, $e_i = e_k$ then

$HDI^{LA}_i > HDI^{LA}_k$

$HDI^{DI}_i > HDI^{DI}_k$

Similarly if

$h_i < h_{k'}$, $e_i = e_k$ *then*

$HDI^{LA}_i < HDI^{LA}_{k'}$

$HDI^{DI}_i < HDI^{DI}_{k'}$

All points to the top-right (zone A) of j will have better HDI values and those to the bottom-left (zone B) will have lower HDI values.

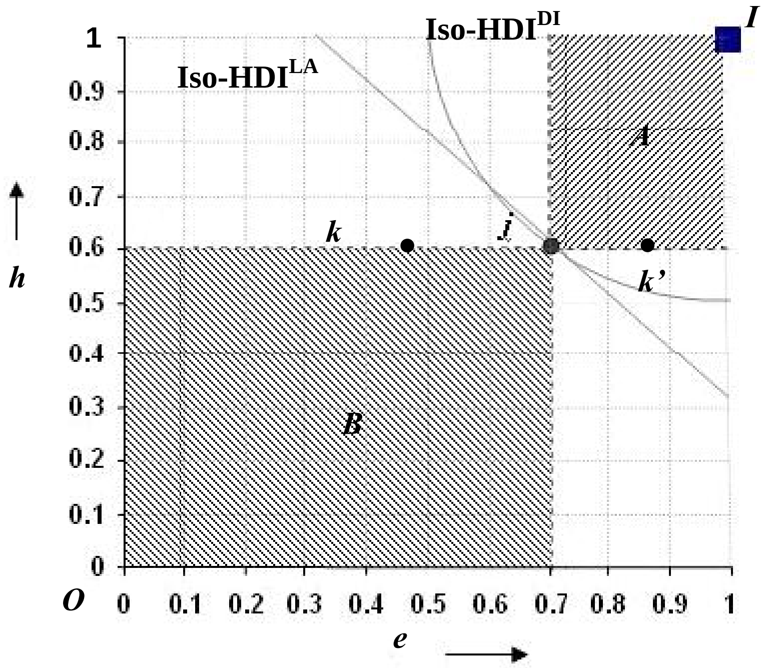

Figure 4: Monotonocity axiom applied to LA and DI

Proximity: The method of DI is itself based on the normalized Euclidian distance from the ideal, and hence, it satisfies this axiom. However, method of LA need not satisfy this. For two countries *j* and *k* with Euclidian distance from the ideal being such that $d_j > d_k$ then $HDI^{DI}_j < HDI^{DI}_k$, but it is possible to have $HDI^{LA}_j > HDI^{LA}_k$. There is a possibility of lower HDI^{LA} being closer to ideal than a higher HDI^{LA}. This has been illustrated in two-dimensional space in Figure 5.

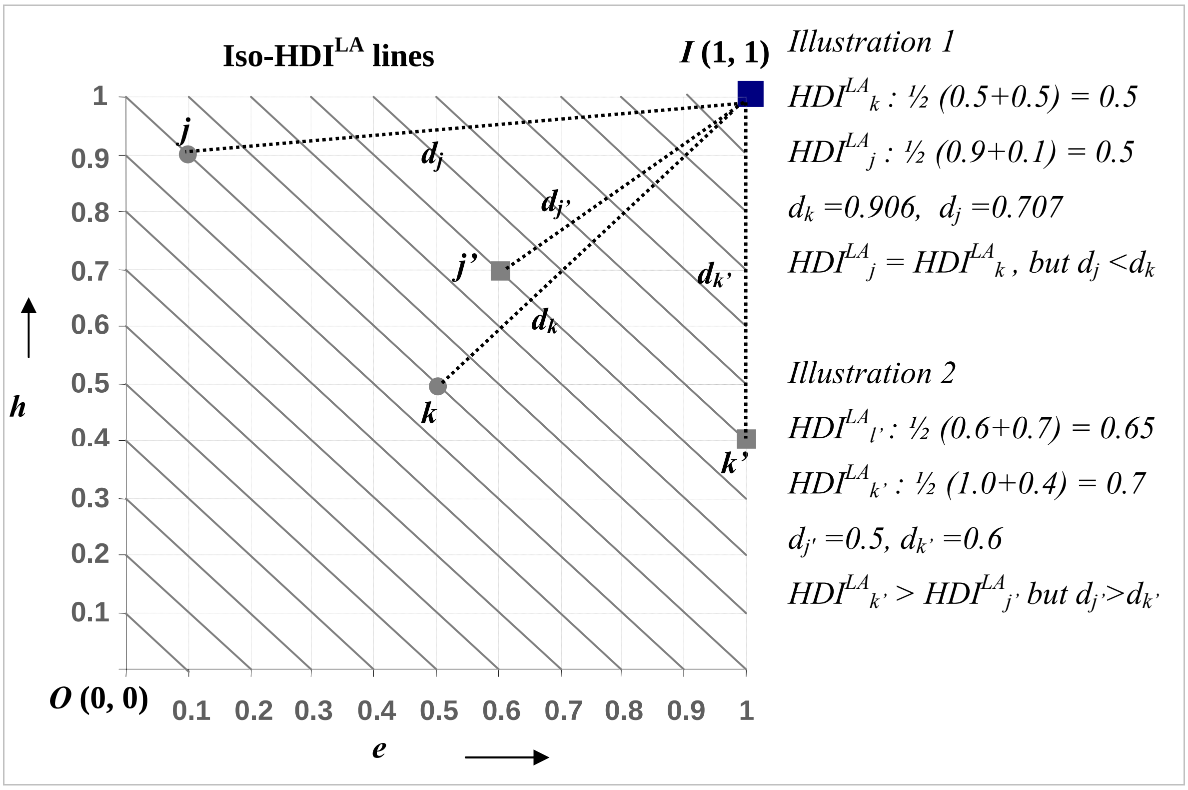

Figure 5: Proximity axiom applied to LA

Uniformity: The DI method satisfies this, but not the LA method. For two countries *j* and *k*, if $\mu_j = \mu_k$ and $\sigma_j > \sigma_k$ then $HDI^{DI}_j < HDI^{DI}_k$, but $HDI^{LA}_j = HDI^{LA}_k$. The LA method is independent of the dispersion. But DI, on the contrary, will have minimum distance from the ideal if and only if the values are equally shared in all the dimensions, that is, if the point lies on the line of equality. Thus, given an iso-HDI^{DI} line the ideal position should be on the line of equality. This means that a position closer to uniformity is rewarded and a position away from uniformity is penalized. This has been illustrated in Figure 6. The proof of the minimization is given in Appendix 1.

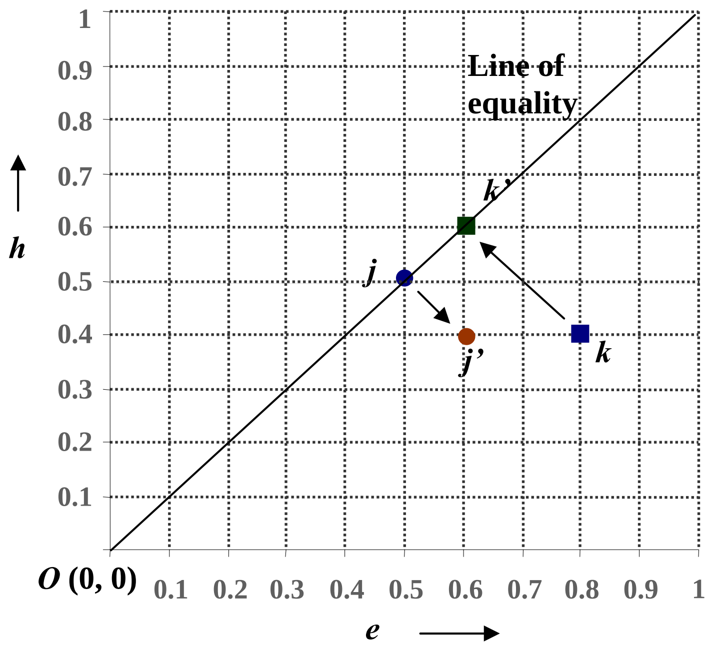

Illustration 1

Uniform to Non-Uniform

j(0.5, 0.5) $d_j = \sqrt{(0.50)}$

j'(0.6,0.4) $d_{j'} = \sqrt{(0.52)}$

Change in HDI:

$HDI^{LA}_{j} = HDI^{LA}_{j'}$

$HDI^{DI}_{j} > HDI^{DI}_{j'}$

Illustration 2

Non-Uniform to Uniform

k(0.8, 0.4) $d_k = \sqrt{0.80}$

k'(0.6,0.6) $d_{k'} = \sqrt{0.72}$

Change in HDI:

$HDI^{LA}_{k} = HDI^{LA}_{k'}$

$HDI^{DI}_{k} < HDI^{DI}_{k'}$

Figure 6: Uniformity applied to DI and LA

Signaling: Given an HDI value, a country should get a unique ideal path to progress to a higher value. This is possible under DI method but not under the LA method. Under LA, given the algebraic sum of magnitude of movement of components, there are multiple directions of movement to reach higher HDI value. Figure 7 gives an illustration in two dimensions, where country *j* can move in four different directions to obtain the same increment in HDI as sum of the change in health and education indices ($\Delta h+\Delta e$) are same in all the four directions. There are infinite such paths. Thus, there exists no unique ideal path of going to the higher value.

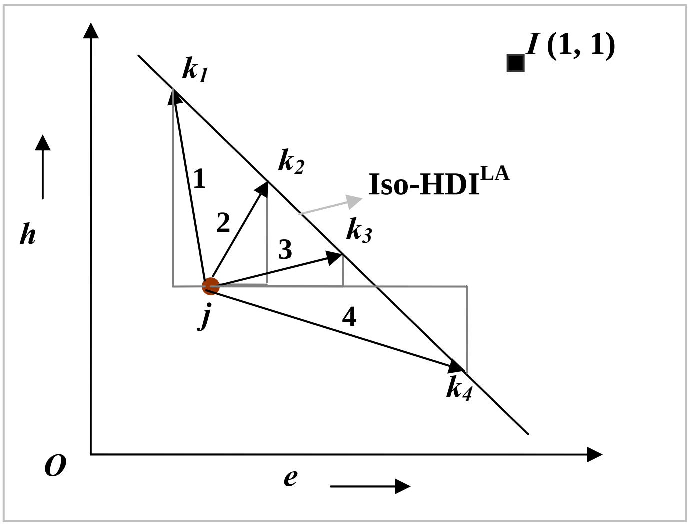

Fig 7: Signaling test on LA

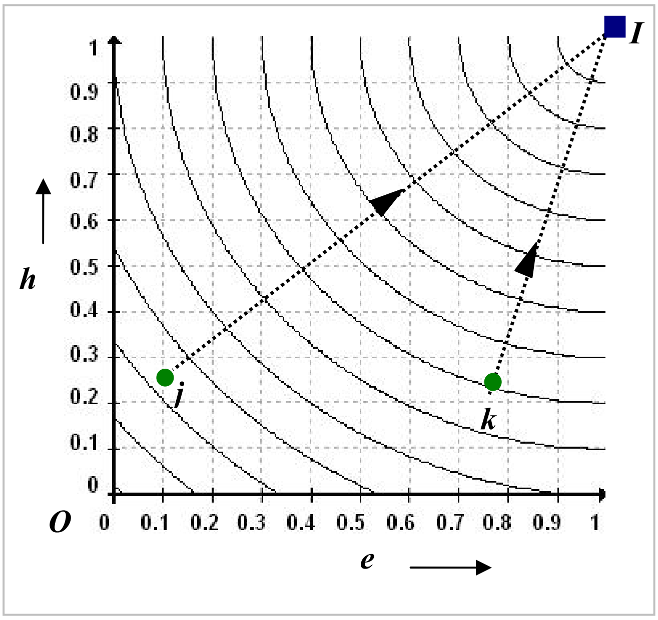

Fig 8: Ideal path under DI

Under DI method the minimum distance to any given higher value should be in the path that joins the actual position with the ideal point. Alternatively, for a given distance from the current actual position, maximum increment in HDI^{DI} happens along the line joining the actual position with the ideal point (Appendix 2). The line joining the actual position and the ideal point is referred to as the ideal path and it is this that will signal the country to attain the maximum increment. As shown in a two-dimensional scenario in Figure 8, the ideal paths for

j and k are jI and kI respectively. These paths should not be confused with the line of equality, OI, which would of course be the ideal path for a society to progress from the origin to the ideal. Between two paths, the one closer to ideal path will give a higher HDI^{DI} (Appendix 3). This indicates that a slower movement or stagnancy in a dimension that has a lower value would be farther from the ideal path, and hence, less preferred.

Thus, DI satisfies all the NAMPUS axioms, whereas LA fails to satisfy the last three axioms of Proximity, Uniformity, and Signaling. The failure arises on account of the fact the LA method assumes perfect substitutability across the three dimensions. Under perfect substitutability if $\mu_j=\mu_k$ then $HDI^{LA}{}_j=HDI^{LA}{}_k$ even if $\sigma_{j\neq}\sigma_k$ or $d_j\neq d_k$. Further, it is silent about any desirable path among the infinite possibilities to improve on HDI. Doing away with perfect substitutability in the DI method means that HDI^{DI} is sensitive to dispersion, $\sigma_\bullet$, and distance from the ideal, $d_\bullet$, and it gives a unique optimal path to move from the actual position to a higher position.

5. Applying DI method in HDI ranking

The *Human Development Report 2006* (UNDP 2006) gives the indices for health, education and income and the rank of countries as per HDI^{LA}. Using the same indices across three dimensions we have also obtained ranks of countries using HDI^{DI}. The difference in ranks indicates that a negative (positive) value implies a worse (better) performance of the country with the DI method when compared with the LA method. As a measure of dispersion, we calculate the range, which is the difference between the maximum and minimum values across the three dimensions – health, education and income. These are given in Appendix 4. The countries that slipped to lower positions (got higher ranks under DI) are referred to as losers. Similarly, those that moved up are referred to as gainers. Following are some observations.

5.1 *Biggest Losers*

The three biggest losers are Kazakhstan (Rank difference: -17), Botswana (Rank difference: -17) and Swaziland (Rank difference: -16). For comparisons, for each of the above three countries, a corresponding country is chosen within ±5 ranks under HDI^{LA} which have improved their ranking under HDI^{DI}. The ranks under LA for China (81) and Kazakhstan (79) are close by. Under DI, the former gained three positions to get a rank of 78 whereas Kazakhstan slipped by 17 positions to 96. Kazakhstan's development in the three dimensions had a greater range (0.32: health-0.64, education 0.94 and income 0.72) wheras China's development was balanced with a relatively lower range (0.12: health: 0.78, education: 0.84, income: 0.68). Between Botswana and India the former had a greater range (0.62) compared to the latter (0.06). This is reflected with rank improvement for India which goes from 126 under LA to 122 under DI whereas Botswana slips by 17 positions from 131 under LA to 148 under DI. In case of Swaziland, the story repeats. Its range of 0.62 is higher than Djibouti's 0.05. Swaxiland's rank slips by 16 positions whereas that of the latter improves by six positions. The DI method favoured countries having uniform development and penalized countries having skewed development.

Table 1: Biggest Losers

Country	Health Index	Education Index	Income Index	Rank LA	Rank DI	Rank Difference	Range
Kazakhstan	0.64	0.96	0.72	79	96	-17	0.32
China	0.78	0.84	0.68	81	78	3	0.16
Botswana	0.16	0.78	0.77	131	148	-17	0.62
India	0.64	0.61	0.58	126	122	4	0.06
Swaziland	0.10	0.72	0.67	146	162	-16	0.62
Djibouti	0.47	0.52	0.50	148	142	6	0.05

5.2 *Biggest Gainers*

The three biggest gainers are Turkey (Rank difference: 12), Belize (Rank difference: 11) and Tunisia (Rank difference: 10). For comparison, for each country a corresponding country is chosen within ±5 ranks under HDI^{LA} which have lost their ranks under HDI^{DI}. Turkey, which was just one rank above Sri Lanka under LA, enjoys 15 positions lead in the HDI rankings constructed through DI. The comparison between these two countries is a classic example. With education index being same for both countries (0.81), the analysis boils down to comparison between health and income. For Turkey, both the health and income indices are 0.73, whereas for Sri Lanka the corresponding values are 0.82 and 0.63 respectively. The DI method, which satisfies the axiom of uniformity, rewards Turkey, which for its uniform development is closer to the ideal than Sri Lanka. Belize was just two positions above Georgia under LA, but is 19 ranks above under DI. Poor income made Georgia fare much worse under DI. In case of Tunisia and Jordan, the later has a higher rank under LA whereas the former is 13 positions above under DI.

Table 2: Biggest Gainers

Country	Health Index	Education Index	Income Index	Rank LA	Rank DI	Rank Difference	Range
Turkey	0.73	0.81	0.73	92	80	12	0.08
Sri Lanka	0.82	0.81	0.63	93	95	-2	0.19
Belize	0.78	0.77	0.70	95	84	11	0.08
Georgia	0.76	0.91	0.56	97	103	-6	0.35
Tunisia	0.81	0.75	0.73	87	77	10	0.08
Jordan	0.78	0.86	0.64	86	90	-4	0.22

5.3 Topsy-turvy at the top

The topsy-turvy among the top rankers throws some interesting observations. The ranks of the first two countries, Norway and Iceland, gets swapped. This is understood from the fact that Norway has a range of 0.08 whereas the same for Iceland is 0.05. Among top rankers, Switzerland because of its holistic development across the three dimensions has gained substantially. From ninth under LA it became third under under DI. On the contrary, USA which was ranked eighth under LA, slipped 11 positions to be at 19 under DI. USA's income is the maximum possible (1.0) but its health index (0.88) is below 27 coutnries. This non-uniformity is captured by DI.

Table 3: Topsy-turvy at Top

Country	Health Index	Education Index	Income Index	Rank LA	Rank DI	Rank Difference	Range
Iceland	0.93	0.98	0.97	2	1	1	0.05
Norway	0.91	0.99	0.99	1	2	-1	0.08
Switzerland	0.93	0.95	0.97	9	3	6	0.04
United States	0.88	0.97	1.00	8	19	-11	0.12

7. Conclusions

The proposed displaced ideal (DI) method of constructing HDI deserves attention for its many advantages over the conventional linear averaging (LA) method. By an axiomatic characterization, and empirical exercise it has been shown that the DI method rates a balanced development higher than an unbalanced or skewed development. Moreover, the DI method also signals the country for movement towards the ideal point through a unique ideal path. Under DI, the complementarity in different dimensions of human development is captured, unlike LA, where perfect substitutability assumption is forced upon. Perfect substitutability has less practical significance. Sub-Saharan African countries like Botswana, Lesotho, Swaziland, and Zimbabwe reeling under a human immunodeficienty virus/acquired immune deficiency syndrome (HIV/AIDS) epidemic resulting in a poor health index could not be compensated by a relatively better education index. Thus, DI method captures uniformity, complementarity and balanced development across the three dimensions; it also signals the countries to give greater focus on those dimensions in which they are lagging behind.

Appendix 1

For a given sum of indices value in the three dimensions, $c=h+e+y$, we can write

$$y=c-h-e \tag{A1.1}$$

Now, minimizing the distance from the ideal can be written as

$$d^2 = (1-h)^2 + (1-e)^2 + (1-c+h+e)^2 \tag{A1.2}$$

Differentiating (A1.2) partially with respect to *h* and *e*, and applying the minimization condition simultaneously,

$$\frac{\partial(d^2)}{\partial h} = 2(1-h)(-1) + 2(1-c+h+e)(1) = 0 \Rightarrow 2h+e=c \tag{A1.3}$$

$$\frac{\partial(d^2)}{\partial e} = 2(1-e)(-1) + 2(1-c+h+e)(1) = 0 \Rightarrow 2e+h=c \tag{A1.4}$$

Solving (A1.3) and (A1.4) simultaneously one gets

$$h=e \tag{A1.5}$$

and if we substitute this in (A1.2) we have

$$d^2 = 2(1-h)^2 + (1-c+2h)^2 \tag{A1.6}$$

Differentiating (1.6) partially with respect to *h* and applying the minimization condition

$$\frac{\partial(d^2)}{\partial h} = 4(1-h)(-1) + 2(1-c+2h)(2) = 0 \Rightarrow 12h = 4c \Rightarrow h = \frac{c}{3} \tag{A1.7}$$

From (A1.1), (A1.5) and (A1.7), $h=e=y$.

Appendix 2

If the actual position is (h_1,e_1,y_1) and the next incremental position is (h,e,y) such that distance between the two is $\sqrt{(h-h_1)^2 + (e-e_1)^2 + (y-y_1)^2} = c$, then by manipulation we have

$$y = \sqrt{c^2 - (h-h_1)^2 - (e-e_1)^2} + y_1 \tag{A2.1}$$

Maximizing increment in HDI between the two positions is equivalent to minimizing the distance to (h,e,y) from the ideal,

$$d^2 = (1-h)^2 + (1-e)^2 + (1-y)^2 \tag{A2.2}$$

Substituting (A2.1) in (A2.2) we have,

$$d^2 = (1-h)^2 + (1-e)^2 + (1-\sqrt{c^2-(h-h_1)^2-(e-e_1)^2}-y_1)^2 \tag{A2.3}$$

Differentiating (A2.3) partially with respect to h and e, and applying the minimization condition simultaneously,

$$\frac{\partial(d^2)}{\partial h} = 2(1-h)(-1) + 2(1-\sqrt{c^2-(h-h_1)^2-(e-e_1)^2} + y_1)\left(\frac{2(h-h_1)}{2\sqrt{c^2-(h-h_1)^2-(e-e_1)^2}}\right) = 0$$

$$\Rightarrow 1-h_1 = (1+y_1)\frac{(h-h_1)}{\sqrt{c^2-(h-h_1)^2-(e-e_1)^2}} \tag{A2.4}$$

$$\frac{\partial(d^2)}{\partial e} = 2(1-e)(-1) + 2(1-\sqrt{c^2-(h-h_1)^2-(e-e_1)^2} + y_1)\left(\frac{2(e-e_1)}{2\sqrt{c^2-(h-h_1)^2-(e-e_1)^2}}\right) = 0$$

$$\Rightarrow 1-e_1 = (1+y_1)\frac{(e-e_1)}{\sqrt{c^2-(h-h_1)^2-(e-e_1)^2}} \tag{A2.5}$$

From, (A2.4) and (A2.5);

$$\frac{1-h_1}{h-h_1} = \frac{1-e_1}{e-e_1} \tag{A2.6}$$

Similarly, proceeding with h and y;

$$\frac{1-h_1}{h-h_1} = \frac{1-y_1}{y-y_1} \tag{A2.7}$$

From (A2.6) and (A2.7);

$$\frac{1-h_1}{h-h_1} = \frac{1-e_1}{e-e_1} = \frac{1-y_1}{y-y_1} \tag{A2.8}$$

This is the equation of three-dimensional line passing through (h_1, e_1, y_1) and (1,1,1). Hence, (h,e, y) is a point along the ideal path, that is, from (h_1, e_1, y_1) to (1,1,1).

Appendix 3

Consider an actual position (h_1,e_1,y_1) such that on the ideal path from this position we have a higher point (h_2,e_2,y_2) and the distance between the higher point and the actual position is $d_{12} = \sqrt{(h_2-h_1)^2+(e_2-e_1)^2+(y_2-y_1)^2}$. The locus of all points at same distance is a sphere having radius d_{12} and the centre at (h_1,e_1,y_1). Consider another point on the sphere at (h_3,e_3,y_3) so that $d_{12}=d_{13}$ and the joining of these two lines make an angle, α. Let d_{jI} denote the distance joining the j^{th} point with the ideal point (note that $d_{1I}=d_{12}+d_{2I}$); see Figure 9.

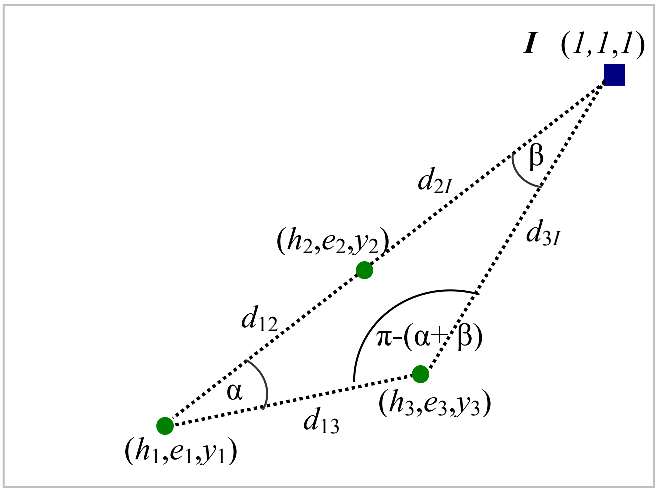

Figure 9: A cross section view of HDI space

Applying the law of sines to the triangle,

$$\frac{d_{3I}}{\sin\alpha} = \frac{d_{12}}{\sin\beta} = \frac{d_{12}+d_{2I}}{\sin(\pi-(\alpha+\beta))} \tag{A3.1}$$

(note that $d_{12}=d_{13}$)

As $\sin(\pi-(\alpha+\beta)) = \sin(\alpha+\beta)$, rewriting (A3.1) gives

$$\frac{d_{12}}{\sin\beta} = \frac{d_{12}+d_{2I}}{\sin(\alpha+\beta)} \tag{A3.2}$$

and by manipulating we get

$$\tan\beta = \frac{d_{12}\sin\alpha}{d_{2I}+d_{12}(1+\cos\alpha)} \tag{A3.3}$$

$$\beta = \tan^{-1}\frac{d_{12}\sin\alpha}{d_{2I}+d_{12}(1-\cos\alpha)} = \sin^{-1}\frac{d_{12}\sin\alpha}{\sqrt{(d_{2I})^2+2d_{12}{}^2(1-\cos\alpha)}} \tag{A3.4}$$

Applying (A3.4) in (A3.1)

$$d_{3I} = \sqrt{d_{2I}{}^2+2d_{12}{}^2(1-\cos\alpha)} \tag{A3.5}$$

Squaring and differentiating with respect to α

$$\frac{\partial(d_{3I}{}^{2})}{\partial\alpha} = 2d_{12}{}^{2}\sin\alpha \qquad \text{(A3.6)}$$

For α (0, π), sin α>0 and d_{3I} is an increasing function of α. Hence, between two paths, the one closer to ideal path will give a higher HDI^{DI}.

Appendix 4

Country	Health Index	Education Index	Income Index	Rank LA	Rank DI	Rank Difference	Range
Iceland	0.93	0.98	0.97	2	1	1	0.05
Norway	0.91	0.99	0.99	1	2	-1	0.08
Switzerland	0.93	0.95	0.97	9	3	6	0.04
Japan	0.95	0.94	0.95	7	4	3	0.01
Canada	0.92	0.97	0.96	6	5	1	0.05
Australia	0.92	0.99	0.95	3	6	-3	0.07
Sweden	0.92	0.98	0.95	5	7	-2	0.06
France	0.91	0.97	0.95	16	8	8	0.06
Italy	0.92	0.96	0.94	17	9	8	0.04
Belgium	0.90	0.98	0.96	13	10	3	0.08
Austria	0.90	0.96	0.96	14	11	3	0.06
Netherlands	0.89	0.99	0.96	10	12	-2	0.10
Ireland	0.88	0.99	1.00	4	13	-9	0.12
United Kingdom	0.89	0.97	0.96	18	14	4	0.08
Finland	0.89	0.99	0.95	11	15	-4	0.10
Spain	0.91	0.98	0.92	19	16	3	0.07
Germany	0.90	0.96	0.94	21	17	4	0.06
Israel	0.92	0.95	0.92	23	18	5	0.03
United States	0.88	0.97	1.00	8	19	-11	0.12
Luxembourg	0.89	0.94	1.00	12	20	-8	0.11
New Zealand	0.90	0.99	0.91	20	21	-1	0.09
Hong Kong, China (SAR)	0.95	0.88	0.96	22	22	0	0.08
Denmark	0.87	0.99	0.96	15	23	-8	0.12
Singapore	0.90	0.91	0.94	25	24	1	0.04
Greece	0.89	0.97	0.90	24	25	-1	0.08
Cyprus	0.90	0.91	0.91	29	26	3	0.01
Korea, Rep. of	0.87	0.98	0.89	26	27	-1	0.11
Slovenia	0.86	0.98	0.89	27	28	-1	0.12
Portugal	0.87	0.96	0.88	28	29	-1	0.09
Czech Republic	0.85	0.93	0.88	30	30	0	0.08
Kuwait	0.87	0.87	0.88	33	31	2	0.01
Brunei Darussalam	0.86	0.88	0.88	34	32	2	0.02
Malta	0.89	0.86	0.87	32	33	-1	0.03
Barbados	0.84	0.96	0.84	31	34	-3	0.12
Hungary	0.80	0.95	0.86	35	35	0	0.15
Argentina	0.83	0.95	0.82	36	36	0	0.13
Bahrain	0.82	0.86	0.89	39	37	2	0.07
Poland	0.83	0.95	0.81	37	38	-1	0.14
Slovakia	0.82	0.92	0.83	42	39	3	0.10
Chile	0.89	0.91	0.78	38	40	-2	0.13
Croatia	0.84	0.90	0.80	44	41	3	0.10
Qatar	0.80	0.85	0.88	46	42	4	0.08
Seychelles	0.80	0.88	0.85	47	43	4	0.08
Estonia	0.78	0.97	0.83	40	44	-4	0.19
Lithuania	0.79	0.97	0.81	41	45	-4	0.18
Uruguay	0.84	0.95	0.76	43	46	-3	0.19
Costa Rica	0.89	0.87	0.76	48	47	1	0.13
Latvia	0.78	0.96	0.79	45	48	-3	0.18
Mexico	0.84	0.86	0.77	53	49	4	0.09
Bahamas	0.75	0.86	0.87	52	50	2	0.12
United Arab Emirates	0.89	0.71	0.92	49	51	-2	0.21
Saint Kitts and Nevis	0.75	0.92	0.81	51	52	-1	0.17
Antigua and Barbuda	0.82	0.80	0.81	59	53	6	0.02
Oman	0.82	0.77	0.84	56	54	2	0.07
							continued

Country	Health Index	Education Index	Income Index	Rank LA	Rank DI	Rank Difference	Range
Malaysia	0.81	0.84	0.77	61	55	6	0.07
Trinidad and Tobago	0.75	0.88	0.80	57	56	1	0.13
Mauritius	0.79	0.81	0.80	63	57	6	0.02
Panama	0.83	0.88	0.72	58	58	0	0.16
Tonga	0.79	0.93	0.73	55	59	-4	0.20
Bulgaria	0.79	0.92	0.73	54	60	-6	0.19
Romania	0.78	0.90	0.74	60	61	-1	0.16
Cuba	0.88	0.93	0.67	50	62	-12	0.26
Bosnia and Herzegovina	0.82	0.87	0.71	62	63	-1	0.16
Libyan Arab Jamahiriya	0.81	0.86	0.72	64	64	0	0.14
Macedonia, TFYR	0.82	0.87	0.70	66	65	1	0.17
Brazil	0.76	0.88	0.74	69	66	3	0.14
Colombia	0.79	0.86	0.72	70	67	3	0.14
Saint Lucia	0.79	0.89	0.69	71	68	3	0.20
Dominica	0.84	0.86	0.67	68	69	-1	0.19
Thailand	0.75	0.86	0.73	74	70	4	0.13
Saudi Arabia	0.78	0.72	0.82	76	71	5	0.10
Venezuela	0.80	0.87	0.68	72	72	0	0.19
Russian Federation	0.67	0.95	0.77	65	73	-8	0.28
Belarus	0.72	0.95	0.71	67	74	-7	0.24
Lebanon	0.79	0.86	0.68	78	75	3	0.18
Albania	0.82	0.88	0.65	73	76	-3	0.23
Tunisia	0.81	0.75	0.73	87	77	10	0.08
China	0.78	0.84	0.68	81	78	3	0.16
Samoa (Western)	0.76	0.90	0.67	75	79	-4	0.23
Turkey	0.73	0.81	0.73	92	80	12	0.08
Suriname	0.74	0.84	0.70	89	81	8	0.14
St Vincent and the Grenadines	0.77	0.81	0.69	88	82	6	0.12
Peru	0.75	0.87	0.67	82	83	-1	0.20
Belize	0.78	0.77	0.70	95	84	11	0.08
Fiji	0.72	0.87	0.69	90	85	5	0.18
Dominican Republic	0.71	0.83	0.72	94	86	8	0.12
Paraguay	0.77	0.86	0.65	91	87	4	0.21
Ukraine	0.69	0.94	0.69	77	88	-11	0.25
Grenada	0.67	0.88	0.73	85	89	-4	0.21
Jordan	0.78	0.86	0.64	86	90	-4	0.22
Iran, Islamic Rep. of	0.76	0.75	0.72	96	91	5	0.04
Philippines	0.76	0.89	0.64	84	92	-8	0.25
Armenia	0.78	0.91	0.62	80	93	-13	0.29
Ecuador	0.82	0.86	0.61	83	94	-11	0.25
Sri Lanka	0.82	0.81	0.63	93	95	-2	0.19
Kazakhstan	0.64	0.96	0.72	79	96	-17	0.32
Algeria	0.77	0.71	0.70	102	97	5	0.07
Maldives	0.70	0.87	0.65	98	98	0	0.22
Cape Verde	0.76	0.73	0.68	106	99	7	0.08
El Salvador	0.77	0.76	0.65	101	100	1	0.12
Jamaica	0.76	0.79	0.62	104	101	3	0.17
Azerbaijan	0.70	0.89	0.62	99	102	-3	0.27
Georgia	0.76	0.91	0.56	97	103	-6	0.35
Syrian Arab Republic	0.81	0.74	0.60	107	104	3	0.21
Occup. Palestinian Territories	0.80	0.89	0.53	100	105	-5	0.36
Turkmenistan	0.63	0.91	0.64	105	106	-1	0.28
Guyana	0.64	0.90	0.63	103	107	-4	0.27
Indonesia	0.70	0.83	0.60	108	108	0	0.23
Egypt	0.75	0.73	0.62	111	109	2	0.13
Nicaragua	0.75	0.75	0.60	112	110	2	0.15
Viet Nam	0.76	0.81	0.55	109	111	-2	0.26
Guatemala	0.71	0.68	0.63	118	112	6	0.08
Honduras	0.72	0.77	0.56	117	113	4	0.21
Bolivia	0.66	0.87	0.55	115	114	1	0.32
Vanuatu	0.73	0.71	0.57	119	115	4	0.16
Kyrgyzstan	0.70	0.92	0.49	110	116	-6	0.43
Moldova, Rep. of	0.72	0.89	0.48	114	117	-3	0.41
Uzbekistan	0.69	0.91	0.49	113	118	-5	0.42
Mongolia	0.66	0.91	0.50	116	119	-3	0.41
Morocco	0.75	0.54	0.63	123	120	3	0.21
Gabon	0.48	0.71	0.70	124	121	3	0.23

continued

Country	Health Index	Education Index	Income Index	Rank LA	Rank DI	Rank Difference	Range
India	0.64	0.61	0.58	126	122	4	0.06
Tajikistan	0.65	0.90	0.41	122	123	-1	0.49
South Africa	0.37	0.80	0.79	121	124	-3	0.43
Solomon Islands	0.63	0.67	0.48	128	125	3	0.19
Namibia	0.37	0.79	0.72	125	126	-1	0.42
São Tomé and Principe	0.64	0.76	0.42	127	127	0	0.34
Cambodia	0.52	0.69	0.53	129	128	1	0.17
Equatorial Guinea	0.30	0.77	0.89	120	129	-9	0.59
Myanmar	0.59	0.76	0.39	130	130	0	0.37
Comoros	0.64	0.53	0.50	132	131	1	0.14
Lao People's Dem. Rep.	0.50	0.66	0.50	133	132	1	0.16
Bhutan	0.64	0.48	0.50	135	133	2	0.16
Pakistan	0.64	0.46	0.52	134	134	0	0.18
Ghana	0.53	0.54	0.52	136	135	1	0.02
Bangladesh	0.64	0.46	0.49	137	136	1	0.18
Papua New Guinea	0.51	0.52	0.54	139	137	2	0.03
Nepal	0.62	0.51	0.45	138	138	0	0.17
Sudan	0.53	0.53	0.50	141	139	2	0.03
Timor-Leste	0.52	0.63	0.39	142	140	2	0.24
Congo	0.46	0.72	0.38	140	141	-1	0.34
Djibouti	0.47	0.52	0.50	148	142	6	0.05
Togo	0.49	0.54	0.46	147	143	4	0.08
Madagascar	0.51	0.66	0.36	143	144	-1	0.30
Uganda	0.39	0.67	0.45	145	145	0	0.28
Cameroon	0.34	0.66	0.51	144	146	-2	0.32
Mauritania	0.47	0.49	0.49	153	147	6	0.02
Botswana	0.16	0.78	0.77	131	148	-17	0.62
Yemen	0.60	0.51	0.36	150	149	1	0.24
Haiti	0.45	0.50	0.49	154	150	4	0.05
Gambia	0.52	0.42	0.50	155	151	4	0.10
Kenya	0.37	0.69	0.41	152	152	0	0.32
Senegal	0.52	0.39	0.47	156	153	3	0.13
Eritrea	0.49	0.50	0.38	157	154	3	0.12
Guinea	0.48	0.34	0.51	160	155	5	0.17
Zimbabwe	0.19	0.77	0.51	151	156	-5	0.58
Rwanda	0.32	0.61	0.42	158	157	1	0.29
Lesotho	0.17	0.77	0.54	149	158	-9	0.60
Nigeria	0.31	0.63	0.41	159	159	0	0.32
Benin	0.49	0.40	0.40	163	160	3	0.09
Angola	0.27	0.53	0.51	161	161	0	0.26
Swaziland	0.10	0.72	0.67	146	162	-16	0.62
Côte d'Ivoire	0.35	0.46	0.46	164	163	1	0.11
Tanzania, U. Rep. of	0.35	0.62	0.32	162	164	-2	0.30
Mozambique	0.28	0.47	0.42	168	165	3	0.19
Congo, Dem. Rep. of the	0.31	0.54	0.33	167	166	1	0.23
Burundi	0.32	0.52	0.32	169	167	2	0.20
Zambia	0.21	0.63	0.37	165	168	-3	0.42
Malawi	0.25	0.64	0.31	166	169	-3	0.39
Ethiopia	0.38	0.40	0.34	170	170	0	0.06
Chad	0.31	0.29	0.51	171	171	0	0.22
Guinea-Bissau	0.33	0.39	0.33	173	172	1	0.06
Central African Republic	0.24	0.42	0.40	172	173	-1	0.18
Burkina Faso	0.38	0.23	0.41	174	174	0	0.18
Mali	0.39	0.24	0.38	175	175	0	0.15
Sierra Leone	0.27	0.45	0.29	176	176	0	0.18
Niger	0.33	0.26	0.34	177	177	0	0.08

Note: LA and DI denote Linear Averaging and Displaced Ideal method respectively.

References

Anand, Sudhir and Amartya Sen (2003a) Concepts of Human Development and Poverty: A Multidimensional Perspective, in Sakiko Fukuda-Parr and A.K. Shiva Kumar (eds.) *Readings in Human Development*, Oxford University Press, New York, pp.127-137. Prepared for the *Human Development Report 1995*.

Anand, Sudhir and Amartya Sen (2003b) Gender Inequality in Human Development: Theories and Measurement, in Sakiko Fukuda-Parr and A.K. Shiva Kumar (eds.) *Readings in Human Development*, Oxford University Press, New York, pp.127-137. First published in *Human Development Papers*, 1997, UNDP, New York.

DEAT (2001) *National Core Set of Environmental Indicators For the State of Environment Reporting in South Africa*, Scoping Report, Phase 1, DEAT, Pretoria.

Fakuda-Parr, Sakiko, Kate Raworth and A. K. Shiva Kuamr (2003) Using the HDI for Policy Analysis, in Sakiko Fukuda-Parr and A.K. Shiva Kumar (eds.) *Readings in Human Development*, Oxford University Press, New York, pp. 177-187.

Haq, Mahbub ul (2003a) The Birth of the Human Development Index, in Sakiko Fukuda-Parr and A.K. Shiva Kumar (eds.) *Readings in Human Development*, Oxford University Press, New York, pp.127-137. First published in Mahbub ul Haq, *Reflections on Human Development*, 1995, Chapter 4, Oxford University Press.

Jahan, Selim (2003) Evolution of the Human Development Index, in Sakiko Fukuda-Parr and A.K. Shiva Kumar (eds.) *Readings in Human Development*, Oxford University Press, New York, pp.152-163.

Raworth, Kate and David Stewart (2003) Critiques of the Human Development Index: A Review, in Sakiko Fukuda-Parr and A.K. Shiva Kumar (eds.) *Readings in Human Development*, Oxford University Press, New York, pp.164-176.

Sen, Amartya (1999) *Development as Freedom*, Oxford University Press, Oxford.

Sen, Amartya (2003a) Development as Capability Expansion, in Sakiko Fukuda-Parr and A.K. Shiva Kumar (eds.) *Readings in Human Development*, Oxford University Press, New York. First published in *Journal of Development Planning*, 1989, No.19, pp.41-58.

Sen, Amartya (2003b) Human Capital and Human Capability, in Sakiko Fukuda-Parr and A.K. Shiva Kumar (eds.) *Readings in Human Development*, Oxford University Press, New York. First published in *World Development*, 1997, Vol.25, No.12, pp.1959-1961.

Subramanium, S and Manabi Majumdar (2002) On Measuring Deprivation Adjusted for Group Disparities, *Social Choice and Welfare*, Vol.19, No.xx, pp.265-280

UNDP (2006) Statistics in the Human Development Report, *Human Development Report, 2006*, http://hdr.undp.org/hdr2006/statistics (accessed on 15 September 2007).

Zeleny, M. A. (1974) Concept of Compromise Solutions and Method of Displaced Ideal, Computational Operation Research, 1: 479-496pp.