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Reverse Mortgages as Retirement Financing Instrument: An Option for "Asset-rich and Cash-poor" Singaporeans

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ABSTRACT

The unique way of financing housing through the mandatory savings system in Singapore has created a class of "asset-rich and cash-poor" Singaporeans. This paper provides a framework to assess the viability of a reverse mortgage (RM) market so that such instruments may be harnessed as a source of financing retirement income for home owners. Based on different cost of capital, we estimate the probability of loss for both the private supplier and public provider of RMs. The probability of loss is computed by three major components: choice of replacement ratio and property growth rate; forecast of cohort survival probability by joint-life; and generation of yield curves to discount the future cash flows. The stochastic forecast of survival probability is estimated using the Lee-Carter demographic model based on the abridged life tables. The discount factor for future cash flows are generated from stochastic interest rates. Our simulation results indicate that based on the benchmark scenario, RM instruments by private providers are likely to achieve about 50% replacement ratio for the 4-room public housing owners. However, the market may be missing if a replacement ratio of 70% is required.

Keywords: incomplete reverse mortgage market, replacement ratio, probability of loss, risk free interest, breakeven annuity

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1. Introduction

Housing equity forms a large fraction of the non-pension wealth for elderly households in many developed countries. For instance, about 33% of the total financial asset in the US is occupied by housing assets in 2000 (Poterba, 2001). It is even higher for Japan at 63.9% (Noguchi, 1997). If there is a mechanism to unlock housing equities, it will potentially help to alleviate poverty among elderly homeowners and to finance retirement expenditures. Reverse mortgage (RM) instrument essentially is designed to allow property owners to obtain loans in the form of retirement annuities, using their residential assets as collaterals; and repaying the loans by selling the house upon their death. It thus affords an alternative means for elderly homeowners to borrow against the financial equity embodied in their homes, while sparing them from the emotional disruption of moving out of or selling their abodes.

There are many forms of RM, but the basic idea is that the property will be reverted to the RM supplier at the end of a period or upon the death of the reverse mortgagor. In conventional mortgages, the loan quantum is dependent on the borrower's ability to pay. However for RM, the loan quantum depends on the appraised value of the property, the projected rate of house price appreciation, the age and sex of the homeowners, the levels of interest rates and the required loan. There are also costs involved in taking out these RM loans. RMs differ in terms of the types of loan advance and the time frame. Loan advance can be taken as a lump sum or as a regular income

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¹ For example, in US the minimum qualifying age is set at 62 years, and the average age is around 72 years. In Singapore, the average qualifying age is 62. Gender plays a role as male and female has different life expectancy.

stream. The terms of the RM can either be fixed-term or tenure. In a fixed term RM, the period of loan advance is fixed, usually 10, 15 or 20 years.

Empirical study by Kutty (1998) indicates that the use of home equity conversion mortgage products could possibly raise about 29% of the poor elderly homeowners in the US above the poverty line. In addition, the equity released could potentially help to finance long term care among the elderly, where relatively large sum of money are required (Gibbs, 1992).² The usefulness of such a RM scheme is conceivably greater in countries where land prices are extremely high (for example, Japan) or where there is a skewed investment portfolio towards home ownership (for example, Australia) or where there is a deliberate public policy towards home ownership (for example, Singapore).³

While some empirical studies support the potential effect of RM, others do not. For example, Hancock (1998) examines the impact of equity release scheme on the net income of older homeowners in Britain and finds that the increased income is not significant for some of the oldest homeowners. Theoretically, there is potential in unlocking housing wealth to help alleviate poverty and meet health care needs of the elderly. However, in reality, RM markets have remained weak. Mayer and Simmons (1994) and Caplin (2002) attribute this to the substantial loads in the RM market because of moral hazard and adverse selection problems. Other major barriers include product designs, availability of information, bequest motives and the desire to keep house equity

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² Sheiner and Weil (1993) found that besides shocks to family status, health shocks also contributed to the decline in home equity at older ages.

³ The share of land assets in real assets is 83.4% in Japan and 36% in US. (see Noguchi, 1997). Australian home ownership is in excess of 70% (Beal, 2001), whereas home ownership in Singapore is in excess of 90%.

as precautionary savings. These authors conclude that to unleash the potential of RM instrument, there is a need for policy makers to provide institutional and legal support for the RM market. For the case of feasibility of RM in Japan, Mitchell and Piggott (2003) highlight the need to facilitate information on housing values and transactions and credit worthiness of borrowers.

Simulations by McCarthy et al. (2002) indicate that a typical Singapore worker would have around 75% of his retirement wealth in housing asset from age 50. Such a concentration surpasses that of an American elderly household who would have 20% of their retirement wealth in housing asset. In fact, some attempts have been made to unleash the housing asset as alternative to financing retirement needs in Singapore. In January 1997, the NTUC Income, a local insurance firm, launches the first RM scheme. But the market has remained thin, with only 180 customer base. The average monthly draw down is \$1800; and the average property value is \$1.6 million. RM remains unpopular as it is only available to private property owners and not to public housing owners. In addition, the high profit margin set by the private provider virtually reduces the monthly annuity payouts generated for the RM buyers. In this paper, we shall explore whether RM can be harnessed as an instrument to finance retirement expenditures for the "asset-rich and cash-poor" elderly Singaporeans.

Most RM studies have focused on the demand side of the market and examine the effect of equity release on net incomes. For examples, Merrill et al. (1994), Hancock

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⁴ See Inter-Ministerial Committee on Ageing Population, 1999, p. 77.

(1998), Venti and Wise (1991, 2001) and Mitchell and Piggott (2003) calculate the tenure or life RM that would provide the homeowners with monthly payments over the borrower's remaining life after retirement. In these studies, the maximum amount of RM loan is calculated and then the lump-sum loan is converted to lifetime annuities with monthly payments. The analyses then focus mainly on the demand for RM to augment the incomes of the elderly homeowners. Our approach in this paper is different because we consider both the demand and supply factors of the RM market. They include the replacement ratio, the initial appraised value of the property, the growth rate of house price appreciation, the age and sex of the homeowners, various costs of capital and the probability of loss.

The organisation of the rest of the paper is as follows. In Section 2, we provide an overview of the housing market in Singapore, focusing on the unique way of financing housing through the compulsory savings mechanism. Section 3 describes the methodology and calibration procedure used in the Monte Carlo experiments. The calibration of the retirement annuity consists of three major components. The first part assesses the adequacy of the monthly annuity payments to finance retirement at the first breakeven month for different initial property values and appreciation rates. Second, the Lee-Carter (1992) model is adopted to forecast cohort survival probability at each post-retirement age for the household using the abridged life tables for Singapore. Third, the discount/accumulation factor for future cash flows are generated from three interest rate models, including two deterministic and one stochastic interest rate environments. Section 4 presents simulation results and analysis on the first breakeven month and the

probability of loss for both the private supplier and the public provider of RM. The final section concludes the study and draws some policy implications on the role of public provider to complete the RM market.

2. Housing Market and Housing Finance in Singapore

After independence from the British in 1959, the new Singapore government was set to solve the housing shortage which saw many living in slums. The Housing Development Board (HDB) was set up in 1960 to build "emergency" public housing on state-owned land. These 1-room to 3-room public housings were leased to citizens at standardised and affordable rates, which averaged \$20 and \$30 per room in suburban and urban zones respectively. However in February 1964, in line with the strategy of making private home ownership an investment in the stake of the country, HDB introduced a scheme to encourage existing tenants to own their flats. Under the Home Ownership Scheme for the People, HDB offered subsided mortgage loans with an attractive repayment scheme. The loan quantum was set at 80% of the price of a new flat with repayment periods of either 5, 10 or 15 years. However, the home-ownership rate remained low at only 5% by December 1965. This was due to the low purchasing power of households at that time.

To further ease financing difficulty on the demand side, the government introduced a unique system which is closely integrated to the compulsory savings scheme or the Central Provident Fund (CPF) system. The CPF, which was instituted in 1955, was originally a retirement savings scheme. It is a fully funded and a defined

contribution system. Under the CPF, every employee and employer is required to contribute a proportion of the wage to the CPF which is credited into the individual's personal accounts. In September 1968, the CPF introduced the Approved Housing scheme which allowed HDB purchasers to withdraw their savings to finance the purchase of public housing. Funds can be withdrawn for down-payment, stamp duties, mortgage payments and interest incurred for the purchase. The CPF Approved Housing Scheme marked the beginning of a series of schemes in which mandatory savings were used in relation to housing finance. It also sets off a gradual liberalisation of CPF from merely a retirement vehicle to instruments to help finance merit goods consumption such as education and health care.

In order to achieve a nation of homeowners, HDB also implemented supply side regulations and subsidies. First, the option to rent was made unattractive or effectively unavailable for the majority due to strict eligibility criteria. Second, the public housing was priced affordably through government supply and price discounts, enabling buyers to purchase HDB flats at below market prices. However, unlike most merit good programs, these subsidies are not financed primarily from taxes or other government revenues, but

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⁵ In 1955, the rate of contributions was only 10%. Since 1968, the rate has increased, rising to a peak of 50% in 1984. The rate is currently graduated according to age, with an average rate of 36%. More details on the CPF and the CPF-HDB link are discussed in the next section. Also, see Chia and Tsui (2003a) for the institutional details regarding the CPF.

⁶ In 1981, the use of CPF savings was extended to the purchase of private residential private property. See Phang and Tan (1991) for a chronological account of the liberalization of the use of retirement savings in the CPF for housing finance in Singapore.

⁷ See Chia and Tsui (2003b) for the link between compulsory savings and the financing of health care in Singapore using the medical savings accounts.

rather from land rents that are captured through state ownership and acquisition.⁸ Indeed, nearly 80% of the land in Singapore is owned by the state. Under the Land Acquisition Act, the government is empowered to acquire land at its discretion from private land owners and at prices below market prices.⁹ Hence, compared to private sector developers who have to purchase or cost land at market rates, producer costs for public sector housing is lower and HDB is able to sell its flats at below market prices.

The success and sustainability of HDB to build public housing and pricing flats at affordable prices is due to the strong institutional support from the government. HDB's annual deficit is fully covered by government grant. The cumulative government grant received since the establishment of HDB in 1960 amounted to \$10,533 million. In the fiscal year 2000/01, HDB received \$920 million to cover the deficit. (HDB, Annual Report 2001). HDB also receives two main government loans to finance its operations. First, the Housing Development Loans is used to finance the development programmes and operations. Interest rate is pegged at two percent points above the floating CPF interest rate with a repayment term of 20 years. The second is the Mortgage Financing Loans to finance the mortgage loans granted to the purchasers of HDB flats. The ability of HDB to obtain loans from the government at below market rates implies that HDB is able to subsidise the mortgage financing rate for its buyers. The mortgage financing rate

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⁸ As such, the supply side price discounts have little impact on government expenditure and is not a significant expenditure item. According to Asher and Phang (1997), receipts from land rent have enabled the government to keep expenditure on housing at no more than 2% of total government expenditure in any fiscal year.

⁹ For example, between 1973 and 1987, the government acquired land under the Land Acquisition Act at 1973 rates rather than at market rates of compensation. For details, see Phang (1996).

is pegged at 0.1% above the interest rates paid by the CPF for the compulsory savings and is about 2% below the housing mortgage interest rates of commercial banks. ¹⁰

The HDB Home Ownership Scheme and its link to the CPF housing financing scheme, together with many supply side instruments, have skewed the housing tenure choice towards owner occupation, particularly towards the owner-occupied public housing. This is evidenced from the fall of HDB rental occupancy of 100% in the early 1960s to 76 % at the end of 1970 and to 38% in 1981 and finally to 7% in 2002. After almost four decades since the inception of Home Ownership Scheme in 1964, 85% of Singapore's population resides in public housing; with 93% of the public housing residents owning the units they occupied. (See Figure 1). Furthermore, HDB flats constitute 80% of the residential housing stock in Singapore. ¹¹

== insert Figure 1 ==

One consequence of the owner-occupied housing policy is that housing becomes the most important non-financial assets for Singaporeans. This can be gleaned from Table 1. Compared to France (47%), Japan (40%), US (28%) and UK (34%), Singapore has the highest ratio of household residential property assets relative to total assets (at 51%). This is also true for housing asset relative to personal disposable income and GDP.

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¹⁰ The CPF interest rate on the ordinary account is pegged to the average of 12-month fixed deposit and month-end savings rates of the local banks rate.

¹¹ See Statistical Highlights (2003).

In the National Survey of Senior Citizens (1995), 63.1% of elderly aged 60 and above reported housing among their assets and 48.4% cited their own house as their most important asset. While the provision of early withdrawal from CPF savings has helped in housing finance, it has also diluted its original intent as a retirement savings scheme, thereby reducing the accumulated amount available for retirement needs. Indeed, the CPF-HDB link has created a class of "asset-rich and cash-poor" households, whose savings are "plastered on the wall". The issue is how to unlock the housing equity and to liquidate these assets for old age consumption purposes.

== insert Table 1 ==

3 Modelling reverse mortgages

Our model builds on the approach by Tse (1995b). It consists of three main parts, namely the treatment of the monthly annuities as replacement ratios, the use of survival ratios and various interest rate models, respectively. However, we make four extensions to Tse's approach. First, we provide an economic interpretation of the pre-supposed level of life annuity which is set according to different target replacement ratios to ensure adequate post-retirement living. Second, like Tse (1995b), Lachance and Mitchell (2002) and Chia and Tsui (2003a), the Cox-Ingersoll-Ross (CIR) model is used to generate yield curves for discounting and accumulating cash flows. But monthly interest rates are generated instead of 6-month rates. This facilitates the computation of the monthly accumulated loan value and expected profit. Our approach also allows death to happen monthly instead of in the middle of the year as assumed by Tse. Third, unlike Tse's

analysis which is based on a single period life table, we estimate and forecast mortality rates using the Lee-Carter (1992) demographic model. Based on the forecasts, we further construct the cohort mortality rates using the approach by Bourbeau et al. (1997). More discussion is given in Section 3.3. Fourth, Tse's assessment of the RM market is mainly based on some assigned profit margin issued by private RM suppliers. However we also assess the viability of RMs based on risk-free interest rates. This implies that some non-profit organisations, for example, the government who has access to low-cost capital, may be able to launch such a financial instrument in the case of an incomplete RM market. Hence our analysis bears significant policy implications for a missing market.

The nature of the RM market entails lenders granting various guarantees to borrowers. First, RM instrument has a residency guarantee in which the mortgagors are allowed to remain in their property until death, regardless of the loan amount. Second, under income guarantee, the monthly annuity payment will continue as long as the homeowner lives in the home. Third, under repayment guarantee, repayment will only occur after the demise of the last couple, thereupon the property is sold. Fourth, being a "non-recourse" loan, the accumulated loan value cannot exceed the accumulated property value and the mortgagor's other assets cannot be used to repay the loan.

All these guarantees inevitably spell different risks for the lender, which are reflected by three different interest rates. The risks for the lender include the longevity of the borrowers. The longer the life expectancy of the lender, the higher is the probability of loss for the lender as the accumulated loan value may exceed the accumulated property

value. ¹² As repayment is over a longer time frame, there are risks associated with volatilities of interest rates and property prices.

It is crucial to distinguish three types of interest rates used in our model.¹³ First, the cost of capital, denoted by r, is a risk-free rate of interest, which represents the opportunity cost of using funds. Second, an interest rate i is used to discount the future value of loan and repayment cash flows. This interest rate can be interpreted as the lending rate which includes risk premium to reflect the uncertainty to the lender in the event that at the time of repayment, the accumulated loan value exceeds the accumulated property value. This is partly due to mortality uncertainty which affects the length of residence and the uncertainty on property appreciation rates. We assume i = r + 0.02. Third, an interest rate y is used to discount the loan balance that incorporates the necessary profit margin. It can be regarded as the cost of borrowing for the lender to finance the RM loan. We assume y = r + 0.01 for the private supplier. The spread between y and i also reflects the intermediary role of the lender who has access to lower cost fund at y but charges the borrower i for the use of fund.

We assume that the elderly do not have any outstanding mortgage and that a tenure joint life RM is taken up by a married elderly, both of the same age. The eligible couple will then receive a fixed monthly annuity at the beginning of each month till the end of life. Furthermore, we assume that the couple will not move out of their home.

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¹² See Phillips and Gwin (1993) and Mitchell and Piggot (2003) for an excellent exposition of the risks facing the RM suppliers. Besides longevity risk, interest rate risk and property appreciation risk, they also discuss specific house appreciation risk and expense risk.

¹³ For example, Boehm and Ehrhardt (1994) show that compared to other types of interest-bearing assets, interest rate changes are riskier for RMs. Both Tse (1995b) and Mitchell and Piggot (2003) also incorporate the different risks when modelling interest rates.

Although death is a random process, for convenience we assume that death occurs at the end of the month. The RM will continue upon the death of one spouse. Only upon the death of the last survivor would the loan be repaid through the sale of the property. However, the property sale will initiate one month after death and that the sale will be completed after three months.

3.1 Present value of estimated profit

The private supplier gives a RM loan to the homeowner in the form of monthly life annuity payout. The monthly payout is accumulated with interest until the repayment period. Let A denotes the fixed monthly payout from the RM. For exposition purposes, we assume that the elderly receive the first monthly annuity payout at age 62 and continue to receive the payout at the beginning of each month till death of the last survivor. We assume a maximum life span of 105 years, so that t can take any value from 1 to 528 months. Upon the death of the last surviving spouse in month t, the total accumulated loan balance (L_t) is:

$$L_t = \left(\sum_{j=1}^t AU_j\right) B_t$$
 $t = 1, ..., 528$ (1)

where U_j is the accumulation factor used to sum up the level monthly annuity A from age 62 to the time of death at time t. Denoting i_j as the nominal interest rate which reflects the supplier's cost of capital at month j, U_i can be expressed as follows:

$$U_j = \prod_{n=j}^t \left(1 + \frac{i_n}{12} \right) \tag{2}$$

As it takes four months to complete the sale, B_t in equation (1) is the additional accumulation factor given by:

$$B_t = \prod_{n=1}^4 \left(1 + \frac{i_{n+t}}{12} \right) \tag{3}$$

In the simulations, deterministic and stochastic interest rates are generated to accumulate and discount the future cash flows.

Besides interest rate charges, the supplier also levies other administrative costs, including an origination fee for initiating the RM instrument. We assume that the origination fees, denoted by λ , are fully borne by the RM buyer and it is borrowed from the supplier who incorporates the amount into the loan. As in Tse (1995b), we also set the origination fee at 1% of the appraised value of the property. Besides the origination fee, closing costs are incurred at the time of sale of the property. Such costs cover fees for title search and title insurance, legal and appraisal services, surveys and inspections, mortgage taxes, credit checks and other related transaction costs. ¹⁴ The closing cost, denoted by τ , is set at 3.5% of the initial appraised value of the property, which is the usual rate charged in Singapore.

The accumulated value of the property net of all transaction costs is given by P_t as follows:

$$P_{t} = P_{0} \Big((1 - \lambda)(1 + \alpha)^{(t+1)/12} - \tau U_{t}^{*} \Big)$$
 (4)

$$U_{t}^{*} = \prod_{j=1}^{t+4} \left(1 + i_{j} / 12 \right) \tag{5}$$

origination and closing fee and 2% insurance premium. Besides these cost, other loading factors include insurance cost at 0.5 % over and above the interest rate charges.

¹⁴ In the United States, the Home Equity Conversion Mortgage (HCEM) imposes 2 % (of home value) for

where P_0 is the initial appraised property value; α represents the rate of property appreciation or the annual growth rate of the appraised value of the property. U_t^* is the accumulation factor to compute the cash flow of the property, including the four-month lags to complete the sale of the house. i_j is the interest rate charged by the supplier at month j.

In what follows, we consider the profit maximizing behaviour of the RM suppliers under uncertainty. There are two major sources of uncertainties. Besides the stochastic interest rate movements, the other source comes from the time of death of the last survivor which in turn determines the time of repayment. At the time of repayment, the supplier will compare the accumulated net value of the property (P_t) with the value of the accumulated loan (L_t) to assess the profitability of supplying the RM instrument. During the initial months of the launched RM, the accumulated loan is definitely smaller than the accumulated value of property. However, as the RM progresses and as the time of residence lengthens, the accumulated loan becomes bigger and subsequently it may exceed the accumulated value the house. As such, it is necessary to compute the first breakeven month m^* such that at $t = m^*$, the accumulated loan (L_{m^*}) is greater than the accumulated net value of the property (P_{m^*}) . Hence, for all $t > m^*$, we have $L_t > P_t$. In other words, if the borrower survives m^* months or longer, the accumulated net property value will fall short of the accumulated annuity payouts, thereby the supplier incurs a loss. As the probability of loss is dependent on the mortality of the RM holder, we define the probability of loss to the supplier (γ) as:

$$\gamma = \sum_{t=m^*}^{528} {}_{t-1|1}q_{62} \tag{6}$$

where $_{t-1|1}q_{62}$ denotes the probability that death occurs within month t given that the borrower has survived (t-1) months from age 62.

We next examine the flow of funds for the supplier in terms of receipts and costs. As a "non-recourse" loan, the total value of the loan cannot exceed the sale value of the property. Thus at any repayment month t, the receipt (Q_t) or the maximum claim amount by the supplier is the minimum of the accumulated loan L_t and the accumulated appraised net value of the property P_t , that is,

$$Q_t = \min \{L_t, P_t\} \tag{7}$$

The present value of the cost to the RM supplier for providing the monthly level payout A up to the period t is C_t , such that:

$$C_t = \sum_{j=1}^t AW_j \tag{8}$$

where $y_0 = 0$ and

$$W_{j} = \prod_{n=1}^{j} \left(1 + \frac{y_{n-1}}{12} \right)^{-1} \tag{9}$$

We assume that the private supplier has access to a lower cost of capital. The spread between y_t and i_t then represents the intermediary role of the RM supplier who charges the borrower at a higher rate for the use of fund, taking into account the risk premium and profit margin.

Hence, the present value of profit (π_t) generated at month t which is discounted to age 62 is given by:

$$\pi_t = Q_t / V_t - C_t \tag{10}$$

where V_t is the appropriate discount factor for Q_t given by:

$$V_{t} = \prod_{n=1}^{t+4} \left(1 + \frac{y_{n}}{12} \right) \tag{11}$$

The mean present value of profit (MPVP) is obtained by weighing π_t in equation (10) by the mortality of the last survivor to obtain:

$$MPVP = \sum_{t=1}^{528} \pi_{t-t-1|1} q_{62}$$
 (12)

Note that the probability of the last survivor who dies within the t^{th} month given survival in the $(t-1)^{th}$ month from age 62 can be computed as follows:¹⁵

$$_{t-1|1}q_{x} = _{t-1}p_{62} - _{t}p_{62} \tag{13}$$

where $_{t}p_{62}$ denotes the probability of survival of the last spouse up to t months from age 62.

The Lee-Carter (1992) demographic model and the Bourbeau and Legare approach are adopted to forecast cohort survival probability at each post-retirement age for the household using the abridged life tables for Singapore. Detailed are given in Chia

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¹⁵ For computational simplicity, we assume that the married couples are of the same age and the events of deaths are independent in probability. See Chapter 10 of Jordan (1975) for details.

and Tsui (2003a). In what follows we highlight the major procedures required to calibrate the discount factors and the joint life survival ratios respectively.

3.2 Interest rates

The present value of the calibrated cash flows of retirement expenses are to be discounted by the appropriate yield curves. As there are no consensus for modeling interest rates¹⁶, we follow Tse (1995a), Lachance and Mitchell (2002), and Chia and Tsui (2003a) to generate stochastic short-term interest rates using the discretized version of the Cox, Ingersoll and Ross (1985) short-term model: ¹⁷

$$r_{t+1} = r_t + \theta (r_a - r_t) + \beta r_t^{1/2} \varepsilon_{t+1}$$
 (14)

where r_t is the short-rate at time t. The second term on the right-hand side of (3) captures the deterministic trend which consists of the long-term average interest rate, r_a , and the speed of mean-reversion, θ , respectively. For $\theta > 0$, r_t is expected to decrease and revert to r_a if the current rate is above the long-run mean, and vice versa. The third term captures the stochastic part, consisting of independently and identically distributed standard normal random variable ε_{t+1} ; and β is the volatility parameter.

For comparison, we also employ two deterministic interest rate models to discount the future cash flows: the constant yield curve (CYC) model and the fixed yield curve (FYC) model. The CYC has a flat annual rate for all durations. Choices include

¹⁶ See "Term-Structure Models" in Campbell, Lo and MacKinlay (1997, Chapter 11).

¹⁷ Tse (1995a) is the only available empirical study on the stochastic behaviours of 3-month Treasury-bill rates in Singapore and finds that the CIR model provides reasonable replicates of the yield curves. .

2%, 3% and 4%, respectively. The FYC is based on the average of the available historical rates for the government bonds since 1988. They comprise 2.3% for 3-month bills, 2.5% for 1-year bills, 3.0% for 2-year bonds, 3.9% for 5-year bonds, 3.8% for 7-year bonds, 4.3% for 10-year bonds and 3.9% for the 15-year bonds, respectively. We use the 15-year rate as proxy for spot rates with longer durations. Spot rates for other durations below 15 years are obtained by the method of interpolation.

3.3 Survival probabilities

Appropriate mortality rates of the last survivor are required to compute the present value of profit for the RM suppliers as given in equations (10) to (12). We follow the approach by Chia and Tsui (2003a) to construct the required survival probabilities for the joint life starting from age 62, up to and including 105.

Basically the construction takes three main steps. First, we predict the future mortality rates of the elderly based on the abridged life tables using the Lee-Carter model (1992, 2000) for the male and female elderly of age 60 to 85 and above:

$$ln m_{xt} = a_x + b_x k_t + \varepsilon_{xt}$$
 (15)

$$k_t = \mu + \varphi k_{t-1} + \eta_t \tag{16}$$

where $m_{x\,t}$ is the central death rate in age class x in year t; a_x is the additive age-specific constant, reflecting the general shape of the age schedule; b_x is the responsiveness of mortality at age class x to variations in the general level; k_t is a time-specific index of the general level of mortality; μ and φ are parameters; ε_{xt} is the error to the actual age schedule, assuming to follow a normal distribution with zero mean and a constant

variance; and η_t is the white noise. The Lee-Carter model has been successfully applied to the G7 countries to forecast life expectancy at birth. See Tuljapurkar et al. (2000). For those elderly aged above 85, the interpolation method proposed by Wilmoth (1995) is used.

Second, we calibrate the abridged cohort life tables based on the predicted mortality rates obtained from Step 1 using techniques developed by Bourbeau et al. (1997). Third, we convert the calibrated abridged cohort mortality rates into annual mortality rates by Pollard (1989)'s methodology, and then to monthly mortality rates. Fourth, transform the mortality rates to survival rates up to fractional ages is transformed to monthly interval.

Finally, we obtain the joint life mortality rates by assuming the events of death are independent in probability, and that the married elderly are of the same age.

$$_{t-1|t} q_{XY} = _{t-1} p_{XY} - _{t} p_{XY}$$

where the probability of last survivor is given by:

$$_{t}p_{XY} = 1 - _{t}q_{X} *_{t}q_{Y} = 1 - (1 - _{t}p_{X}) - (1 - _{t}p_{Y}) = _{t}p_{X} + _{t}p_{Y} - _{t}p_{X} + _{t}p_{Y}$$

4. Calibration and simulation results

Monte Carlo experiments are conducted to explore the feasibility of launching RM instruments for public housing owners in Singapore. We obtain values of L_t and P_t by simulations using equations (1) and (4) under different model parameterisation, including the initial home value (P_0), the different rates of property appreciation (α), the different interest rate paths to reflect different risk premium, and under different joint cohort survival probabilities. All computations and estimations are coded in Gauss. Despite the infinite interest-rate paths generated by the CIR model as described in equation (13), we have confined our simulations to 5,000 runs.

In our simulation, we choose 4-room public housing owners as the benchmark household. This is supported by the empirical evidence that the greatest proportions of HDB owners (about 39%) in year 2002 are in 4-room flats. In fact, 68% of the residents are living in HDB 4-room or larger flats or private housing, up from 52% in 1990. While a decade ago, the greatest proportions of the HDB owners (about 35%) were in 3-room flats. Because of greater affluence, 42% of these 3-room HDB owners have moved to 4-room or larger flats or private properties in 2000. ¹⁸ Table 2 shows the price range of the new 4-room flats offered by HDB for different residential zone and town in 2000. In pricing new flats for sale, HDB takes into consideration the affordability factor. The prices of new 4-room flat are pegged to the average household income levels to ensure

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¹⁸ See Singapore Department of Statistics, (2001), Table 10.

that at least 70% of all household can afford to purchase a new 4-room flat. It also shows the average valuation of resale flat at market prices, which varies according to location. ¹⁹

== insert Table 2 ===

Table 3 describes the average floor area of the different public housing types and the profile of the households in terms of the average annual household income for different housing types. The viability of the RM instrument to finance retirement is assessed by comparing monthly annuity payments to the replacement ratios. These ratios are expressed against pre-retirement household incomes. We set the benchmark mean household income at \$3719 and the median household income at \$3000.

== Insert Table 3 ===

Table 4 summarises the value of parameters used in the benchmark scenario. We first estimate the accumulated loan amount (L_t) in equation (1) by setting various monthly annuity payouts, starting from \$900 and increasing it in steps of \$100. For each level of payout, we compute the associated net accumulated property value P_t which is given in

¹⁹ Since March 1971, a resale market in HDB flats emerged when owners of HDB flats were allowed to sell their flats at market prices, subject to eligibility requirements. The government intervenes in the HDB resale market by setting the minimum occupancy period requirement before resale is possible. In 1971, the period was set at three years and was extended to five years in 1973. It has remained so until 1979 when it was relaxed to two and half year. The active resale HDB market has led to the sentiment that public housing is "a cash cow for the milking of housing subsidies". (*The Straits Times*, April 19, 1997). As of January 2003, the minimum occupancy period was further reduced to one year.

For the general population, the Census of Population 2000 indicates that the average household income in 2000 is \$4,943 and the median household income is \$3,607. But for HDB dwellers, the average household income is lower at \$3,719 and the median household income is \$3000. (HDB, 2000).

equation (4). As long as P_t exceeds L_t , the supplier will make a profit. However at a particular breakeven month m^* , such that $t > m^*$, L_t will exceed P_t . As can be observed from Table 5, our simulation results are consistent with the intuition that the first breakeven month for the supplier is sooner for higher than lower annuity payouts. Table 5 also tabulates the probability of loss for different levels of monthly annuity payouts, using equation (6).

== Insert Table 4 ===

The present value of profit (PVP) in equation (10) depends on stochastic interest rate fluctuations. Using the CIR model as described in Section 3.3, we generate a yield curve to obtain the corresponding PVP, and run 5000 replications to obtain the mean value of PVP (MPVP). Table 5 also tabulates the values of MPVP, the standard deviation of PVP and its values at the 5^{th} and 95^{th} percentiles as well as the probability of loss and the first breakeven months at various monthly level annuities.

== insert Table 5 ==

Table 5 shows that at the an initial property value of \$240,000, and a monthly annuity level of \$1500, the probability of loss for the supplier is only 0.0367, with a later first breakeven month, which occurs at $m^* = 478$. However, when the monthly annuity payout is increased to \$1600, the probability of loss will be much higher at 0.5374, with the first breakeven month occurring sooner at $m^* = 350$. If the private RM supplier is

risk averse and prefers a smaller probability of loss, then he will pick a lower level of monthly annuity, say at \$900. But there may be no demand for the RM as the replacement ratio will be much lower than the expected 70%.²¹ Hence, the completeness of the RM market depends on setting an annuity payout that is adequate to finance retirement expenditure, while minimising the probability of loss to the supplier.

Table 5 also indicates that when the monthly payout is increased to \$1700 to achieve a replacement ratio of 46% of the pre-retirement average income, the probability of loss to the supplier is almost certain at 0.8024. This is in stark contrast to the negligible probability of loss when the monthly annuity payout is \$1400. Hence we may interpret the difference of \$300 as the loading factor levied by the RM supplier. In this case the loading factor can be as high as 21% (\$300/\$1400). Our findings are consistent with Caplin and many other authors who accord the incompleteness of the reverse mortgage market to the high loading factor.

We next evaluate the RM market when the supplier is a non-profit motivated supplier, for example the government. As profit is not a major consideration for the public provider, in our computation of the accumulated loan value (L_t) in equation (1), risk-free interest rate r is used instead of the risk-embedded rate y which incorporates the risk premium and profit margin. For easy comparison, we define the *breakeven annuity* as the annuity which yields zero MPVP for the private supplier. We repeat the simulation

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²¹ There is no single acceptable replacement ratio. McGill et al., (1996) recommend using a replacement rate of 73%. In Canada, financial planners typically give a ratio of 70% of post-retirement income as being required to maintain a comparable standard of living experienced before retirement.

process to obtain the new breakeven month for the public provider using the breakeven annuity.

Table 6 tabulates simulation results which compare the adequacy of the RM instruments provided by profit or non-profit motivated suppliers of RM. These include the mean monthly payout at the breakeven annuity, its standard deviation, the 5th and 95th percentiles, the probability of loss, the first breakeven month, and replacement ratio based on the mean and median monthly income of the household. We repeat the simulations using different initial property values assumption, ranging from \$220,000 to \$300,000.

== insert Table 6 ==

As can be observed from Table 6, for a given property value of \$220,000, the mean breakeven monthly payout is \$1475. This level of breakeven annuity represents a replacement ratio of 49% of the average household income. However, there may be no private supplier as the probability of loss is close to 0.581, implying that it is highly probable that the accumulated loan balance is higher than the net value of the property at the time of repayment. Hence the RM market fails. Can the government intervene in this market? Table 6 indicates that at the same average breakeven annuity, but the probability of loss for the public provider is now almost negligible as the public provider has lower cost of fund and is able to charge the loan balance at the risk free interest rate.

In addition, at a higher initial property value of \$300,000, the homeowner can expect to unlock the housing equity to yield an income which is almost 67% of the pre-retirement average income. But again the market may be missing as the probability of loss for the private supplier is still high at 0.586. However, the probability of loss is only 0.003 for a public provider. Furthermore, compared to the private supplier, the first breakeven month for the public provider occurs 176 months later.

We concur that profitability is vital for the private RM suppliers but not so for the public supplier. Figure 2 displays the probability of loss for both the public and private supplier using different levels of monthly annuity payouts. It also compares the probability of loss for various property appreciation rates. As can be gleaned from Figure 2, the probability of loss is smaller at higher appreciation rates. For example, when the annuity level is at \$1600, the probability of loss for the private provider is 0.586 when α is 5%. But at a higher appreciation rate at 6%, the probability of loss is lower at 0.476. In addition, the probability of loss for a private supplier always lies to the left of that for a public provider, implying that the public provider is able to support a reasonably higher level of life annuity compared to the private supplier. Hence the market is more efficient under a public than a private supplier.

== insert Figure 2 ==

Both Figure 2 and Table 7 show that the completeness of the RM market depends on the appreciation rates of the property (α). Table 7 shows that at $\alpha = 6\%$, the mean

monthly breakeven payout is \$2154, hence implying a high replacement ratio of 72%. This no doubt will attract a demand for RM. However, there may be no private supply as the probability of loss is 0.476. This is due to the non-discourse nature of RM. However at a lower appreciation rate of 3%, the mean monthly breakeven payout is \$879, representing a low replacement ratio of 30%. The probability of loss for the private supplier is 0.5 and 0.296 for a public provider. This is undesirable as even the public provider will suffer a loss.

== insert Table 7 ==

Figure 3 plots the simulated first breakeven month m^* at different monthly annuity levels for both the public provider and the private supplier. As can be gleaned, the higher is the appreciation rate, the higher is the breakeven month. For example, using the benchmark parameters, when $\alpha = 5\%$, the average first breakeven month for the public provider is at the 338th month compared to 511th month for the private supplier. With $\alpha = 6\%$, the first breakeven month for the public provider and the private supplier is at the 490th and 364th month, respectively.

== insert Figure 3 ==

Table 8 presents simulation results using different risk-free interest rates, 3%, 4% and 5%. As the RM supplier charges a higher interest rate, the lower is the mean breakeven payout annuity and RM becomes less adequate to finance retirement. At the

benchmark risk-free interest rate of 3%, the monthly annuity payout is \$1609 which implies a replacement ratio of 54%. However at a higher interest rate of 5%, the monthly payout is \$1067, thereby implying a mere replacement ratio of 36%.

== insert Table 8 ==

Moreover, values of the breakeven annuity are affected by various yield curves used in the simulation exercises. Table 9 tabulates the simulated mean breakeven monthly annuity payout with the yield curves generated by the Cox, Ingersoll and Ross (CIR) stochastic model, fixed yield rate (FYC) and the constant yield curve (CYC) models, respectively. In general both the CIR and FYC models give consistent values of probability of loss and the first breakeven month. However, those values generated from the CYC model deviate substantially from those generated from the CIR and FYC models. This may due to using flat rates to discount cash flows. However, the simulated values are relatively robust to interest rates generated by the CIR and FYC models.

== insert Table 9 ==

5. Conclusion

We have examined the monthly annuity payouts in terms of replacement ratios and drawn implications on the adequacy of RM in financing retirement needs. Using Monte Carlo simulations, we compute the mean present value of profit for different levels of annuity, the probability of loss and the first breakeven month. We compare the

viability of the RM under a private supplier and a public provider based on different costs of capital. Such a comparison yields policy implications on the role of HDB. Our simulation results suggest a transformed role for HDB from a public housing provider to a RM provider. This is because HDB has access to lower-cost funds, thereby enabling it more able to function as a supplier of RM in cases when the private supplier fails.

Our simulation findings indicate that although most public housing homeowners have their wealth tied up in their flats, property values are inadequate to support retirement expenditure at the 70% replacement ratio. If these home owners are willing to lower their expectation to attain at about 54% of the retirement income, then it is possible to convert their house into a stream of future income by borrowing from the public supplier. However, a replacement ratio at 54% may be moderately low compared to the recommended ratio of 70% in most of the developed countries. But there are alternative sources of retirement income in Singapore. For example, an important source is family support, particularly from the children. The living arrangement among the elderly testifies to strong family support. As can be observed from Appendix 1, less than 13% of the elderly are not living with spouse of children. Only 6.6% of the elderly live alone. In 2000, about 60% of the elderly live with their working children. This is especially so for the elderly aged 75 and over. About 50% of these old-old live with the children only.

Our study is not without caveats. For instance, we have not factored in the tenure of HDB flats. As most HDB flats are on lease for 99 years, finance companies may not

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²² In Singapore, there is a Parents Maintenance Bill, whereby children are legally obliged to care for their elderly parents.

be willing to take on a flat with less than 60 years left on the tenure. A more viable financing option is for home owners to down-grade to smaller flats and then to convert the financial gains from their existing flats into some forms of annuity instruments. In fact, in March 1998, HDB introduced the Studio Apartment Scheme, whereby the elderly flat owners could sell off their flats in the resale market and use part of the proceeds to buy a smaller studio apartment from HDB.²³ The remaining fund may be used to top up their medical savings account to ensure that the elderly have adequate funds to meet their medical needs or it may be invested in annuities which yield regular monthly income. Furthermore, since October 2003, many HDB-imposed restrictions on public housing were lifted, thereby allowing flat owners to monetise their assets to sublet the entire flats.

Another caveat is that we have excluded other social dimensions in this paper which could possibly affect the success of RM. These factors include bequest motives and the psychological reluctance among the elderly to mortgage their homes to financial institutions. Such social norms are likely to prevail so that even with a public provider, the RM market may remain thin. These issues are left for future investigation.

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²³ The studio apartments are smaller, which come in two sizes - 35 to 45 sq m. They are specially equipped with elderly-friendly features and are sold on a 30-year lease.

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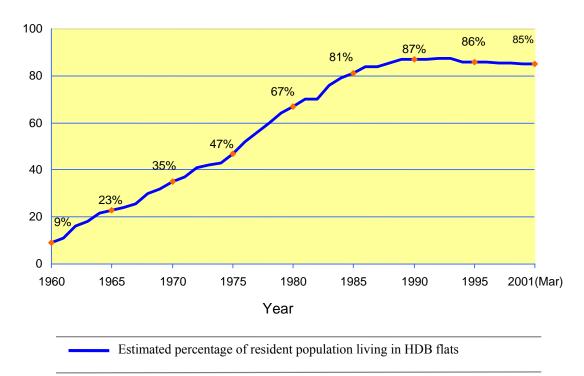
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Figure 1
Percentage of population housed in HDB flats



Note: Data from 1960 to 1979 refer to Total Population (including both residents and non-residents. From 1980 onwards, the data refer to Resident Population only (i.e. Singaporeans and permanent residents) and exclude non-residents.

Source: HDB, Research & Planning Department.

Table 1 Household residential property asset ratios in 2000

	Housing Assets/ Total Assets (%)	Housing Assets/ Personal Disposable Income (%)	Housing Assets/ GDP (%)
Singapore	51	452	230
United States	28	155	113
Japan	40	294	198
France	47	271	176
United Kingdom	39	292	197

Source: Singapore, Department of Statistics (2003) Wealth and Liabilities of Singapore Household Occasional Paper on Economic Statistics.

Table 2
Sample of price range of flats offered by HDB and
Average valuation of resale flat in 2000

Town	HDB list price (\$)	Resale price(\$)
Sembawang	98,000 - 162,000	204,000
Jurong West	99,000 - 56,000	186,100
Woodlands	104,000 - 151,000	192,500
Choa Chu Kang	110,000 - 162,000	217,200
Bukit Panjang	110,000 - 166,000	187,400
Seng Kang	120,000 – 186,000	228,100

Source: Singapore, Housing Development Board, Annual Report. 2000/01 and HDB Average Valuation by town and flat types, available at: http://www.hdb.gov.sg

Table 3
Profile of household by housing types in Singapore

	3-room HDB	4-room HDB	5-room HDB	Exec HDB	Private
Average Floor Area (sq ft)	550-900	750-1100	1200-1500	1500-1900	1300-2000
Average Household Income per annum (\$)	42,200	52,900	77,200	95,100	149,700

Source: Adapted from: http://www.mof.gov.sg/budget/budget 2002/budget2002 p4 19 a10.html

Note: Household incomes are estimated based on the 1997/98 Department of Statistics (DOS) Household Expenditure Survey (HES) and adjusted to 2001 levels.

Table 4 Values of parameters for the benchmark scenario

- (i) Age of mortgagor at the first monthly annuity payout = 62
- (ii) Typical mortgagor is a 4-room HDB dweller Average pre-retirement household income = \$3,719 Median pre-retirement household income = \$3,000
- (iii) Initial appraised property value of the 4-room flat (P_0) = \$240,000 Annual growth rate of the property (α) = 5% Initiation fee (λ) = 1% of the appraised value of the property Closing fee (τ) = 3.5% of the selling price of the property
- (iv) Risk-free interest rate or cost of capital (r) = 3 %Cost of capital to RM private supplier (y) = r + 1%Cost of capital to RM buyer (i) = r + 2%
- (v) Cox-Ingersoll-Ross model of stochastic interest rates: Initial interest rate = 3%Average interest rate (r_a) = 3%

Table 5
Present value of profit, probability of loss for private RM suppliers using benchmark parameters

Level of	Breakeven	Probability	Present value of profit (P			'VP)	
Annuity month (m*)	month	of loss	Mean	Standard deviation	5 th percentile	95 th percentile	
900	528	0.0000	56679	821	55328	58014	
1000	528	0.0000	62924	961	61476	64460	
1100	528	0.0000	66105	2571	66751	68097	
1200	528	0.0000	60292	4433	63485	62726	
1300	528	0.0000	48439	5173	53181	51088	
1400	528	0.0000	33868	5386	39319	36362	
1500	478	0.0367	18061	5404	23942	20352	
1520	455	0.0871	14783	5423	8279	29965	
1540	429	0.1742	11507	5414	5064	26660	
1560	403	0.2911	8209	5403	1828	23332	
1580	375	0.4203	4892	5391	-1431	19986	
1600	350	0.5374	1578	5353	-4709	3594	
1700	273	0.8024	-15322	5270	-8726	-13603	
1800	233	0.8747	-32486	5172	-25544	-31085	
1900	206	0.9121	-49818	5070	-42520	-48766	
2000	186	0.9350	-67265	4972	-59586	-66578	
2100	170	0.9495	-84793	4879	-76707	-84481	
2200	157	0.9595	-102377	4794	-93876	-102455	
2300	146	0.9665	-120004	4720	-111086	-120474	
2400	136	0.9716	-137664	4656	-128326	-138529	
2500	128	0.9756	-155348	4604	-145585	-156613	

Table 6 Probability of loss and first breakeven month under private and public RM suppliers using different initial property values

	Initial property value					
	\$ 220K	\$ 240K	\$ 260K	\$ 280K	\$ 300K	
Mean payout of breakeven annuity	1475	1609	1743	1876	2011	
Standard deviation	5010	5472	5894	6260	6675	
5 th percentile	-8106	-9091	-9752	-10233	-11274	
95 th percentile	8368	8913	9448	9791	10887	
Probability of loss Private supplier Public provider	0.581 0.004	0.586 0.004	0.585 0.004	0.584 0.004	0.586 0.003	
Breakeven month Private supplier Public provider	339 511	338 511	338 511	338 511	338 514	
Replacement ratio ^a (3000 per month)	49%	54%	58%	63%	67%	
Replacement ratio ^b (\$3719 per month)	40%	43%	47%	50%	54%	

Note:

a.

Replacement ratio based on the median monthly income of HDB households. Replacement ratio based on the average monthly income of HDB households. b.

Table 7
Probability of loss and first breakeven month under private and public RM suppliers using different property appreciation values

	Rates of property appreciation (α)				
	3%	4%	5%	6%	
Mean payout of breakeven annuity	879	1194	1609	2154	
Standard deviation	2714	3814	5472	7321	
5 th percentile	-4595	-6066	-9091	-12202	
95 th percentile	4534	6087	8913	11919	
Probability of loss Private supplier Public provider Breakeven month Private supplier Public provider	0.500 0.296 358 401	0.535 0.154 350 435	0.586 0.004 338 511	0.476 0.019 364 490	
Replacement ratio ^a (\$3000 per month)	30%	40%	54%	72%	
Replacement ratio ^b (\$3719 per month)	24%	32%	43%	58%	

Note:

a. Replacement ratio based on the median monthly income of HDB households.

b. Replacement ratio based on the average monthly income of HDB households.

Table 8 Sensitivity of simulation results to alternative risk-free interest rates

		Interest rate (r)
	3%	4%	5%
Mean breakeven annuity	1609	1316	1067
Standard deviation	5472	4615	3714
5 th percentile	-9091	-7356	-5841
95 th percentile	8913	7670	6275
Probability of loss			
Private supplier	0.586	0.545	0.512
Public provider	0.004	0.048	0.149
Breakeven month			
Private supplier	338	348	355
Public provider	511	471	436
Replacement ratio ^a (Y=\$3000 per month)	54%	44%	36%
Replacement ratio ^b (Y=\$3719 per month)	43%	35%	29%

a. Replacement ratio based on the median monthly income of HDB households.b. Replacement ratio based on the average monthly income of HDB households

Table 9
Sensitivity of simulation results to different interest rate models

Interest rate	Private supplier		Publi	c provider
models	m*	Prob (loss)	<i>m</i> *	Prob (loss)
CIR	358	0.500	401	0.296
FYC	360	0.492	405	0.280
CYC	297	0.735	320	0.657
CIR	350	0.535	435	0.154
FYC	352	0.527	440	0.135
CYC	272	0.801	296	0.738
CIR	338	0.586	511	0.004
FYC	330	0.619	526	0.000
CYC	233	0.874	254	0.838
CIR	364	0.476	490	0.019
				0.000
CYC	181	0.940	193	0.927
	CIR FYC CYC CIR FYC CYC CIR FYC CYC CIR FYC CYC CIR FYC CYC	models m* CIR 358 FYC 360 CYC 297 CIR 350 FYC 352 CYC 272 CIR 338 FYC 330 CYC 233 CIR 364 FYC 367	models m* Prob (loss) CIR 358 0.500 FYC 360 0.492 CYC 297 0.735 CIR 350 0.535 FYC 352 0.527 CYC 272 0.801 CIR 338 0.586 FYC 330 0.619 CYC 233 0.874 CIR 364 0.476 FYC 367 0.458	models m* Prob (loss) m* CIR 358 0.500 401 FYC 360 0.492 405 CYC 297 0.735 320 CIR 350 0.535 435 FYC 352 0.527 440 CYC 272 0.801 296 CIR 338 0.586 511 FYC 330 0.619 526 CYC 233 0.874 254 CIR 364 0.476 490 FYC 367 0.458 528

Note:

The mean first breakeven month at the corresponding breakeven annuity are computed using three different yield curve models. CIR represents the yield curve generated by the Cox, Ingersoll and Ross model; CYC is the flat 4% yield curve and FYC is the fixed yield curve based on the mean spot rates of government bonds with various durations. A is the mean level monthly payout, α is the property appreciation rates.

Appendix 1 Living arrangements of elderly Singaporeans in 2000

	Total	65-74	75 & over
Living with spouse	50.4	58.5	35.0
No children in household	13.9	15.8	10.3
With working children in household	33.1	38.9	22.2
With non-working children in household	3.4	3.9	2.4
Living with Children only	37.2	29.8	51.4
With working children in household	33.2	27.6	43.8
With non-working children in household	4.1	2.2	7.6
Not living with spouse or children	12.3	11.7	13.6
Alone	6.6	6.5	6.9
With other elderly persons only	1.2	1.3	1.1
Others	4.5	3.9	5.7

Source: Singapore, Department of Statistics (2002), Singapore Census of Population, 2000, Advance Data Release No. 6, Households and Housing.