Direction-of-Change Forecasts Based on Conditional Variance, Skewness and Kurtosis Dynamics: International Evidence

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Abstract: Recent theoretical work has revealed a direct connection between asset return *volatility* forecastability and asset return *sign* forecastability. This suggests that the pervasive volatility forecastability in equity returns could, via induced sign forecastability, be used to produce direction-of-change forecasts useful for market timing. We attempt to do so in an international sample of developed equity markets, with some success, as assessed by formal probability forecast scoring rules such as the Brier score. An important ingredient is our conditioning not only on conditional mean and variance information, but also conditional skewness and kurtosis information, when forming direction-of-change forecasts.

JEL Codes: G10, G12

Keywords: Volatility, variance, skewness, kurtosis, market timing, asset management, asset allocation, portfolio management.

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1. Introduction

Recent work by Christoffersen and Diebold (2004) has revealed a direct connection between asset return *volatility* dependence and asset return *sign* dependence (and hence sign forecastability). This suggests that the pervasive volatility dependence in equity returns could, via induced sign dependence, be used to produce direction-of-change forecasts useful for market timing.

To see this, let R_t be a series of returns, and Ω_t be the information set available at time t. $\Pr[R_t > 0]$ is the probability of a positive return at time t. The conditional mean and variance are denoted, respectively, as $\mu_{t+1|t} = \mathrm{E}[R_{t+1} | \Omega_t]$ and $\sigma^2_{t+1|t} = \mathrm{Var}[R_{t+1} | \Omega_t]$. The return series is said to display conditional mean predictability if $\mu_{t+1|t}$ varies with Ω_t; conditional variance predictability is defined similarly. If $\Pr[R_t > 0]$ exhibits conditional dependence, i.e., $\Pr[R_{t+1} > 0 | \Omega_t]$ varies with Ω_t, then we say the return series is sign predictable (or the price series is direction-of-change predictable).

For clearer exposition, and to emphasize the role of volatility in return sign predictability, suppose that there is no conditional mean predictability in returns, so $\mu_{t+1|t} = \mu$ for all *t*. In contrast, suppose that $\sigma^2_{t+1|t}$ varies with t in a predictable manner, in keeping with the huge literature on volatility predictability reviewed in Andersen, Bollerslev, Christoffersen and Diebold (2005). Denoting $D(\mu, \sigma^2)$ as a generic distribution dependent only on its mean μ and variance σ^2, assume

$$R_{t+1} | \Omega_t \sim D(\mu, \sigma^2_{t+1|t})$$

Then the conditional probability of positive return is

$$\begin{aligned}\Pr(R_{t+1} > 0 | \Omega_t) &= 1 - \Pr(R_{t+1} \leq 0 | \Omega_t) \\ &= 1 - \Pr\left(\frac{R_{t+1} - \mu}{\sigma_{t+1|t}} \leq \frac{-\mu}{\sigma_{t+1|t}}\right) \\ &= 1 - F\left(\frac{-\mu}{\sigma_{t+1|t}}\right) \qquad (1)\end{aligned}$$

where F is the distribution function of the "standardized" return $(R_{t+1}-\mu)/\sigma_{t+1|t}$. If conditional volatility is predictable, then the sign of the return is predictable even if conditional mean is unpredictable, provided $\mu \neq 0$.

In practice, interaction between mean and volatility can weaken or strengthen the link between conditional volatility predictability and return sign predictability. For instance, time-variation in conditional means of the sort documented in recent work by Brandt and Kang (2004) and Lettau and Ludvigson (2005) would strengthen our results. Interaction between volatility and higher-ordered conditional moments can similarly affect the potency of conditional volatility as a predictor of return signs.

In this paper, we use

$$\Pr(R_{t+1} > 0 \mid \Omega_t) = 1 - F\left(\frac{-\mu_{t+1|t}}{\sigma_{t+1|t}}\right) \quad (2)$$

to explore the sign predictability of one-, two-, and three-month returns in twenty stock markets, in which we examine out-of-sample predictive performance. We also use an extended version of (2) that explicitly considers the interaction between volatility and higher-ordered conditional moments. We estimate the parameters of the models recursively, and we evaluate the performance of sign probability forecasts. We proceed as follows. In Section 2, we discuss our data and its use for the construction of volatility forecasts. In Section 3, we discuss our direction-of-change forecasting models and evaluation methods. In Section 4 we present our empirical results, and in Section 5 we conclude.

2. Data and Volatility Forecasts

Estimates and forecasts of realized volatility are central to our analysis; for background see Andersen, Bollerslev, Diebold and Labys (2003) and Andersen, Bollerslev, Christoffersen and Diebold (2005). Daily values for the period 1980:01 through 2004:06 of the MSCI index for Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Singapore, Sweden, Switzerland, UK, USA, as well as MSCI Europe, Far East, and World indexes, were collected from Datastream. From these, we constructed one-, two-, and three-month returns and realized volatility. The latter is computed as the sum of squared daily returns within each one-, two-, and three-month period. We use data from 1980:01 to 1993:12 as the starting estimation sample, which will be recursively expanded as more data becomes available. We reserve the period 1994:01 to 2004:06 for our forecasting application.

Table 1 summarizes some descriptive statistics of the return and the log of the square root of realized volatility (hereafter "log realized volatility") for the two markets. All markets have low positive mean returns for the period. With a few exceptions, returns have negative skewness and are leptokurtic at all three frequencies. The p-values of the Jarque-Bera statistics indicate non-normality of all returns series except Denmark at all frequencies, and Japan, Sweden, and Far East for two-month returns. Log realized volatility is positively skewed in all except Austria, Denmark, and Japan, and are slightly leptokurtic in most markets. As with the returns series, the p-values of the Jarque-Bera statistics indicate non-normality of most of the volatility series, the main exceptions being Austria, Denmark, Japan, and Far East.

Figure 1 presents the plots of the log realized volatilities for three of the twenty markets, namely Hong Kong, UK, and US. There appears to be some clustering of return volatility. Predictability of volatility is indicated by the corresponding correlograms. As we move from the monthly frequency to the quarterly frequency, the autocorrelations diminish, but still indicate predictability. These comments extend to the other seventeen series. The correlograms (none of which are reported here) of the return series of the twenty indexes show that they are all serially uncorrelated.

Our method for forecasting the probability of positive returns will require forecasts of volatility, which we discuss here. We use the data from 1980:01 through 1993:12 as the base estimation sample. Out-of-sample one-step ahead forecasts are generated for the period 1994:01 through 2004:06, with recursive updating of parameter estimates, i.e., a volatility forecast for period $t+1$ made at time t will use a model estimated over the period 1980:1 to t. In addition, we also choose our models recursively: at each period we select ARMA models for log-volatility by minimizing either the AIC or the SIC.

Broadly speaking, both criteria tend to choose AR(1) and ARMA(1,1) models, with SIC favoring the former, especially at the quarterly frequency, and AIC favoring the latter, particular at the monthly frequency. In Figure 2 we display the volatility forecasts (with actual log realized volatilities included for comparison) for the Hong Kong, UK, and US markets. Both the forecasts generated by the AIC and the SIC criteria produce fairly similar forecasts, and both track actual log realized volatility fairly reasonably. This is also true for the other seventeen markets. The ratios of the mean square prediction errors (MSPEs) to the sample variance of log realized volatility are given in Table 2. For all markets, the performance of the AIC and SIC models are very similar, and no one criterion outperforms the other systematically. More than half of all the ratios reported are 0.6 or less, so the forecasts capture a substantial amount of the variation in actual log realized volatility. Because the probability forecasts generated by both criteria, and the corresponding evaluation results, are very similar, we will report results only for the models selected by the AIC.

A comment on our notation: throughout the paper we use $\hat{\sigma}_t$ to represent the square root of realized volatility. The symbol $\hat{\sigma}_{t+1|t}$ will represent the period t forecast of the square root of period t + 1 realized volatility. Note also that our volatility forecasting models use (and forecast) the log of these objects, so that $\hat{\sigma}_{t+1|t}$ actually represents the exponent of the forecasts of (log) realized volatility. Finally, for simplicity of notation, we will also write $\Pr[R_{t+1|t} > 0]$ for $\Pr[R_{t+1} > 0 \mid \Omega_t]$.

3. Forecasting Models and Evaluation Methods

Forecasting Models

We will evaluate the forecasting performance of two sets of forecasts, and compare their performance against forecasts from a baseline model. Our baseline forecasts are generated using the empirical cdf of the R_t using data from the beginning of our sample period right up to the time the forecast is made, i.e., at period k, we compute

$$\widehat{\Pr}(R_{k+1|k} > 0) = 1/k \sum\nolimits_{t=1}^{k} I(R_t > 0) \tag{3}$$

where $I(\cdot)$ is the indicator function.

Our first forecasting model makes direct use of (2). Using all available data at time k, we first regress R_t on a constant, $\log(\hat{\sigma}_t)$, and $[\log(\hat{\sigma}_t)]^2$, and compute

$$\hat{\mu}_t = \hat{\beta}_0 + \hat{\beta}_1 \log(\hat{\sigma}_t) + \hat{\beta}_2 [\log(\hat{\sigma}_t)]^2, \; t = 1,...,k \tag{4}$$

where $\hat{\sigma}_t$ is the square root of (actual, not forecasted) realized volatility . The period $k+1$ forecast is then generated by

$$\widehat{\Pr}(R_{k+1|k} > 0) = 1 - \hat{F}\left(-\hat{\mu}_{k+1|k} / \hat{\sigma}_{k+1|k}\right)$$
$$= 1 - 1/k \sum_{t=1}^{k} I\left(\frac{R_t - \hat{\mu}_t}{\hat{\sigma}_t} \leq -\frac{\hat{\mu}_{k+1|k}}{\hat{\sigma}_{k+1|k}}\right) \quad (5)$$

i.e., $\widehat{F}$ is the empirical cdf of $(R_t - \hat{\mu}_t)/\hat{\sigma}_t$. The one-step ahead volatility forecast $\hat{\sigma}_{t+1|t}$ is generated from a recursively estimated model selected, at each period, by minimizing AIC or SIC, as described in the previous section. The one-step ahead mean forecast $\hat{\mu}_{t+1|t}$ is computed as

$$\hat{\mu}_{t+1|t} = \hat{\beta}_0 + \hat{\beta}_1 \log(\hat{\sigma}_{t+1|t}) + \hat{\beta}_2[\log(\hat{\sigma}_{t+1|t})]^2 . \quad (6)$$

The coefficients $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are recursively estimated using (4). A quadratic specification is used as the quadratic term is significant for almost all series in the starting estimation sample. Although the coefficients are recursively estimated, at each recursion no attempt is made to refine the model. We refer to forecasts from (5) as nonparametric forecasts (even though the realized volatility forecasts are generated using fully parametric models) to differentiate it from forecasts from our next model.

The second model is an extension of (1), and explicitly considers the interaction between volatility, skewness and kurtosis. This is done by using the Gram-Charlier expansion:

$$1 - F\left(\frac{-\mu_{t+1|t}}{\sigma_{t+1|t}}\right) \approx 1 - \Phi\left(\frac{-\mu_{t+1|t}}{\sigma_{t+1|t}}\right)$$
$$+\Phi\left(\frac{-\mu_{t+1|t}}{\sigma_{t+1|t}}\right)\left(\frac{\gamma_{3,t+1|t}}{3!}\left[\frac{\mu^2_{t+1|t}}{\sigma^2_{t+1|t}} - 1\right] + \frac{\gamma_{4,t+1|t}}{4!}\left[\frac{-\mu^3_{t+1|t}}{\sigma^3_{t+1|t}} + \frac{3\mu_{t+1|t}}{\sigma_{t+1|t}}\right]\right)$$

where $\Phi(\cdot)$ is the distribution function of a standard normal, and γ_3 and γ_4 are, respectively, the skewness and excess kurtosis, with the usual notation for conditioning on Ω_t. This equation can be rewritten as

$$1 - F(-\mu_{t+1|t} x_{t+1}) \approx 1 - \Phi(-\mu_{t+1|t} x_{t+1})(\beta_{0t} + \beta_{1t} x_{t+1} + \beta_{2t} x^2_{t+1} + \beta_{3t} x^3_{t+1})$$

with $\beta_{0t} = 1 + \gamma_{3,t+1|t}/6$, $\beta_{1t} = -\gamma_{4,t+1|t}\mu_{t+1|t}/8$, $\beta_{2t} = -\gamma_{3,t+1|t}\mu^2_{t+1|t}/6$, and $\beta_{3t} = \gamma_{4,t+1|t}\mu^3_{t+1|t}/24$, where for notational convenience, we denote $x_{t+1} = 1/\sigma_{t+1|t}$,

Several points should be noted. The sign of returns is predictable for nonzero $\mu_{t+1|t}$ even when there is no volatility clustering, as long as the skewness and kurtosis are time varying. On the other hand, even if $\mu_{t+1|t}$ is zero, sign predictability arises as long as conditional skewness dynamics is present, regardless of whether volatility dynamics is present. If there is no conditional skewness and excess kurtosis, the above equation is reduced to

$$1 - F(-\mu_{t+1|t}x_{t+1}) \approx 1 - \Phi(-\mu_{t+1|t}x_{t+1})$$

so that normal approximation applies. If returns are conditionally symmetric but leptokurtic (i.e., $\gamma_{3,t+1|t} = 0$ and $\gamma_{4,t+1|t} > 0$), then $\beta_{0t} = 1$ and $\beta_{2t} = 0$, and we have

$$1 - F(-\mu_{t+1|t}x_{t+1}) \approx 1 - \Phi(-\mu_{t+1|t}x_{t+1})(1 + \beta_{1t}x_{t+1} + \beta_{3t}x^3_{t+1}) .$$

Furthermore, if $\mu_{t+1|t} > 0$, we have $\beta_{1t} < 0$ and $\beta_{3t} > 0$; and the converse is true for $\mu_{t+1|t} < 0$. Finally, if $\mu_{t+1|t}$ is small, as in the case of short investment horizons, then β_{2t} and β_{3t} can safely be ignored, resulting in

$$1 - F(-\mu_{t+1|t}x_{t+1}) \approx 1 - \Phi(-\mu_{t+1|t}x_{t+1})(\beta_{0t} + \beta_{1t}x_{t+1}) .$$

Thus, conditional skewness affects sign predictability through β_{0t}, and conditional kurtosis affects sign predictability through β_{1t}. When there is no conditional dynamics in skewness and kurtosis, the above equation is reduced to

$$1 - F\left(-\mu_{t+1|t}x_{t+1}\right) \simeq 1 - \Phi\left(-\mu_{t+1|t}x_{t+1}\right)\left(\beta_0 + \beta_1 x_{t+1}\right) \tag{7}$$

for some time-invariant quantities β_0 and β_1.

We use equation (7) as our second model for sign prediction, i.e., we generate forecasts of the probability of positive returns as

$$\widehat{\Pr}(R_{t+1|t} > 0) = 1 - \Phi(-\hat{\mu}_{t+1|t}\hat{x}_{t+1})(\hat{\beta}_0 + \hat{\beta}_1\hat{x}_{t+1}) \tag{8}$$

where $\hat{x}_{t+1|t} = 1/\hat{\sigma}_{t+1|t}$, and where $\hat{\mu}_{t+1|t}$ and $\hat{\sigma}_{t=t+1|t}$ are as defined earlier. We refer to these as forecasts from the "extended" model. The parameters β_0 and β_1 are estimated by regressing $1 - I(R_t > 0)$ on $\Phi(-\hat{\mu}_t\hat{x}_t)$ and $\Phi(-\hat{\mu}_t\hat{x}_t)\hat{x}_t$ for t = 1, …, *k*. Although we have not explicitly placed any constraints on this model to require $\Phi(-\hat{\mu}_t\hat{x}_t)(\hat{\beta}_0 + \hat{\beta}_1\hat{x}_t)$ to lie between 0 and 1, this was inconsequential as all our predicted probabilities turn out to lie between 0 and 1.

Forecast Evaluation

We perform post-sample comparison of the forecast performance of (5) and (8) for the sign of return. Both are compared against baseline forecasts (3). This is done for one-month, two-month, and three-month returns. We assess the performance of the forecasting models using Brier scores; for background see Diebold and Lopez (1996).

Two Brier scores are used:

$$\text{Brier(Abs)} = \frac{1}{T-k}\sum_{t=k}^{T} |\widehat{\Pr}(R_{t+1|t} > 0) - z_{t+1}|$$

$$\text{Brier(Sq)} = \frac{1}{T-k}\sum_{t=k}^{T} 2(\widehat{\Pr}(R_{t+1|t} > 0) - z_{t+1})^2$$

where $z_{t+1} = I(R_{t+1} > 0)$. The latter is the traditional Brier score for evaluating the performance of probability forecasts, and is analogous to the usual mean square prediction error. A score of zero for Brier(Sq) occurs when perfect forecasts are made: where at each period correct probability forecasts of 0 or 1 are made. The worst score is two and occurs if at each period probability forecasts of 0 or 1 are made, but turn out to be wrong each time. Note that if we follow the usual convention where a correct probability forecast of $I(R_{t+1} > 0)$ is one that is greater than 0.5, then correct forecasts will have an individual Brier(Sq) score between 0 and 0.5, whereas incorrect forecasts have individual scores between 0.5 and 2. A few incorrect forecasts can therefore dominate a majority of correct forecasts.

For this reason, we also consider a modified version of the Brier score, which we call Brier(Abs). Like Brier(Sq), the best possible score for Brier(Abs) is 0. The worst score is one. Here correct forecasts have individual scores between 0 and 0.5, whereas incorrect forecasts carry scores between 0.5 and 1.

4. Empirical Results

Figures 3a, 3b and 3c show, for the Hong Kong, UK, and US markets, the predicted probabilities generated by the baseline model, the nonparametric model, and the extended model (columns 1, 2, and 3 respectively) for the one-month, two-month, and quarterly frequencies (rows 1, 2, and 3 respectively). For the nonparametric and extended models, forecasts based on both AIC and SIC volatility forecasts are plotted, although visually these are mostly indistinguishable. For all three markets, the baseline forecasts are very flat, at values slightly above 0.5. The nonparametric and extended forecasts show more variability, especially in the later periods. The figures for the other seventeen markets show similar characteristics.

Before reporting our main results, we highlight some interesting regularities in the Brier scores. In Table 3 we report the mean and standard deviation of the Brier(Abs) scores from the AIC based probability forecasts for the Hong Kong, UK, and US markets. Results are reported for three "subperiods". The 'low volatility subperiod' comprises all dates for which realized volatility falls in the 1st to 33rd percentile range. The 'medium' and 'high' volatility subperiods comprise all dates for which realized volatility falls in the 34th to 66th, and 67th to 100th percentile ranges. In all three markets, the

Brier score for the low volatility period is lower than the corresponding Brier score for the high volatility period. In contrast, the standard deviation of the Brier scores for the nonparametric and extended models are higher in the low volatility periods than in the high volatility periods. For instance, the mean Brier score for the extended model in the US market at the one-month frequency is 0.378 for the low volatility period and 0.532 in the high volatility period. The standard deviation of the same Brier scores fall from 0.143 in the low volatility period, to 0.088 in the high volatility period. The patterns described here generalize to all markets in our study (except Japan), regardless of whether AIC or SIC volatility forecasts are used in generating probability forecasts, and regardless of whether Brier(Abs) or Brier(Sqr) is used to evaluate the forecasts. These findings seem perfectly reasonable: we should expect our models to have more to say in periods of low volatility, and little to say in periods of high volatility. In high volatility periods, the models tend to generate probability forecasts that are close to 0.5. The corresponding Brier scores in turn tend to be close to 0.5, resulting in the lower standard deviation of Brier scores in high volatility periods.

Our main results are reported in Tables 4a to 4d. Table 4a contains our results for the full forecast period. Tables 4b to 4d contain the results for the low, medium, and high-volatility subperiods. In all four tables, both Brier(Abs) and Brier(Sq) are given for the baseline model. The Brier scores for the nonparametric and extended models are expressed relative to the baseline Brier scores. A relative measure of less than 1 therefore implies improvement in forecast performance.

Table 4a shows improvement in the performance of the nonparametric or extended models over the baseline forecasts in just under half of all the cases considered, when using Brier(Abs) as a measure of performance. All of the improvements, however, are very small. The best performance is for Sweden at the two-month frequency. The situation is worse when the forecasts are evaluated using Brier(Sq) instead.

The fact that the nonparametric and extended models perform better during low volatility periods than during high volatility periods suggests that their performance relative to the baseline model might be better during low volatility periods than during high volatility periods. This is verified by the relative performances reported in Tables 4b to 4d. In Table 4b, there is improvement in all except two markets (Japan and Far East, which is heavily weighted on Japan). Although in some cases the improvements are small, there are many instances where the improvement is substantial. In a number of cases, the ratio of the Brier(Abs) scores for the extended / parametric models to the baseline model is less than 0.9. In the case of Denmark at the quarterly frequency, the ratio is 0.772 for the extended model. When Brier(Sq) is used to measure forecast performance, there are fewer instances where the nonparametric and extended models perform better than the baseline. The notable differences between the Brier(Sq) scores and Brier(Abs) scores occur for France, Hong Kong, and Singapore, where Brier(Sq) shows no improvements from the nonparametric and extended models. Nonetheless, over half of all the ratios reported under Brier(Sq) are less than one, and a number are less than 0.9.

The ratios under Brier(Abs) also show that in a few cases, the extended model performs much better than the nonparametric model. The notable case is Denmark at the quarterly frequency, where the ratio for the nonparametric model is 0.937, whereas the ratio for the extended model is 0.772. Other instances include Belgium, France, Germany, Netherlands, UK, and USA at the quarterly frequency. Interestingly, the largest improvements by the extended model over the nonparametric model tend to occur at the quarterly frequency. Using Brier(Sq), however, the nonparametric and extended models appear to perform similarly.

We note also that for both Brier(Abs) and Brier(Sq), the performance of the nonparametric and extended models, relative to the baseline, is better at the quarterly frequency than at the monthly frequency. This occurs in just under half of the markets for Brier(Abs), and just over a third of the markets for Brier(Sq). This is to be expected, as the theory indicates that volatility-aided prediction depends on a sizable mean return, and the mean return increases in all markets as we go from monthly to quarterly frequencies. Finally, we note that the worst performances by the nonparametric and extended models occur for Japan, Singapore, and the Far East, and these are among the markets in our sample with the lowest mean returns over the period under consideration.

The ratios in Tables 4c and 4d are qualitatively the same as Table 4a. In the medium volatility period only Sweden and Australia show any improvements in the performance of the nonparametric and extended models, using Brier(Abs). As expected, the performance of the nonparametric and extended models relative to baseline is even worse during the high volatility period. It appears that volatility in these periods is simply too large relative to the mean to be useful in guiding direction-of-change forecasts.

Figures 4a and 4b show for the Hong Kong, UK, and US markets a clear picture of the forecast performance of the nonparametric and extended models compared to the baseline forecasts. At each frequency, we show a scatterplot of the Brier(Abs) scores of individual forecasts. We include only observations when volatility is low, as previously defined. In both figures the horizontal axis measures the Brier(Abs) scores for individual baseline forecasts. In Figure 4a, the vertical axis measures the Brier(Abs) scores for individual nonparametric forecasts. In Figure 4b, the vertical axis measures the Brier(Abs) scores for individual forecasts from the extended model. We plot only the Brier scores for probability forecasts that use the AIC-based volatility forecasts. In addition to the scatterplots, we include a horizontal and vertical gridline at 0.5, and a 45 degree line. As a Brier(Abs) score below 0.5 indicates a correct forecast, points in the lower left quadrant indicate that both competing forecasts are correct, whereas a point in the lower right quadrant indicates that the baseline forecast for this observation is incorrect, with the competing forecast correct. Points below the 45 degree line indicate improvements in the Brier(Abs) scores over the baseline.

In all three markets illustrated here, the nonparametric and extended models clearly provide better signals than the baseline model when both the baseline and the competing forecasts are correct. However, for Hong Kong and UK, the performance of the nonparametric and extended model is worse than the baseline model when the baseline and the competing forecasts are wrong. Note that the upper left and lower right quadrants of Figure 4a and 4b are mostly empty, which implies that the models by and large make predictions that are similar to the baseline forecasts. Nonetheless, there is evidence that when volatility is low, forecasts of volatility can improve the quality of the signal, in the sense of providing probability forecasts with improved Brier scores. All of these remarks hold in general for the other seventeen markets.

5. Summary and Directions for Future Research

Methodologically, we have extended the Christoffersen-Diebold (2004) direction-of-change forecasting framework to include the potentially important effects of higher-ordered conditional moments. Empirically, in an application to an international sample of equity markets, we have verified the importance of allowing for higher-ordered conditional moments and taken a step toward evaluating the real-time predictive performance. In future work, we look forward to using our direction-of-change forecasts to formulate and evaluate actual trading strategies, and to exploring their relationships to the "volatility timing" strategies recently studied by Fleming, Kirby and Ostdiek (2003), in which portfolio shares are dynamically adjusted based on forecasts of the variance-covariance matrix of the underlying assets.

References

Andersen, T.G., Bollerslev, T., Christoffersen, P.F. and Diebold, F.X. (2005), "Volatility Forecasting," in G. Elliott, C.W.J. Granger and A. Timmermann (eds.), *Handbook of Economic Forecasting*. Amsterdam: North-Holland, forthcoming.

Andersen, T.G., Bollerslev, T. Diebold, F.X. and Labys, P. (2003), "Modeling and Forecasting Realized Volatility," *Econometrica*, 71, 579-626.

Brandt, M.W. and Kang, Q. (2004), "On the Relationship Between the Conditional Mean and Volatility of Stock Returns: A Latent VAR Approach," *Journal of Financial Economics*, 72, 217-257.

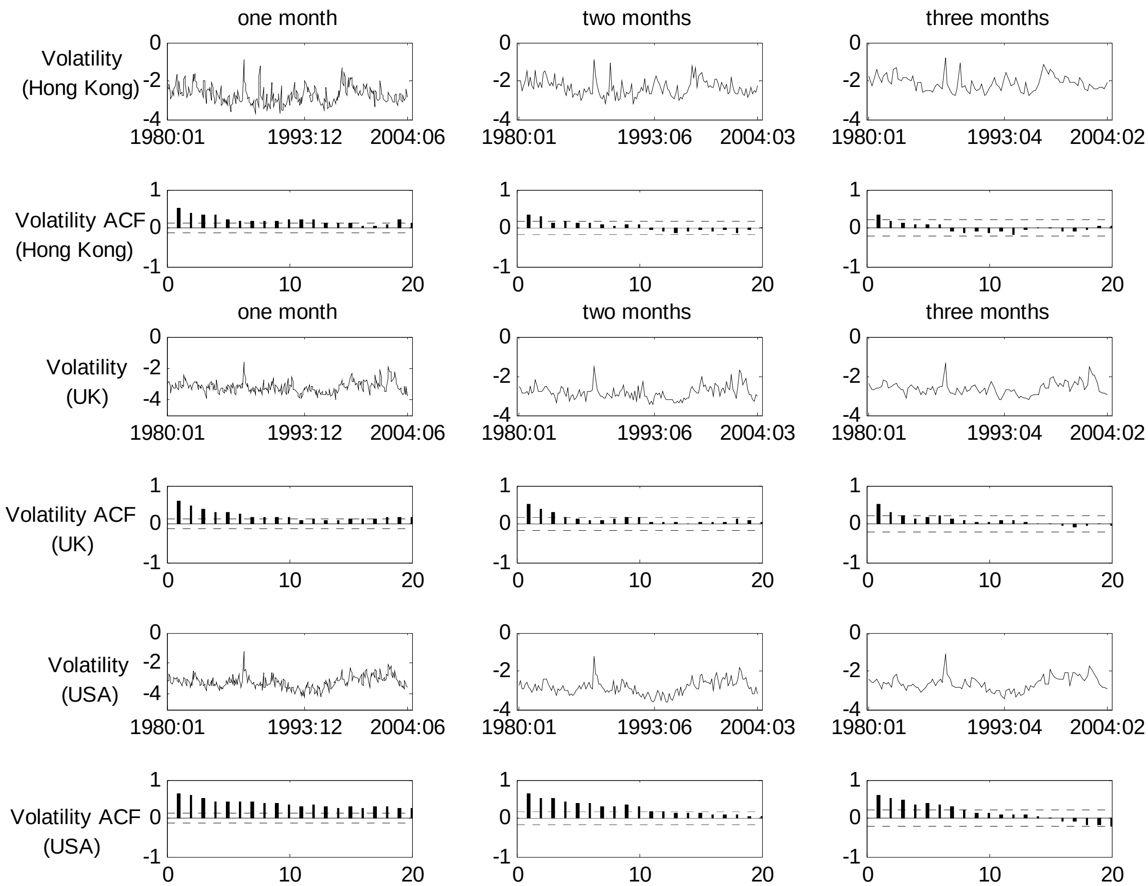
Christoffersen, P.F. and Diebold, F.X. (2004), "Financial Asset Returns, Direction-of-Change Forecasting, and Volatility Dynamics," *Management Science*, forthcoming.

Diebold, F.X. and Lopez, J. (1996), "Forecast Evaluation and Combination," in G.S. Maddala and C.R. Rao (eds.), *Handbook of Statistics*, 241-268. Amsterdam: North-Holland.

Fleming, J., Kirby, C. and Ostdiek, B. (2003), "The Economic Value of Volatility Timing Using Realized Volatility," *Journal of Financial Economics*, 67, 473-509.

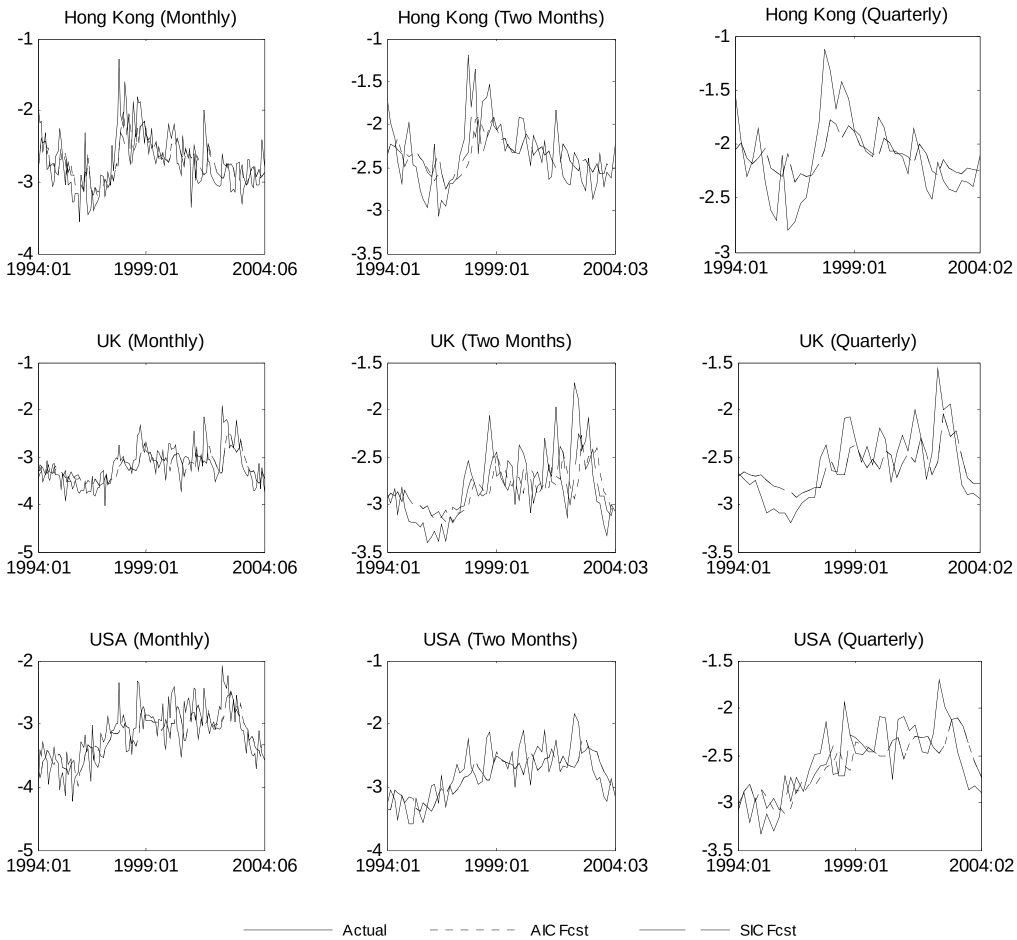
Lettau, M. and Ludvigson, S. (2005), "Measuring and Modeling Variation in the Risk-Return Tradeoff," in Y. Ait-Shalia and L.P. Hansen (eds.), *Handbook of Financial Econometrics*, forthcoming.

Figure 1 Realized Volatility



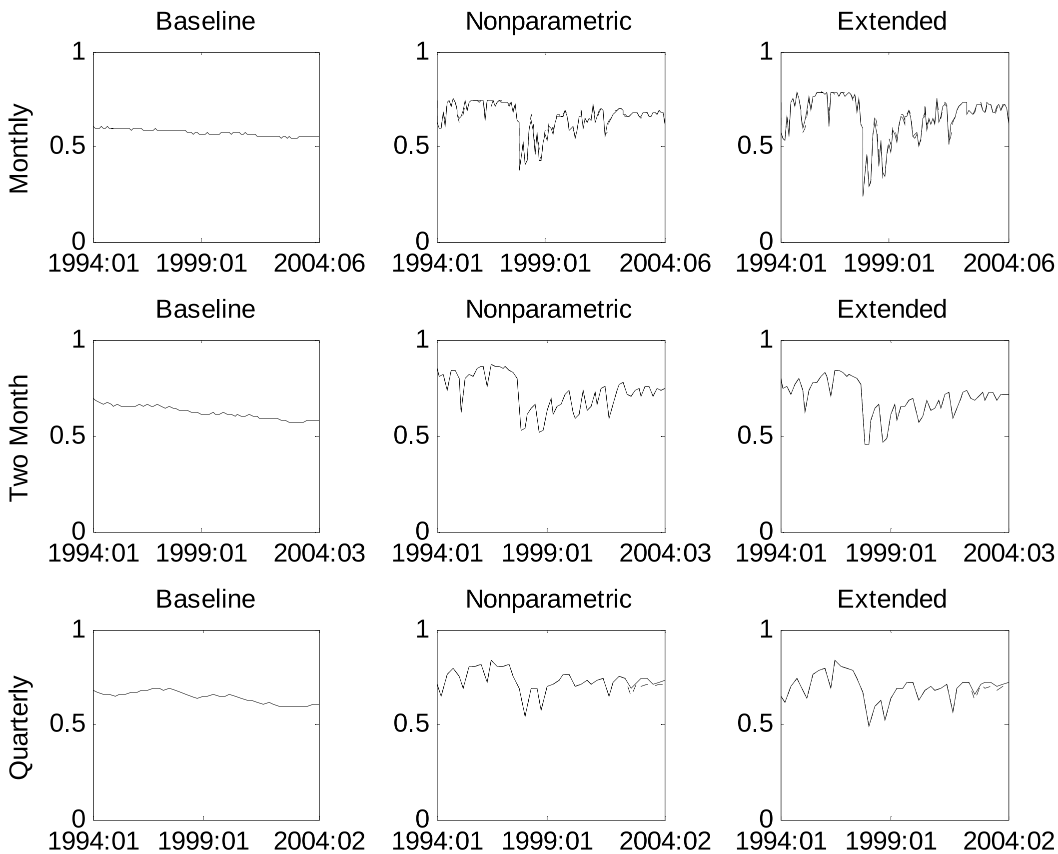
Notes: "Volatility" refers to the log of the square root of realized volatility constructed from daily returns.

Figure 2   Realized Volatility and Recursive Realized Volatility Forecasts



Notes: "Volatility" refers to log of the square root of realized volatility constructed from daily returns. "AIC Forecasts" and "SIC Forecasts" are one-step ahead forecasts of volatility generated from recursively estimated ARMA models chosen (recursively) using the AIC and SIC criteria, respectively.

Figure 3a Predicted Probabilities (Hong Kong)



Notes: "Nonparametric" forecasts (second column) refer to forecasts generated using

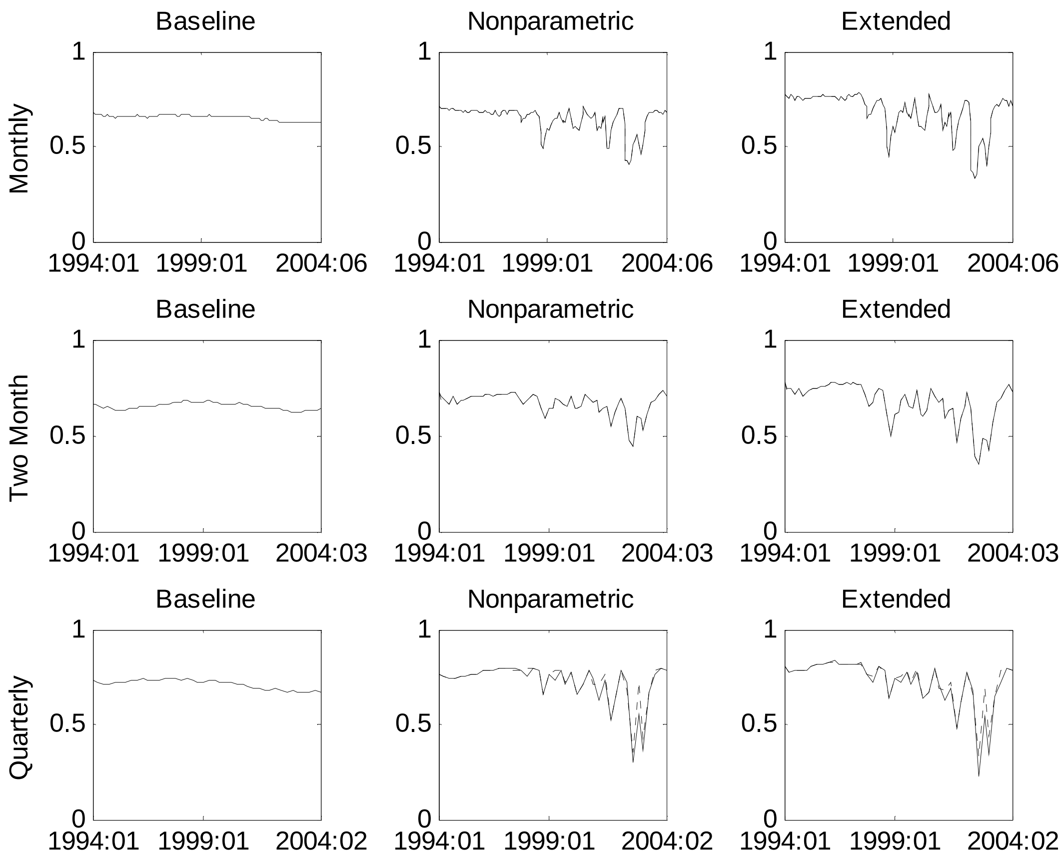
$$\widehat{\Pr}(R_{t+1} > 0) = 1 - \hat{F}\left(-\hat{\mu}_{t+1|t} / \hat{\sigma}_{t+1|t}\right)$$

where $\widehat{F}$ is the empirical cdf of $(R_t - \hat{\mu}_t)/\hat{\sigma}_t$. "Extended" (third column) refers to forecasts generated from the extended model

$$\widehat{\Pr}(R_{t+1} > 0) = 1 - \Phi\left(-\hat{\mu}_t \hat{x}_{t+1}\right)\left(\hat{\beta}_0 + \hat{\beta}_1 \hat{x}_{t+1}\right).$$

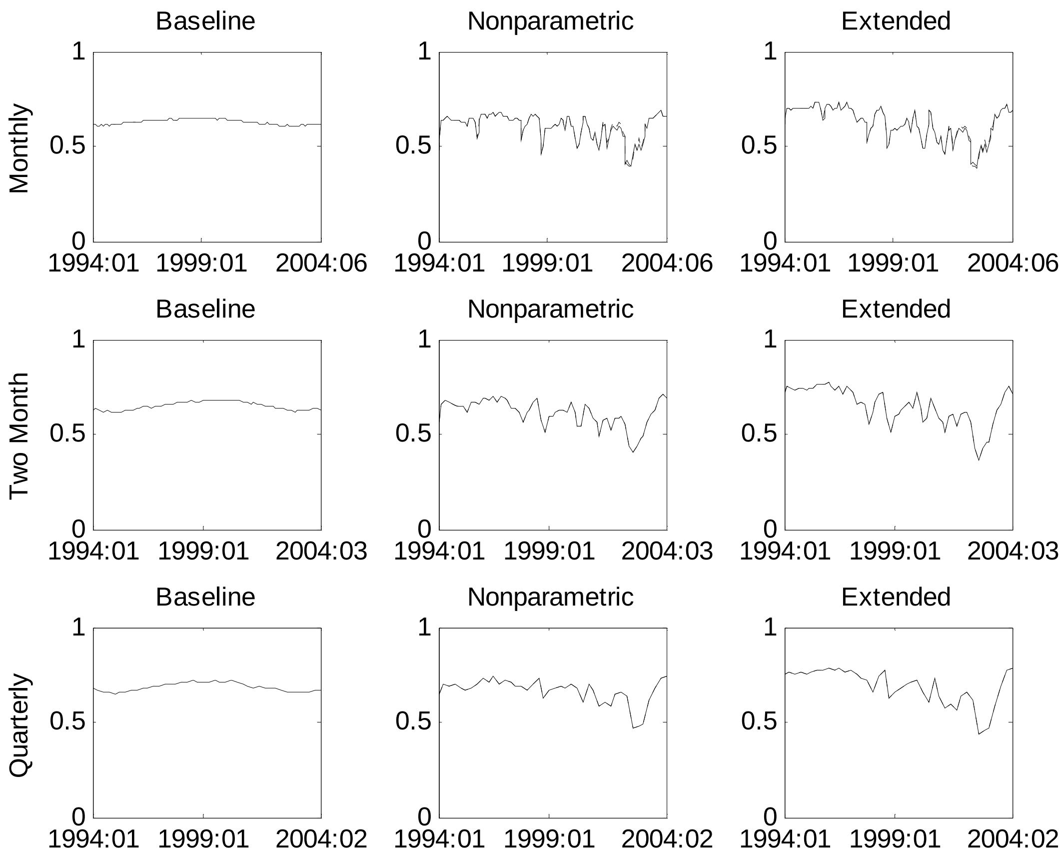
In both columns, the forecasts generated using AIC and SIC volatility forecasts are given (solid line vs. dotted line respectively), although in some cases these are visually indistinguishable.

Figure 3b Predicted Probabilities (UK)



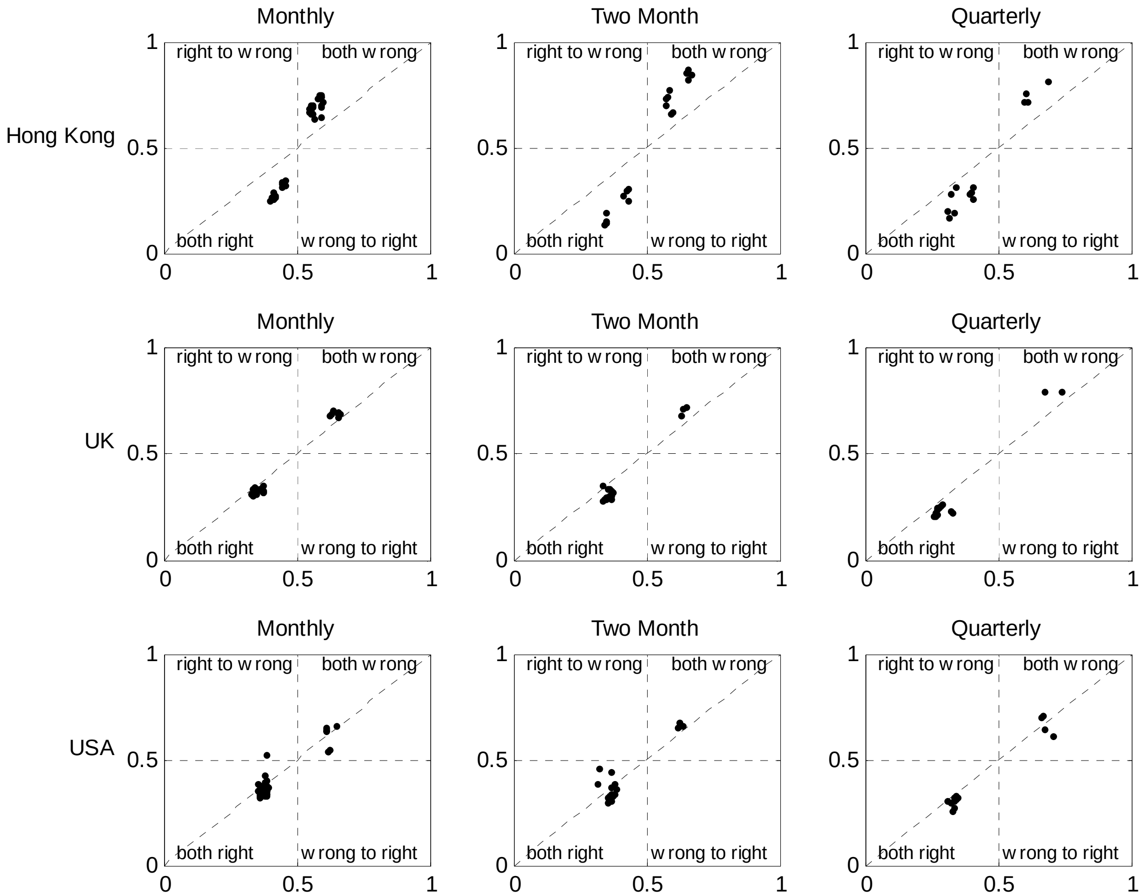
Notes: See notes to Figure 3a.

Figure 3c   Predicted Probabilities (USA)



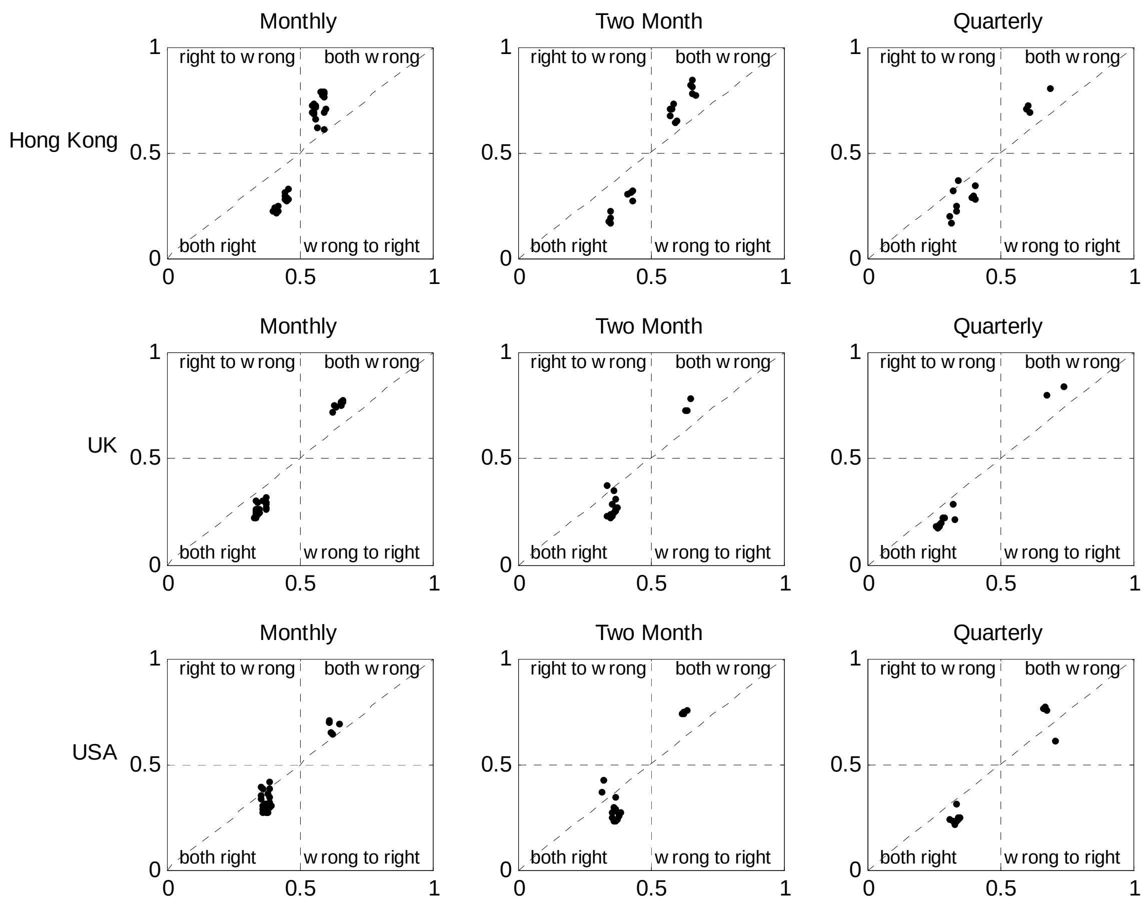
Notes: See notes to Figure 3a.

Table 4a Comparative Brier (Abs) Scores, Low Volatility (Nonparametric vs. Baseline)



Notes: Horizontal axis measures individual Brier(Abs) scores for baseline forecasts. Vertical axis measures corresponding Brier(Abs) scores for the nonparametric forecasts. A score below 0.5 indicates a correct forecast. Only observations with volatility in the 1st to 33rd percentile range are included.

Figure 4b   Comparative Brier (Abs) Scores, Low Volatility (Extended Model vs. Baseline)



Notes: Horizontal axis measures individual Brier(Abs) scores for baseline forecasts. Vertical axis measures corresponding Brier(Abs) scores for the extended forecasts. A score below 0.5 indicates a correct forecast. Only observations with volatility in the 1st to 33rd percentile range are included.

Table 1a Summary Statistics of the Full Sample of Returns, 1980:01 – 2004:06

	Mean	Std.Dev.	Skewness	Kurtosis	JB p-val	Mean	Std.Dev.	Skewness	Kurtosis	JB p-val
			Australia					*Austria*		
1 mth	0.007	0.058	–2.837	28.000	0.000	0.005	0.062	–0.065	6.171	0.000
2 mth	0.013	0.083	–1.466	11.523	0.000	0.010	0.087	0.635	5.177	0.000
3 mth	0.020	0.096	–1.602	10.104	0.000	0.016	0.119	0.073	6.369	0.000
			Belgium					*Canada*		
1 mth	0.007	0.054	–0.450	6.787	0.000	0.006	0.050	–0.935	6.852	0.000
2 mth	0.014	0.079	–0.272	5.975	0.000	0.011	0.073	–0.829	5.615	0.000
3 mth	0.020	0.107	–0.593	5.006	0.000	0.017	0.090	–0.713	4.191	0.001
			Denmark					*France*		
1 mth	0.009	0.055	–0.287	3.147	0.124	0.008	0.060	–0.626	4.341	0.000
2 mth	0.019	0.073	–0.083	2.981	0.911	0.016	0.084	–0.604	4.331	0.000
3 mth	0.028	0.103	–0.433	3.143	0.224	0.024	0.119	–0.988	5.173	0.000
			Germany					*Hong Kong*		
1 mth	0.006	0.063	–0.901	5.812	0.000	0.008	0.091	–1.029	8.902	0.000
2 mth	0.012	0.082	–0.496	3.885	0.006	0.016	0.125	–0.422	5.046	0.000
3 mth	0.018	0.126	–1.361	6.300	0.000	0.025	0.164	–0.684	3.712	0.011
			Italy					*Japan*		
1 mth	0.010	0.072	0.216	3.628	0.035	0.004	0.055	–0.284	4.211	0.000
2 mth	0.020	0.101	0.449	3.498	0.048	0.007	0.077	0.166	2.938	0.696
3 mth	0.030	0.134	0.262	4.160	0.055	0.011	0.108	–0.875	4.474	0.000
			Netherlands					*Norway*		
1 mth	0.008	0.054	–0.929	5.915	0.000	0.006	0.073	–0.932	5.718	0.000
2 mth	0.015	0.069	–0.774	4.869	0.000	0.012	0.101	–0.438	3.277	0.085
3 mth	0.023	0.101	–1.594	6.976	0.000	0.018	0.138	–1.303	6.082	0.000
			Singapore					*Sweden*		
1 mth	0.004	0.076	–1.457	12.166	0.000	0.014	0.073	–0.278	4.392	0.000
2 mth	0.008	0.114	–1.011	7.532	0.000	0.027	0.099	0.264	3.432	0.275
3 mth	0.011	0.145	–0.719	4.599	0.000	0.041	0.149	–0.606	3.920	0.013
			Switzerland					*UK*		
1 mth	0.007	0.050	–1.101	6.829	0.000	0.008	0.049	–1.228	8.236	0.000
2 mth	0.014	0.067	–0.543	3.773	0.006	0.015	0.064	–0.611	4.186	0.000
3 mth	0.021	0.103	–1.215	6.881	0.000	0.023	0.085	–1.047	5.254	0.000
			USA					*Europe*		
1 mth	0.008	0.045	–0.841	6.124	0.000	0.008	0.047	–1.366	7.559	0.000
2 mth	0.016	0.058	–0.884	5.875	0.000	0.015	0.061	–0.918	4.844	0.000
3 mth	0.024	0.082	–0.799	4.237	0.000	0.023	0.092	–1.461	6.628	0.000
			Far East					*World*		
1 mth	0.004	0.053	–0.389	4.287	0.000	0.007	0.041	–1.140	6.541	0.000
2 mth	0.007	0.074	0.126	3.007	0.825	0.013	0.054	–0.893	4.913	0.000
3 mth	0.011	0.105	–0.915	4.543	0.000	0.020	0.083	–1.189	5.298	0.000

Notes: Returns are per time interval (one month, two months or one quarter, not annualized). 'JB p-val' refers to the p-values of the Jarque-Bera statistic.

Table 1b Summary Statistics of the Full Sample of Realized Volatility, 1980:01 – 2004:06

	Mean	Std.Dev.	Skewness	Kurtosis	JB p-val	Mean	Std.Dev.	Skewness	Kurtosis	JB p-val
			Australia					*Austria*		
1 mth	–3.241	0.354	0.814	6.961	0.000	–3.435	0.601	–0.256	2.967	0.200
2 mth	–2.860	0.322	1.103	7.759	0.000	–3.041	0.566	–0.331	2.863	0.244
3 mth	–2.648	0.302	1.408	9.920	0.000	–2.819	0.545	–0.280	2.708	0.420
			Belgium					*Canada*		
1 mth	–3.357	0.485	0.697	3.564	0.000	–3.358	0.460	0.616	3.230	0.000
2 mth	–2.970	0.453	0.763	3.494	0.001	–2.976	0.432	0.699	2.920	0.003
3 mth	–2.740	0.431	0.735	3.567	0.008	–2.766	0.428	0.664	2.830	0.028
			Denmark					*France*		
1 mth	–3.207	0.403	–0.034	3.362	0.482	–3.064	0.408	0.593	3.718	0.000
2 mth	–2.827	0.360	–0.043	3.418	0.642	–2.687	0.376	0.713	3.470	0.001
3 mth	–2.598	0.329	–0.204	3.036	0.717	–2.470	0.362	0.709	3.451	0.014
			Germany					*Hong Kong*		
1 mth	–3.074	0.493	0.449	2.916	0.007	–2.737	0.451	0.682	3.835	0.000
2 mth	–2.695	0.470	0.404	2.620	0.083	–2.357	0.427	0.719	3.659	0.001
3 mth	–2.475	0.454	0.404	2.544	0.163	–2.126	0.405	0.727	3.437	0.011
			Italy					*Japan*		
1 mth	–2.923	0.401	0.392	3.128	0.023	–3.136	0.484	–0.076	3.057	0.861
2 mth	–2.547	0.370	0.356	2.876	0.201	–2.753	0.455	–0.223	3.057	0.549
3 mth	–2.331	0.359	0.267	2.590	0.367	–2.517	0.417	–0.239	2.758	0.531
			Netherlands					*Norway*		
1 mth	–3.123	0.455	0.635	3.764	0.000	–2.939	0.376	0.855	4.627	0.000
2 mth	–2.745	0.425	0.768	3.655	0.000	–2.550	0.333	1.077	4.707	0.000
3 mth	–2.523	0.406	0.830	3.567	0.003	–2.331	0.307	1.249	5.896	0.000
			Singapore					*Sweden*		
1 mth	–3.074	0.459	0.607	3.925	0.000	–2.949	0.452	0.319	2.869	0.074
2 mth	–2.683	0.428	0.599	3.889	0.002	–2.567	0.416	0.319	2.645	0.185
3 mth	–2.451	0.398	0.703	4.012	0.003	–2.352	0.402	0.348	2.670	0.283
			Switzerland					*UK*		
1 mth	–3.314	0.483	0.537	3.763	0.000	–3.213	0.362	0.821	4.677	0.000
2 mth	–2.928	0.462	0.433	3.438	0.066	–2.843	0.341	0.909	4.496	0.000
3 mth	–2.703	0.439	0.500	3.444	0.104	–2.626	0.323	0.936	4.799	0.000
			USA					*Europe*		
1 mth	–3.226	0.400	0.566	4.619	0.000	–3.434	0.468	0.836	3.621	0.000
2 mth	–2.851	0.377	0.658	4.476	0.000	–3.055	0.448	0.807	3.388	0.000
3 mth	–2.637	0.367	0.679	4.523	0.000	–2.835	0.436	0.789	3.335	0.006
			Far East					*World*		
1 mth	–3.184	0.468	0.128	2.854	0.569	–3.539	0.416	0.718	3.896	0.000
2 mth	–2.801	0.440	0.015	2.679	0.675	–3.164	0.394	0.766	3.623	0.000
3 mth	–2.566	0.404	–0.027	2.522	0.562	–2.947	0.379	0.794	3.727	0.003

Notes. "Volatility" refers to log of the square root of realized volatility computed from daily returns. 'JB p-val' refers to the p-values of the Jarque-Bera statistic.

Table 2 Ratio of MSPE of Forecasts to Sample Variance, Realized Volatility

	AIC Forecast	SIC Forecast	AIC Forecast	SIC Forecast
	Australia		*Austria*	
1 mth	0.626	0.626	0.306	0.305
2 mth	0.656	0.656	0.276	0.274
3 mth	0.791	0.791	0.359	0.338
	Belgium		*Canada*	
1 mth	0.574	0.602	0.497	0.491
2 mth	0.785	0.751	0.522	0.521
3 mth	0.696	0.730	0.552	0.547
	Denmark		*France*	
1 mth	0.546	0.547	0.500	0.479
2 mth	0.656	0.651	0.687	0.642
3 mth	0.756	0.757	0.938	0.931
	Germany		*Hong Kong*	
1 mth	0.436	0.436	0.479	0.486
2 mth	0.477	0.483	0.632	0.631
3 mth	0.505	0.478	0.575	0.586
	Italy		*Japan*	
1 mth	0.592	0.592	0.435	0.437
2 mth	0.665	0.665	0.405	0.405
3 mth	0.686	0.656	0.481	0.512
	Netherlands		*Norway*	
1 mth	0.531	0.544	0.716	0.716
2 mth	0.741	0.739	0.712	0.769
3 mth	0.695	0.721	0.756	0.742
	Singapore		*Sweden*	
1 mth	0.558	0.554	0.448	0.426
2 mth	0.613	0.615	0.516	0.531
3 mth	0.579	0.617	0.657	0.694
	Switzerland		*UK*	
1 mth	0.548	0.547	0.648	0.648
2 mth	0.604	0.604	0.756	0.757
3 mth	0.566	0.570	0.855	0.909
	USA		*Europe*	
1 mth	0.537	0.531	0.515	0.518
2 mth	0.592	0.592	0.675	0.691
3 mth	0.563	0.563	0.710	0.757
	Far East		*World*	
1 mth	0.459	0.464	0.491	0.502
2 mth	0.430	0.430	0.565	0.565
3 mth	0.474	0.479	0.557	0.587

Notes: AIC and SIC indicate the criterion used to recursively select the model for forecasting $\sigma_{t+1|t}$. Numbers reported are the ratio of the Mean Square Prediction Error (MSPE) of realized volatility to the sample variance of realized volatility.

Table 3 Forecast Performance, Brier(Abs), Selected Markets

		Baseline		Nonparametric		Extended	
		Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
				Hong Kong			
	Low Vol.	0.507	0.075	0.510	0.208	0.504	0.239
1 mth	Med.Vol.	0.491	0.070	0.475	0.168	0.475	0.182
	High Vol.	0.520	0.076	0.537	0.137	0.535	0.151
	Low Vol.	0.512	0.126	0.518	0.298	0.512	0.265
2 mth	Med.Vol.	0.487	0.121	0.487	0.230	0.488	0.203
	High Vol.	0.579	0.107	0.640	0.173	0.622	0.145
	Low Vol.	0.435	0.132	0.390	0.241	0.402	0.223
3 mth	Med.Vol.	0.515	0.144	0.526	0.256	0.524	0.222
	High Vol.	0.551	0.154	0.548	0.202	0.534	0.159
				UK			
	Low Vol.	0.430	0.137	0.418	0.164	0.388	0.224
1 mth	Med.Vol.	0.507	0.157	0.516	0.165	0.519	0.215
	High Vol.	0.500	0.151	0.522	0.123	0.531	0.153
	Low Vol.	0.398	0.106	0.362	0.147	0.333	0.181
2 mth	Med.Vol.	0.511	0.160	0.526	0.178	0.546	0.193
	High Vol.	0.517	0.160	0.514	0.154	0.518	0.158
	Low Vol.	0.343	0.156	0.308	0.204	0.288	0.226
3 mth	Med.Vol.	0.470	0.220	0.484	0.247	0.491	0.254
	High Vol.	0.566	0.201	0.638	0.173	0.638	0.162
				USA			
	Low Vol.	0.419	0.093	0.404	0.104	0.378	0.143
1 mth	Med.Vol.	0.468	0.129	0.488	0.127	0.489	0.144
	High Vol.	0.525	0.130	0.536	0.088	0.532	0.088
	Low Vol.	0.417	0.109	0.412	0.134	0.375	0.196
2 mth	Med.Vol.	0.451	0.149	0.466	0.125	0.461	0.157
	High Vol.	0.518	0.165	0.499	0.107	0.502	0.131
	Low Vol.	0.433	0.162	0.405	0.172	0.379	0.231
3 mth	Med.Vol.	0.366	0.143	0.380	0.139	0.364	0.158
	High Vol.	0.524	0.199	0.529	0.158	0.528	0.165

Notes: Brier(Abs) = $1/T\sum_{t=k}^{T}|\,p_{t+1|t}-z_{t+1}\,|$ where k is the start of the estimation sample, $p_{t+1|t}$ is the one-step ahead forecast of $\Pr(R_{t+1}>0)$ made at time t, and $z_{t+1}=I(R_{t+1}>0)$. At each time t, data from the first period up to time t is used to estimate the forecasting model. "Baseline" refers to forecasts generated from the unconditional empirical distribution of R_t. "Nonparametric" refers to forecasts generated using $\widehat{\Pr}(R_{t+1|t}>0)$ = $1-\hat{F}\left(-\hat{\mu}_{t+1|t}/\hat{\sigma}_{t+1|t}\right)$ where $\hat{F}$ is the empirical cdf of $(R_t-\hat{\mu}_t)/\hat{\sigma}_t$. "Extended" refers to forecasts generated from $\widehat{\Pr}(R_{t+1|t}>0)$ = $1-\Phi\left(-\hat{\mu}_{t+1|t}\hat{x}_{t+1}\right)\left(\hat{\beta}_0+\hat{\beta}_1\hat{x}_{t+1}\right)$.

Table 4a Relative Forecast Performance (Full Sample)

	Brier(Abs)						Brier(Sq)					
	Bsln	Npar	Ext	Bsln	Npar	Ext	Bsln	Npar	Ext	Bsln	Npar	Ext
		Australia			*Austria*			*Australia*			*Austria*	
1 mth	0.484	**0.995**	**0.978**	0.501	**0.992**	**0.994**	0.489	1.038	1.066	0.501	1.000	1.002
2 mth	0.496	**0.999**	**0.997**	0.506	**0.976**	**0.981**	0.509	1.040	1.076	0.516	**0.952**	**0.963**
3 mth	0.477	**0.993**	**0.985**	0.497	1.001	1.026	0.496	1.051	1.023	0.497	1.052	1.133
		Belgium			*Canada*			*Belgium*			*Canada*	
1 mth	0.486	**0.993**	**0.995**	0.491	**0.988**	**0.978**	0.482	1.008	1.033	0.488	1.002	1.038
2 mth	0.477	1.013	1.009	0.472	**0.982**	**0.968**	0.479	1.043	1.084	0.472	1.014	1.029
3 mth	0.477	1.026	**0.998**	0.457	**0.983**	**0.985**	0.491	1.052	1.148	0.452	1.036	1.059
		Denmark			*France*			*Denmark*			*France*	
1 mth	0.482	1.012	1.007	0.483	1.017	1.017	0.489	1.013	1.024	0.485	1.038	1.073
2 mth	0.464	1.007	**0.969**	0.490	**0.996**	**0.997**	0.482	**0.989**	1.002	0.500	1.034	1.083
3 mth	0.438	**0.990**	**0.925**	0.435	1.076	1.014	0.434	**0.973**	**0.957**	0.456	1.144	1.165
		Germany			*Hong Kong*			*Germany*			*Hong Kong*	
1 mth	0.489	1.010	1.017	0.506	1.005	1.001	0.493	1.058	1.102	0.524	1.102	1.123
2 mth	0.491	**0.997**	**0.994**	0.526	1.043	1.028	0.504	1.028	1.091	0.584	1.231	1.158
3 mth	0.469	1.022	**0.984**	0.500	**0.975**	**0.973**	0.479	1.093	1.088	0.544	1.081	1.027
		Italy			*Japan*			*Italy*			*Japan*	
1 mth	0.504	1.015	1.037	0.502	1.002	1.004	0.512	1.051	1.124	0.514	**0.998**	1.003
2 mth	0.482	**0.993**	**0.992**	0.509	1.017	1.009	0.486	1.011	1.016	0.520	1.044	1.051
3 mth	0.494	1.002	1.020	0.497	1.080	1.084	0.506	1.040	1.121	0.548	1.095	1.155
		Netherlands			*Norway*			*Netherlands*			*Norway*	
1 mth	0.478	1.023	1.023	0.490	**0.989**	**0.973**	0.479	1.068	1.106	0.493	1.016	1.032
2 mth	0.479	1.003	**0.998**	0.496	**0.984**	**0.974**	0.513	1.023	1.036	0.497	1.000	1.009
3 mth	0.437	1.089	1.042	0.462	**0.973**	**0.961**	0.451	1.175	1.152	0.455	1.014	1.034
		Singapore			*Sweden*			*Singapore*			*Sweden*	
1 mth	0.501	1.016	1.020	0.479	**0.986**	**0.970**	0.506	1.073	1.129	0.482	**0.973**	**0.961**
2 mth	0.514	1.033	1.055	0.469	**0.939**	**0.927**	0.539	1.112	1.207	0.479	**0.916**	**0.894**
3 mth	0.524	1.019	1.049	0.463	**0.958**	**0.947**	0.569	1.051	1.172	0.482	**0.992**	1.057
		Switzerland			*UK*			*Switzerland*			*UK*	
1 mth	0.473	1.016	1.027	0.482	1.013	1.003	0.472	1.047	1.119	0.510	1.031	1.084
2 mth	0.479	1.014	1.017	0.477	**0.984**	**0.981**	0.498	1.040	1.089	0.500	**0.999**	1.031
3 mth	0.465	1.031	1.044	0.460	1.036	1.027	0.482	1.114	1.205	0.510	1.124	1.129
		USA			*Europe*			*USA*			*Europe*	
1 mth	0.469	1.011	**0.991**	0.458	1.053	1.036	0.471	1.015	1.003	0.492	1.037	1.068
2 mth	0.462	**0.994**	**0.967**	0.479	1.008	1.029	0.470	**0.964**	**0.969**	0.525	1.009	1.030
3 mth	0.441	**0.993**	**0.961**	0.405	1.103	1.081	0.450	**0.972**	**0.966**	0.440	1.122	1.102
		Far East			*World*			*Far East*			*World*	
1 mth	0.501	1.008	1.012	0.474	**0.999**	**0.991**	0.516	1.006	1.017	0.501	**0.996**	1.021
2 mth	0.512	1.009	**0.999**	0.475	**0.989**	**0.994**	0.529	1.032	1.030	0.501	**0.991**	1.009
3 mth	0.504	1.050	1.052	0.424	1.026	1.007	0.569	1.055	1.100	0.458	1.027	1.043

Notes: Brier(Abs) = $1/T\sum_{t=k}^{T}|\,p_{t+1|t} - z_{t+1}\,|$ and Brier(Sq) = $1/T\sum_{t=k}^{T}2(p_{t+1|t} - z_{t+1})^2$ where k is the start of the estimation sample, $p_{t+1|t}$ is the one-step ahead forecast of $\Pr(R_{t+1} > 0)$ made at time t, and $z_{t+1} = I(R_{t+1} > 0)$. At each time t, data from the first period up to time t is used to estimate the forecasting model. "Bsln" refer to forecasts generated from the unconditional empirical distribution of R_t. "Npar" refers to forecasts generated using $\widehat{\Pr}(R_{t+1|t} > 0) = 1 - \hat{F}\left(-\hat{\mu}_{t+1|t}/\hat{\sigma}_{t+1|t}\right)$ where $\hat{F}$ is the empirical cdf of $(R_t - \hat{\mu}_t)/\hat{\sigma}_t$. "Ext" refers to forecasts generated from $\widehat{\Pr}(R_{t+1|t} > 0) = 1 - \Phi\left(-\hat{\mu}_{t+1|t}\hat{x}_{t+1}\right)\left(\hat{\beta}_0 + \hat{\beta}_1\hat{x}_{t+1}\right)$. Actual Brier scores are reported for the baseline forecasts. All other scores are Brier scores for the given model divided by the Brier score for the baseline forecast. Ratios below 1 are in bold print.

Table 4b   Forecast Performance, Low Volatility Periods (1st to 33rd percentile)

	Brier(Abs)						Brier(Sq)					
	Bsln	Npar	Ext	Bsln	Npar	Ext	Bsln	Npar	Ext	Bsln	Npar	Ext
	Australia			*Austria*			*Australia*			*Austria*		
1 mth	0.494	**0.994**	**0.987**	0.501	**0.988**	**0.984**	0.508	1.040	1.087	0.503	**0.997**	**0.985**
2 mth	0.486	**0.983**	**0.973**	0.517	**0.939**	**0.943**	0.487	1.011	1.025	0.540	**0.879**	**0.887**
3 mth	0.441	**0.941**	**0.941**	0.494	**0.961**	**0.971**	0.426	**0.965**	**0.957**	0.490	**0.977**	1.036
	Belgium			*Canada*			*Belgium*			*Canada*		
1 mth	0.474	**0.964**	**0.943**	0.470	**0.907**	**0.837**	0.455	**0.962**	**0.951**	0.445	**0.860**	**0.799**
2 mth	0.446	**0.968**	**0.925**	0.447	**0.882**	**0.857**	0.417	**0.972**	**0.956**	0.417	**0.879**	**0.906**
3 mth	0.446	1.023	**0.917**	0.427	**0.899**	**0.891**	0.425	1.059	1.029	0.388	**0.925**	**0.924**
	Denmark			*France*			*Denmark*			*France*		
1 mth	0.477	1.004	**0.989**	0.469	1.006	**0.991**	0.478	1.015	1.027	0.456	1.039	1.066
2 mth	0.442	1.004	**0.927**	0.474	**0.968**	**0.953**	0.428	1.017	1.001	0.466	1.003	1.039
3 mth	0.385	**0.937**	**0.772**	0.388	1.036	**0.913**	0.320	**0.912**	**0.775**	0.354	1.117	1.025
	Germany			*Hong Kong*			*Germany*			*Hong Kong*		
1 mth	0.475	**0.941**	**0.921**	0.507	1.004	**0.993**	0.464	**0.966**	**0.970**	0.526	1.148	1.178
2 mth	0.460	**0.968**	**0.953**	0.512	1.011	1.000	0.442	1.014	1.090	0.555	1.269	1.186
3 mth	0.446	**0.882**	**0.806**	0.435	**0.896**	**0.923**	0.426	**0.889**	**0.881**	0.411	1.002	1.010
	Italy			*Japan*			*Italy*			*Japan*		
1 mth	0.511	1.019	1.038	0.490	1.017	1.027	0.525	1.063	1.143	0.492	1.039	1.056
2 mth	0.484	**0.970**	**0.953**	0.509	1.042	1.045	0.488	**0.991**	**0.968**	0.522	1.100	1.124
3 mth	0.480	**0.954**	**0.922**	0.483	1.120	1.120	0.475	**0.960**	**0.956**	0.534	1.170	1.272
	Netherlands			*Norway*			*Netherlands*			*Norway*		
1 mth	0.466	**0.956**	**0.923**	0.468	**0.957**	**0.908**	0.452	**0.970**	**0.973**	0.448	**0.962**	**0.928**
2 mth	0.434	**0.948**	**0.929**	0.490	**0.968**	**0.933**	0.427	**0.981**	**0.990**	0.484	**0.970**	**0.946**
3 mth	0.422	1.017	**0.955**	0.417	**0.886**	**0.828**	0.417	1.090	1.083	0.362	**0.855**	**0.801**
	Singapore			*Sweden*			*Singapore*			*Sweden*		
1 mth	0.498	**0.995**	**0.981**	0.438	**0.965**	**0.932**	0.500	1.073	1.123	0.397	**0.956**	**0.927**
2 mth	0.508	**0.993**	**0.986**	0.408	**0.895**	**0.890**	0.531	1.090	1.178	0.348	**0.877**	**0.885**
3 mth	0.514	1.011	1.026	0.411	**0.940**	**0.903**	0.560	1.053	1.204	0.374	1.029	1.061
	Switzerland			*UK*			*Switzerland*			*UK*		
1 mth	0.448	**0.922**	**0.890**	0.430	**0.972**	**0.904**	0.420	**0.893**	**0.874**	0.406	**0.990**	**0.985**
2 mth	0.418	**0.941**	**0.926**	0.398	**0.909**	**0.836**	0.374	**0.956**	**0.963**	0.338	**0.896**	**0.839**
3 mth	0.393	**0.947**	**0.911**	0.343	**0.897**	**0.839**	0.333	1.035	1.027	0.281	**0.950**	**0.929**
	USA			*Europe*			*USA*			*Europe*		
1 mth	0.419	**0.964**	**0.903**	0.393	1.031	**0.969**	0.368	**0.944**	**0.887**	0.363	1.066	1.072
2 mth	0.417	**0.989**	**0.899**	0.414	1.000	1.053	0.370	1.011	0.956	0.397	1.050	1.152
3 mth	0.433	**0.934**	**0.875**	0.357	**0.994**	**0.990**	0.423	**0.902**	**0.911**	0.343	**0.961**	**0.977**
	Far East			*World*			*Far East*			*World*		
1 mth	0.491	1.010	1.015	0.412	**0.944**	**0.893**	0.500	1.023	1.040	0.375	**0.961**	**0.946**
2 mth	0.505	1.043	1.040	0.424	**0.940**	**0.931**	0.520	1.117	1.124	0.397	**0.999**	**1.008**
3 mth	0.484	1.061	1.073	0.367	**0.889**	**0.826**	0.546	1.076	1.178	0.346	**0.899**	**0.890**

Notes: See notes to Table 4a.

Table 4c   Forecast Performance , Medium Volatility Periods (34th to 66th percentile)

	Brier(Abs)						Brier(Sq)					
	Bsln	Npar	Ext	Bsln	Npar	Ext	Bsln	Npar	Ext	Bsln	Npar	Ext
		Australia			*Austria*			*Australia*			*Austria*	
1 mth	0.464	**0.983**	**0.945**	0.500	**0.967**	**0.981**	0.450	1.018	1.004	0.501	**0.949**	**0.972**
2 mth	0.476	**0.984**	**0.961**	0.501	1.000	1.003	0.469	1.014	1.016	0.505	1.001	1.007
3 mth	0.444	**0.979**	**0.975**	0.501	1.011	1.039	0.431	1.025	1.004	0.504	1.072	1.162
		Belgium			*Canada*			*Belgium*			*Canada*	
1 mth	0.477	**0.983**	**0.975**	0.501	1.006	1.025	0.465	**0.987**	**0.983**	0.509	1.037	1.119
2 mth	0.476	1.029	1.022	0.476	1.003	**0.989**	0.479	1.074	1.106	0.479	1.053	1.065
3 mth	0.464	1.011	**0.963**	0.461	1.000	1.005	0.468	1.021	1.070	0.463	1.079	1.112
		Denmark			*France*			*Denmark*			*France*	
1 mth	0.466	1.017	1.010	0.477	1.013	1.002	0.458	1.020	1.023	0.475	1.028	1.039
2 mth	0.460	1.021	**0.988**	0.482	**0.993**	**0.999**	0.477	1.008	1.022	0.484	1.040	1.101
3 mth	0.402	1.027	**0.962**	0.386	1.025	**0.906**	0.365	1.028	**0.991**	0.362	1.013	**0.957**
		Germany			*Hong Kong*			*Germany*			*Hong Kong*	
1 mth	0.484	1.030	1.045	0.491	**0.967**	**0.966**	0.485	1.090	1.128	0.492	1.029	1.046
2 mth	0.484	1.038	1.043	0.487	1.000	1.002	0.492	1.105	1.158	0.502	1.144	1.104
3 mth	0.437	1.164	1.108	0.515	1.022	1.019	0.419	1.370	1.324	0.568	1.187	1.129
		Italy			*Japan*			*Italy*			*Japan*	
1 mth	0.500	1.004	1.010	0.506	1.003	1.005	0.504	1.033	1.081	0.521	**0.995**	**0.999**
2 mth	0.466	1.005	1.007	0.511	1.001	0.977	0.454	1.042	1.053	0.523	1.005	0.984
3 mth	0.502	1.014	1.076	0.527	**0.997**	1.023	0.523	1.069	1.239	0.597	**0.950**	1.021
		Netherlands			*Norway*			*Netherlands*			*Norway*	
1 mth	0.495	1.031	1.043	0.486	**0.983**	**0.965**	0.514	1.078	1.132	0.483	1.007	1.023
2 mth	0.494	1.067	1.075	0.481	**0.966**	**0.950**	0.543	1.130	1.158	0.467	**0.965**	**0.961**
3 mth	0.417	1.136	1.068	0.448	**0.955**	**0.942**	0.414	1.301	1.235	0.425	**0.976**	**0.987**
		Singapore			*Sweden*			*Singapore*			*Sweden*	
1 mth	0.504	**0.991**	**0.981**	0.480	**0.972**	**0.948**	0.511	1.008	1.020	0.485	**0.956**	**0.934**
2 mth	0.518	1.036	1.068	0.437	**0.939**	**0.951**	0.544	1.096	1.192	0.416	**0.935**	**0.954**
3 mth	0.514	1.023	1.033	0.452	**0.936**	**0.902**	0.541	1.061	1.127	0.464	**0.954**	**0.929**
		Switzerland			*UK*			*Switzerland*			*UK*	
1 mth	0.458	1.045	1.054	0.507	1.017	1.023	0.443	1.090	1.136	0.563	1.040	1.116
2 mth	0.466	1.078	1.096	0.511	1.028	1.068	0.475	1.153	1.220	0.572	1.072	1.168
3 mth	0.480	1.011	1.029	0.470	1.029	1.045	0.516	1.045	1.146	0.532	1.093	1.133
		USA			*Europe*			*USA*			*Europe*	
1 mth	0.468	1.043	1.047	0.477	1.033	1.015	0.470	1.080	1.106	0.533	**0.986**	1.003
2 mth	0.451	1.032	1.023	0.513	1.023	1.043	0.449	1.031	1.052	0.595	1.027	1.030
3 mth	0.366	1.040	0.996	0.360	1.310	1.238	0.306	1.065	1.021	0.351	1.494	1.424
		Far East			*World*			*Far East*			*World*	
1 mth	0.505	1.006	1.010	0.480	1.025	1.037	0.523	**0.998**	1.004	0.514	1.043	1.084
2 mth	0.523	**0.958**	**0.920**	0.479	1.012	1.041	0.550	**0.925**	**0.869**	0.510	1.009	1.063
3 mth	0.501	1.049	1.050	0.337	1.176	1.155	0.554	1.046	1.070	0.285	1.311	1.334

Notes: See notes to Table 4a.

Table 4d Forecast Performance , High Volatility Periods (66th to 100th percentile)

	Brier(Abs)						Brier(Sq)					
	Bsln	Npar	Ext	Bsln	Npar	Ext	Bsln	Npar	Ext	Bsln	Npar	Ext
		Australia			*Austria*			*Australia*			*Austria*	
1 mth	0.487	1.000	**0.988**	0.501	1.017	1.014	0.496	1.041	1.077	0.501	1.045	1.041
2 mth	0.526	1.027	1.051	0.500	**0.989**	**0.997**	0.570	1.086	1.166	0.504	**0.978**	**0.995**
3 mth	0.545	1.045	1.028	0.497	1.031	1.069	0.632	1.126	1.081	0.497	1.104	1.200
		Belgium			*Canada*			*Belgium*			*Canada*	
1 mth	0.509	1.034	1.067	0.500	1.042	1.061	0.529	1.072	1.155	0.509	1.084	1.159
2 mth	0.507	1.036	1.066	0.493	1.049	1.045	0.539	1.068	1.159	0.517	1.081	1.089
3 mth	0.520	1.042	1.098	0.482	1.042	1.050	0.580	1.072	1.298	0.507	1.083	1.113
		Denmark			*France*			*Denmark*			*France*	
1 mth	0.506	1.014	1.020	0.500	1.030	1.054	0.540	1.002	1.019	0.522	1.041	1.105
2 mth	0.490	**0.996**	**0.988**	0.513	1.025	1.034	0.539	**0.949**	**0.985**	0.547	1.053	1.102
3 mth	0.526	1.001	1.008	0.532	1.142	1.167	0.618	**0.973**	1.032	0.651	1.231	1.356
		Germany			*Hong Kong*			*Germany*			*Hong Kong*	
1 mth	0.506	1.054	1.076	0.520	1.032	1.029	0.529	1.106	1.189	0.552	1.110	1.118
2 mth	0.527	**0.982**	**0.983**	0.579	1.105	1.075	0.576	**0.973**	1.034	0.692	1.265	1.176
3 mth	0.523	1.024	1.033	0.551	**0.995**	**0.970**	0.591	1.045	1.070	0.652	1.040	0.949
		Italy			*Japan*			*Italy*			*Japan*	
1 mth	0.501	1.021	1.056	0.510	**0.985**	**0.981**	0.507	1.053	1.139	0.528	**0.962**	**0.956**
2 mth	0.496	1.002	1.013	0.506	1.009	1.008	0.515	1.003	1.028	0.514	1.028	1.049
3 mth	0.501	1.035	1.058	0.482	1.129	1.114	0.520	1.084	1.154	0.512	1.184	1.188
		Netherlands			*Norway*			*Netherlands*			*Norway*	
1 mth	0.479	1.081	1.101	0.520	1.026	1.043	0.482	1.151	1.207	0.553	1.072	1.133
2 mth	0.505	**0.986**	**0.979**	0.517	1.017	1.035	0.566	**0.950**	**0.953**	0.540	1.057	1.104
3 mth	0.471	1.110	1.096	0.522	1.058	1.084	0.523	1.142	1.142	0.576	1.142	1.217
		Singapore			*Sweden*			*Singapore*			*Sweden*	
1 mth	0.500	1.053	1.087	0.522	1.014	1.018	0.503	1.124	1.220	0.571	**0.997**	1.002
2 mth	0.515	1.067	1.106	0.560	**0.969**	**0.933**	0.540	1.148	1.249	0.666	**0.924**	**0.860**
3 mth	0.545	1.024	1.086	0.526	**0.992**	1.019	0.605	1.041	1.182	0.609	**0.998**	1.152
		Switzerland			*UK*			*Switzerland*			*UK*	
1 mth	0.510	1.075	1.127	0.500	1.044	1.063	0.547	1.139	1.301	0.544	1.053	1.121
2 mth	0.549	1.013	1.017	0.517	**0.995**	1.002	0.638	1.003	1.062	0.583	**0.984**	1.002
3 mth	0.521	1.114	1.158	0.566	1.127	1.126	0.598	1.217	1.356	0.717	1.214	1.204
		USA			*Europe*			*USA*			*Europe*	
1 mth	0.525	1.021	1.015	0.509	1.075	1.094	0.583	1.009	**0.997**	0.593	1.046	1.106
2 mth	0.518	**0.964**	**0.969**	0.507	1.000	**0.996**	0.588	**0.885**	**0.912**	0.577	**0.964**	**0.950**
3 mth	0.524	1.009	1.008	0.498	1.031	1.034	0.622	**0.974**	**0.977**	0.626	1.000	0.990
		Far East			*World*			*Far East*			*World*	
1 mth	0.506	1.007	1.011	0.521	1.017	1.024	0.525	**0.999**	1.009	0.592	**0.977**	1.008
2 mth	0.507	1.027	1.040	0.519	1.007	**0.999**	0.518	1.064	1.110	0.590	**0.972**	**0.962**
3 mth	0.529	1.040	1.034	0.568	1.027	1.036	0.607	1.043	1.056	0.743	**0.978**	1.003

Notes: See notes to Table 4a.