

# Technology Adoption in a Differentiated Duopoly: Cournot versus Bertrand

Rupayan Pal



Indira Gandhi Institute of Development Research, Mumbai  
January 2009

<http://www.igidr.ac.in/pdf/publication/WP-2009-001.pdf>

# Technology Adoption in a Differentiated Duopoly: Cournot versus Bertrand

Rupayan Pal

Indira Gandhi Institute of Development Research (IGIDR)  
General Arun Kumar Vaidya Marg  
Goregaon (E), Mumbai- 400065, INDIA  
Email (corresponding author): [rupayan@igidr.ac.in](mailto:rupayan@igidr.ac.in)

## Abstract

*This paper compares equilibrium technology adoption in a differentiated duopoly under two alternative modes of product market competition, Cournot and Bertrand. It shows that the cost of technology has differential impact on technology adoption, that is, on cost-efficiency of the industry, under two alternative modes of product market competition. The possibility of ex post cost asymmetry between firms is higher under Bertrand competition than under Cournot competition. If the cost of technology is high, Bertrand competition leads to higher cost-efficiency than Cournot competition provided that the cost reducing effect of the technology is high. On the other hand, if the technology reduces the marginal cost of production by a very low amount, Cournot competition may lead to higher cost-efficiency than Bertrand competition.*

Key words: Differentiated duopoly, limit-pricing, price effect, selection effect, technology adoption

JEL Code(s): L13, L11, O31, D43

# Technology Adoption in a Differentiated Duopoly: Cournot versus Bertrand

Rupayan Pal

## 1 Introduction

This paper analyses the incentives to adopt cost-reducing technology by firms in a horizontally differentiated industry under two alternative categories of product market competition, Cournot and Bertrand. When the cost of production is endogenously determined, whether Bertrand or Cournot competition leads to more cost-efficiency is an essential concern. Our framework allows us to address the question of how the cost of technology affects this comparison in a more general setting that does not rely on the assumption of ‘positive primary outputs’.<sup>1</sup>

Whether higher or lower intensity of product market competition provides greater incentive to adopt cost reducing technology is of perennial interest. A large literature, dating at least as far back as Schumpeter (1943), emphasizes the role of the intensity of competition on innovation. Schumpeter (1943) argues that, since the possibility to realize returns from technological advancement is higher in concentrated markets, market concentration stimulates innovation. In contrast, Arrow (1962) shows, comparing a perfectly

---

<sup>1</sup>The assumption of positive primary outputs, that is, both firms sell positive outputs, even if prices are set at respective marginal costs, is crucial for ranking of the equilibrium outputs and profits under the two categories of competition (Zanchettin, 2006; Amir and Jin, 2001).

competitive industry with a monopoly, that the gain from adopting cost-reducing technology is higher under competitive environment. It indicates that more competitive environment provides higher incentive to innovate. Recently, the attention has turned to the comparison of two oligopolistic industries. A number of recent studies, considering different scenarios, compare firms' incentives to innovate cost-reducing technologies under alternative modes of product market competition. It helps us to understand a variety of issues: role of the nature of product differentiation (Bester and Petrakis, 1993; Bonnano and Haworth, 1998), speed of technological progress (Aghion et al., 1997) conflict between static and dynamic efficiency (Delbono and Denicolo, 1990), impact of competition intensity (Boone, 2001), incentives in mixed oligopoly (Lin and Ogawa, 2005), so on so forth. While characterising equilibrium outcomes, these studies subscribe to the assumption of 'positive primary outputs' and thus restrict the space of the parameter values, which is likely to distort equilibrium outcomes. Also, to the best of our knowledge, existing studies does not analyse the impact of the cost of technology on technology adoption decision explicitly. This paper attempts to fill these gaps.

We consider a two stage non-cooperative game between two firms. Initially, both firms have symmetric cost functions. In the first stage, each firm simultaneously and independently decide whether to adopt a cost-reducing technology, by incurring some given cost, or not. In the second stage, firms engage either in Cournot competition or in Bertrand competition. The analysis shows that, if the cost of technology is high or moderate, Bertrand competition provides stronger incentive to adopt cost-reducing technology by a firm than Cournot competition unless the cost reducing effect of the technology is very low. The intuition behind our result is as follows. Fol-

lowing technology adoption, Bertrand competition not only leads to lower prices (price effect), but also a lower market share of the non-adopting firm (selection effect) than Cournot competition. While price effect generates more disincentive to adopt technology under Bertrand competition than under Cournot competition, the selection effect works in the opposite direction. The selection effect dominates the price effect, and the net effect is higher under Bertrand competition than under Cournot competition.

In equilibrium, only one firm adopts the technology under both Cournot and Bertrand competition, if the cost of technology is moderate. But, if the cost of technology is high, unless the cost reducing effect of the technology is very low, none of the firms adopt the technology under Cournot competition whereas one firm adopts the technology under Bertrand competition - an 'Arrow-like' result, as the gain from technology adoption is higher under Bertrand competition. On the other hand, if the cost of technology is low, both firms adopt the technology under Cournot competition whereas only one firm adopts the technology under Bertrand competition - a 'Schumpeter-type' result. The reason is, since the price effect is smaller under Cournot competition than under Bertrand competition, both firms find technology adoption to be gainful under Cournot competition whereas under Bertrand competition a firm gains by adopting the technology when the other firm does not adopt the technology. Under Bertrand competition, the cost of technology needs to be reduced even further to a very low level to induce both firms to adopt the technology. Clearly, the cost of technology has differential impact on the decision to adopt technology under two alternative modes of competition. Moreover, it shows that the possibility of *ex post* cost asymmetry between firms is higher under Bertrand competition than under Cournot competition. When the cost reducing effect of the technology

is high, the industry becomes more cost-efficient under Bertrand competition than under Cournot competition provided that the cost of technology is high; otherwise, both Bertrand and Cournot competition leads to the same level of cost-efficiency of the industry. On the other hand, if the technology reduces the marginal cost of production by a very low amount, Cournot competition leads to more cost-efficiency of the industry than Bertrand competition unless the cost of technology is moderate. These results has implications to ‘technology subsidy’ policies.

The rest of the paper proceeds as follows. The next section presents the model and characterises Bertrand and Cournot equilibria. Section 3 presents the comparison of equilibrium outcomes under alternative modes of competition. Section 4 concludes.

## 2 The Model

Let us consider an economy with an oligopolistic sector, consisting of two firms - firm 1 and firm 2, that produce a differentiated good and a competitive numeraire sector. Initially, the marginal costs of production of firm 1 and firm 2 are equal to  $c$ . That is, we start with a situation where there is no asymmetry in terms of cost of production between firm 1 and firm 2.

On the demand side of the market, we consider the following utility function of the representative consumer.

$$U = aq_1 + aq_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + m,$$

where  $q_1$  and  $q_2$  are the quantities of the two differentiated products produced by firm 1 and firm 2, respectively, and  $m$  is the quantity of the numeraire good.<sup>2</sup> The degree of product differentiation is measured by the parameter  $\gamma$

---

<sup>2</sup>This specification of the representative consumer’s utility function is similar to that

( $0 < \gamma < 1$ ), lower value of  $\gamma$  denotes higher degree of product differentiation, i.e., lower degree of substitutability between products. This specification of  $U(\cdot)$  generates the following linear demand structure.

$$q_i = \frac{1}{1-\gamma^2}[a(1-\gamma) - p_i + \gamma p_j], \quad i, j = 1, 2; i \neq j \quad (1)$$

Inverting (1), we get the following system of linear inverse demand functions.

$$p_i = a - q_i - \gamma q_j, \quad i, j = 1, 2; i \neq j \quad (2)$$

If prices leads to positive demand for both goods, demand is given by equations (1) and (2). But, if prices are such that  $q_j = \frac{1}{1-\gamma^2}[a(1-\gamma) - p_j + \gamma p_i] \leq 0$ , the demand of good  $i$  reduces to  $q_i = a - p_i$ , as in Zanchettin (2006).

Now, before undertaking production decision, firms can adopt a new technology by incurring the exogenously determined fixed cost  $r$  ( $> 0$ ) to reduce the cost of production. If a firm adopts the technology, its marginal cost of production reduces to  $c - x$  ( $0 < x < c$ ), whereas the non-adopting firm's marginal cost remains at  $c$ .<sup>3</sup> That is, we consider a two stage non-cooperative game between the firms. The stages of the game are as follows.

Stage 1: Firm 1 and Firm 2 simultaneously and independently decide whether to adopt the technology or not.

Stage 2: Firms engage either in Cournot competition or in Bertrand competition in the product market.

---

of Singh and Vives (1984), except that we consider same coefficients of linear terms of  $q_1$  and  $q_2$  and normalise the coefficients of the squared terms to one, to simplify the analysis.

<sup>3</sup>Alternatively, we can say that firms require to invest the amount  $r$  in R&D to get the next best technology that reduces cost of production by the amount  $x$ . There is no other possible intermediate technology, which can reduce cost by less than  $x$ , that requires less investment in R&D. Also, the required investment to obtain more superior technology, which reduces cost by more than  $x$ , is infinite. There is no spillover effect of technology adoption/R&D.

Clearly, in stage 1, there are three possible cases for the decision to adopt the technology: (1) no firm adopts the technology, (2) only one firm, either firm 1 or firm 2, adopts the technology, and (3) both firm 1 and firm 2 adopt the technology. The mode of product market competition, Cournot or Bertrand, in the second stage is exogenously determined. We solve this game by the backward induction method.

*Cournot Competition:* We begin with the scenario where firms are engaged in Cournot competition in the product market. First, we characterise the product market equilibrium, given the choice of technology adoption of firms in stage 1. When none of the firms adopts the technology, the equilibrium outcomes are as follows.

$$\begin{aligned} q_1^C(0,0) = p_1^C(0,0) - c &= \frac{(a-c)}{2+\gamma} = p_2^C(0,0) - c = q_2^C(0,0), \\ \pi_1^C(0,0) = \pi_2^C(0,0) &= \frac{(a-c)^2}{(2+\gamma)^2}, \end{aligned} \quad (3)$$

where  $q_i^C(0,0)$ ,  $p_i^C(0,0)$ , and  $\pi_i^C(0,0)$  are equilibrium quantity, price and profit, respectively, of firm  $i$  ( $= 1, 2$ ), under Cournot competition (denoted by superscript  $C$ ) when no firm adopts the technology (denoted by  $(0,0)$ ). If both firms adopt the technology, the equilibrium outcomes are

$$\begin{aligned} q_1^C(1,1) = p_1^C(1,1) - c + x &= \frac{(a-c+x)}{2+\gamma} = p_2^C(1,1) - c + x = q_2^C(1,1), \\ \pi_1^C(1,1) = \pi_2^C(1,1) &= \frac{(a-c+x)^2}{(2+\gamma)^2} - r, \end{aligned} \quad (4)$$

where  $(1,1)$  denotes that both firms adopt the technology. Finally, we consider the situation when only one firm adopts the technology. Since initially firms have equal marginal cost of production, without any loss of generality, let us consider that only firm 1 adopts the technology. We denote this case by  $(1,0)$ . In this case, the marginal cost of production of firm 1 and firm 2 are  $c-x$  and  $c$  respectively. The equilibrium outputs, price-cost margins



and profits are as follows.

$$\begin{aligned}
q_1^C(1,0) &= p_1^C(1,0) - c + x = \frac{(2-\gamma)(a-c) + 2x}{4-\gamma^2}, \\
q_2^C(1,0) &= p_2^C(1,0) - c = \frac{(2-\gamma)(a-c) - \gamma x}{4-\gamma^2}, \\
\pi_1^C(1,0) &= \frac{[(2-\gamma)(a-c) + 2x]^2}{(4-\gamma^2)^2} - r, \\
\pi_2^C(1,0) &= \frac{[(2-\gamma)(a-c) - \gamma x]^2}{(4-\gamma^2)^2}, \tag{5}
\end{aligned}$$

where  $(1,0)$  denotes that only firm 1 adopts the technology. Alternatively, if only firm 2 adopts the technology, the equilibrium outcomes are symmetric to that in (5):  $q_1^C(0,1) = q_2^C(1,0)$ ,  $q_2^C(0,1) = q_1^C(1,0)$ ,  $\pi_1^C(0,1) = \pi_2^C(1,0)$  and  $\pi_2^C(0,1) = \pi_1^C(1,0)$ , where  $(0,1)$  denotes that only firm 2 adopts the technology.

Note that when only one firm, say firm 1, adopts the technology, the mode of competition in the product market matters only if firm 1 cannot engage in monopoly pricing without bearing any competitive pressure from firm 2 (the non-adopting firm). Now, if the technology reduces marginal cost of firm 1 beyond a certain level, i.e., if the *ex post* efficiency gap between the two firms becomes sufficiently high, then irrespective of the mode of competition, firm 2 is driven out of the market and firm 1 enjoys absolute monopoly power. Now, firm 1 sets the monopoly price  $p_1^M = \frac{a-c+x}{2}$ , if at prices  $p_1 = p_1^M$  and  $p_2 = c$  the demand of firm 2's product is zero, i.e.,  $a(1-\gamma) - c + \gamma p_1^M \leq 0 \Rightarrow x \geq \frac{(a-c)(2-\gamma)}{\gamma}$ .<sup>4</sup> Since the mode of product market competition does not matter when  $x \geq \frac{(a-c)(2-\gamma)}{\gamma}$ , we consider the following.

$$x < \frac{(a-c)(2-\gamma)}{\gamma} \tag{6}$$

In other words, the relevant parameter space, in which the mode of product market competition matters, is  $S = \{0 < \gamma < 1; 0 < x < \frac{(a-c)(2-\gamma)}{\gamma}\}$ . Note

---

<sup>4</sup>Firm 2, i.e., the non-adopting firm, cannot engage in monopoly pricing unless products are completely different ( $\gamma = 0$ ).

that, in our case, the assumption of positive primary outputs is binding only for the non-adopting firm (firm 2). If both prices are set at marginal costs,  $p_1 = c - x$  and  $p_2 = c$ , the demand for non-adopting firm is positive only if  $x < \frac{(a-c)(1-\gamma)}{\gamma}$ . Clearly, the assumption of positive primary outputs curtails the parameter space.

From (5), it is evident that higher  $x$  leads to higher output, price-cost margin and profit of the technology adopting firm, but lower output, price-cost margin and profit of the non-adopting firm. However, it is easy to check that the non-adopting firm remains active under Cournot competition for all  $x \in S$ .

Next, we turn to the technology adoption decision in stage 1 of the game, when firms are engaged in Cournot competition in stage 2. If firm 2 does not adopt the technology, firm 1 adopts the technology provided that  $\pi_1^C(1, 0) > \pi_1^C(0, 0) \Rightarrow r < \frac{4x^2 + 4(2-\gamma)(a-c)x}{(4-\gamma^2)^2} = \bar{r}^C$ , say. On the other hand, if firm 1 adopts the technology, firm 2 does not adopt the technology provided that  $\pi_2^C(1, 0) > \pi_2^C(1, 1) \Rightarrow r > \frac{4(1-\gamma)x^2 + 4(2-\gamma)(a-c)x}{(4-\gamma^2)^2} = \underline{r}^C$ , say. It is easy to observe that  $\underline{r}^C < \bar{r}^C$ . Therefore, since firms are *ex ante* symmetric in terms of cost of production, only one firm (either firm 1 or firm 2) adopts the technology in equilibrium when  $\underline{r}^C < r < \bar{r}^C$ . Alternatively, in equilibrium, no firm adopts the technology when  $r > \bar{r}^C$ , and both firms adopt the technology provided that  $r < \underline{r}^C$ .

**Lemma 1:** *Under Cournot competition in the product market, the equilibrium technology adoption is as follows. (a) If the cost of technology ( $r$ ) is in the intermediate range, i.e., if  $\underline{r}^C < r < \bar{r}^C$ , only one firm adopts the cost-reducing technology; (b) if  $r < \underline{r}^C$  ( $r > \bar{r}^C$ ), both firms (no firm) adopt(s) the technology, where  $\underline{r}^C = \frac{4(1-\gamma)x^2 + 4(2-\gamma)(a-c)x}{(4-\gamma^2)^2}$  and  $\bar{r}^C = \frac{4x^2 + 4(2-\gamma)(a-c)x}{(4-\gamma^2)^2}$ . Both firms remain active in the market irrespective of the cost of technology.*

*Bertrand Competition:* We first characterise the equilibrium outcomes of the product market competition, where firms are competing in terms of price, given the technology adoption decision of firms. When none of the firms adopt the technology, each firm has the marginal cost equal to  $c$ ; therefore, the Bertrand equilibrium is as follows.

$$\begin{aligned} q_1^B(0,0) &= \frac{p_1^B(0,0) - c}{1 - \gamma^2} = \frac{(a - c)}{(2 - \gamma)(1 + \gamma)} = \frac{p_2^B(0,0) - c}{1 - \gamma^2} = q_2^B(0,0), \\ \pi_1^B(0,0) &= \pi_2^B(0,0) = \frac{(a - c)^2(1 - \gamma)}{(2 - \gamma)^2(1 + \gamma)}, \end{aligned} \quad (7)$$

where  $q_i^B(0,0)$ ,  $p_i^B(0,0)$ , and  $\pi_i^B(0,0)$  are equilibrium quantity, price and profit, respectively, of firm  $i$  ( $= 1, 2$ ), under Bertrand competition (denoted by superscript  $B$ ) when no firm adopts the technology (denoted by  $(0,0)$ ). On the other hand, if both firms adopt the technology, each firm's marginal cost reduces to  $c - x$  and the Bertrand equilibrium is

$$\begin{aligned} q_1^B(1,1) &= \frac{p_1^B(1,1) - c + x}{1 - \gamma^2} = \frac{(a - c + x)}{(2 - \gamma)(1 + \gamma)} = \frac{p_2^B(1,1) - c + x}{1 - \gamma^2} = q_2^B(1,1), \\ \pi_1^B(1,1) &= \pi_2^B(1,1) = \frac{(a - c + x)^2(1 - \gamma)}{(2 - \gamma)^2(1 + \gamma)} - r, \end{aligned} \quad (8)$$

where  $(1,1)$  denotes that both firms adopt the technology. Note that, under Bertrand competition, both firms adopt the technology provided the price effect does not lead to  $\pi_i^B(1,1) \leq 0$ ,  $i = 1, 2$ .

If only one firm (say, firm 1) adopts the technology and both firms are active in deliriums, the Bertrand equilibrium is

$$\begin{aligned} q_1^B(1,0) &= \frac{p_1^B(1,0) - c + x}{1 - \gamma^2} = \frac{(2 - \gamma - \gamma^2)(a - c) + (2 - \gamma^2)x}{(4 - \gamma^2)(1 - \gamma^2)}, \\ q_2^B(1,0) &= \frac{p_2^B(1,0) - c}{1 - \gamma^2} = \frac{(2 - \gamma - \gamma^2)(a - c) - \gamma x}{(4 - \gamma^2)(1 - \gamma^2)}, \\ \pi_1^B(1,0) &= \frac{[(2 - \gamma - \gamma^2)(a - c) + (2 - \gamma^2)x]^2}{(4 - \gamma^2)^2(1 - \gamma^2)} - r, \\ \pi_2^B(1,0) &= \frac{[(2 - \gamma - \gamma^2)(a - c) - \gamma x]^2}{(4 - \gamma^2)^2(1 - \gamma^2)}, \end{aligned} \quad (9)$$

where  $(1, 0)$  denotes that only firm 1 adopts the technology. Alternatively, if only firm 2 adopts the technology, the equilibrium outcomes are symmetric to that in (9):  $q_1^B(0, 1) = q_2^B(1, 0)$ ,  $q_2^B(0, 1) = q_1^B(1, 0)$ ,  $\pi_1^B(0, 1) = \pi_2^B(1, 0)$  and  $\pi_2^B(0, 1) = \pi_1^B(1, 0)$ , where  $(0, 1)$  denotes that only firm 2 adopts the technology.

However, the non-adopting firm (say, firm 2) is active in Bertrand equilibrium ( $q_2^B(1, 0) > 0$ ) provided that  $x < \frac{(a-c)(2-\gamma-\gamma^2)}{\gamma}$ . If, on the contrary,

$$\frac{(a-c)(2-\gamma-\gamma^2)}{\gamma} \leq x < \frac{(a-c)(2-\gamma)}{\gamma}, \quad (10)$$

the non-adopting firm is driven out of the market. However, the technology adopting firm (say, firm 1) cannot engage in monopoly pricing without bearing any competitive pressure from the non-adopting firm, since we consider that the efficiency gain through technology adoption is not drastic (i.e.,  $x < \frac{(a-c)(2-\gamma)}{\gamma}$ ). That is, the technology adopting firm cannot enjoy absolute monopoly power. In other words, though the non-adopting firm is driven out of the market, it exerts competitive pressure on the technology adopting firm. If the amount of marginal cost reduction ( $x$ ) due to technology adoption is in the range as specified in (10), in equilibrium, under Bertrand competition the technology adopting firm engages in limit-pricing, which keeps the non-adopting firm out of the market. On the contrary, the non-adopting firm remains active under Cournot competition for all  $x \in S$ . The limit-pricing equilibrium under Bertrand competition is as follows.

$$\begin{aligned} q_2^L(1, 0) &= p_2^L(1, 0) - c = \pi_2^L(1, 0) = 0, \\ q_1^L(1, 0) &= \frac{a-c}{\gamma}, \\ p_1^L(1, 0) - c + x &= \frac{\gamma x - (a-c)(1-\gamma)}{\gamma}, \\ \pi_1^L(1, 0) &= \frac{(a-c)\{\gamma x - (a-c)(1-\gamma)\}}{\gamma^2} - r, \end{aligned} \quad (11)$$

where the superscript  $L$  denotes limit-pricing under Bertrand equilibrium

and  $(1, 0)$  denotes that only firm 1 adopts the technology. If only firm 2 adopts the technology, the equilibrium outcomes are symmetric to that in (11), that is,  $q_2^L(0, 1) = q_1^L(1, 0)$ ,  $q_1^L(0, 1) = q_2^L(1, 0)$ ,  $p_2^L(0, 1) = p_1^L(1, 0)$ ,  $p_1^L(0, 1) = p_2^L(1, 0)$ ,  $\pi_2^L(0, 1) = \pi_1^L(1, 0)$  and  $\pi_1^L(0, 1) = \pi_2^L(1, 0)$ . Note that the possibility of limit-pricing increases with a decrease in the degree of product differentiation (increase in  $\gamma$ ).<sup>5</sup> Nonetheless, even if the degree of product differentiation is high ( $\gamma$  is low), it is optimum for the technology adopting firm to engage in limit-pricing.

Next, we analyse the technology adoption decision of firms in stage 1, when firms are engaged in Bertrand competition in the product market. From the above discussion, it is clear that there are two scenarios: (a)  $\frac{(a-c)(2-\gamma-\gamma^2)}{\gamma} \leq x < \frac{(a-c)(2-\gamma)}{\gamma}$ , i.e., limit-pricing occurs in equilibrium in stage 2 when only one firm (say, firm 1) adopts the technology; and (b)  $0 < x < \frac{(a-c)(2-\gamma-\gamma^2)}{\gamma}$ , i.e., limit-pricing does not occur in equilibrium.

In the first scenario, if firm 2 does not adopt the technology, firm 1 adopts the technology provided that  $\pi_1^L(1, 0) > \pi_1^B(0, 0)$ , i.e.,  $r < \frac{(a-c)\{\gamma x - (a-c)(1-\gamma)\}}{\gamma^2} - \frac{(a-c)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} = \bar{r}^L$ , say. On the other hand, if firm 1 adopts the technology, firm 2 does not adopt the technology provided that  $\pi_2^L(1, 0) > \pi_2^B(1, 1)$ , i.e.,  $r > \frac{(a-c+x)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} = \underline{r}^L$ , say. It implies that, since firms are *ex ante* symmetric, in equilibrium both firms adopt the technology, if  $r < \underline{r}^L$ . But, if  $r > \bar{r}^L$ , no firm adopts the technology in equilibrium. For intermediate costs of the technology,  $\underline{r}^L < r < \bar{r}^L$ , only one firm adopts the technology in equilibrium. In the second scenario ( $0 < x < \frac{(a-c)(2-\gamma-\gamma^2)}{\gamma}$ ) also we get similar equilibrium outcomes: if  $\underline{r}^B < r < \bar{r}^B$ , only one firm adopts the technology; but, if  $r < \underline{r}^B$  ( $r > \bar{r}^B$ ), both firms (none) adopt(s) the technology, where  $\underline{r}^B =$

<sup>5</sup>From (10), the range of limit-pricing region is  $(a-c)\gamma$ , which is positively related to  $\gamma$ .

$\frac{(a-c+x)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} - \frac{[(2-\gamma-\gamma^2)(a-c)-\gamma x]^2}{(4-\gamma^2)^2(1-\gamma^2)}$  and  $\bar{r}^B = \frac{[(2-\gamma-\gamma^2)(a-c)+(2-\gamma^2)x]^2}{(4-\gamma^2)^2(1-\gamma^2)} - \frac{(a-c)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)}$ .  
 Clearly,  $\underline{r}^B < \underline{r}^L$  and  $\bar{r}^B < \bar{r}^L$ .

**Lemma 2:** *When firms are engaged in Bertrand competition in the product market, if the amount of marginal cost reduction ( $x$ ) due to technology adoption is such that  $\frac{(a-c)(2-\gamma-\gamma^2)}{\gamma} \leq x < \frac{(a-c)(2-\gamma)}{\gamma}$ , in equilibrium only one firm adopts the cost-reducing technology and the technology adopting firm engages in limit-pricing provided that  $\underline{r}^L < r < \bar{r}^L$ ; but, when  $r < \underline{r}^L$  ( $r > \bar{r}^L$ ), both firms (no firm) adopt(s) the technology, where  $\underline{r}^L = \frac{(a-c+x)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)}$  and  $\bar{r}^L = \frac{(a-c)\{\gamma x - (a-c)(1-\gamma)\}}{\gamma^2} - \frac{(a-c)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)}$ . On the other hand, if  $0 < x < \frac{(a-c)(2-\gamma-\gamma^2)}{\gamma}$ , both firms (no firm) adopt(s) the technology in equilibrium provided that  $r < \underline{r}^B$  ( $r > \bar{r}^B$ ); but, only one firm adopts the technology when  $\underline{r}^B < r < \bar{r}^B$ , where  $\underline{r}^B = \frac{(a-c+x)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)} - \frac{[(2-\gamma-\gamma^2)(a-c)-\gamma x]^2}{(4-\gamma^2)^2(1-\gamma^2)}$ ,  $\bar{r}^B = \frac{[(2-\gamma-\gamma^2)(a-c)+(2-\gamma^2)x]^2}{(4-\gamma^2)^2(1-\gamma^2)} - \frac{(a-c)^2(1-\gamma)}{(2-\gamma)^2(1+\gamma)}$ .*

### 3 Comparison of Cournot and Bertrand Equilibria

In this section, we compare Cournot and Bertrand equilibria. Let us begin with the scenario in which the marginal cost reduction ( $x$ ) through the technology adoption is relatively high, that is,  $\frac{(a-c)(2-\gamma-\gamma^2)}{\gamma} \leq x < \frac{(a-c)(2-\gamma)}{\gamma}$ . In this case, it is straight forward to observe that the relevant critical values of the cost of technology, as given in Lemma 1 and Lemma 2, satisfy the following relation.

$$0 < \underline{r}^L < \underline{r}^C < \bar{r}^C < \bar{r}^L \quad (12)$$

It implies that, if the cost of technology ( $r$ ) is high ( $\bar{r}^C < r < \bar{r}^L$ ), one firm adopts the technology under Bertrand competition whereas none adopts un-

der Cournot competition. That is, we get very asymmetric outcomes under two alternative modes of product market competition. However, if the cost of technology is moderate ( $\underline{r}^C < r < \bar{r}^C$ ), under both Cournot and Bertrand competition one firm adopts the technology in equilibrium. Nonetheless, the incentive to adopt technology by a single firm is higher under Bertrand competition than under Cournot competition, since  $\pi_1^L(1, 0) - \pi_1^B(0, 0) > \pi_1^C(1, 0) - \pi_1^C(0, 0)$ . This is because, following technology adoption, Bertrand competition not only leads to lower prices (price effect), but also a lower market share of the non-adopting firm (selection effect) than Cournot competition. The selection effect dominates the price effect and the net gain, from these two opposing effects, of the technology adopting firm is higher under Bertrand competition than under Cournot competition. As a result, when the cost of technology is high, firms do not find it profitable to adopt the technology under Cournot competition even when the other firm does not adopt the technology. But, under Bertrand competition one firm gains by adopting the technology when the other does not opt to adopt it. As a result, cost-efficiency of the industry improves under Bertrand competition through technology adoption, whereas the industry remains less efficient under Cournot competition as no firms adopts the technology - an ‘Arrow-like’ result. However, if the cost of technology reduces to the moderate level ( $\underline{r}^C < r < \bar{r}^C$ ), the net gain from two opposing effects, price effect and selection effect, becomes higher than the cost of technology under Cournot competition also. As a result, when the cost of technology is moderate, we get symmetric equilibrium outcomes in terms of technology adoption under Cournot and Bertrand competition: one firm adopts the technology irrespective of the mode of competition. If the cost of technology reduces further to the low level,  $\underline{r}^L < r < \underline{r}^C$ , both firms find the technology adoption to be

gainful under Cournot competition whereas under Bertrand competition a firm gains by adopting the technology only if the other firm does not adopt the technology, since the price effect is smaller under Cournot than under Bertrand competition. If under Bertrand competition both firms adopt the technology, each firm incurs loss. Therefore, when the cost of technology is low, in equilibrium, both firms adopt the technology under Cournot competition whereas only one firm adopts the technology under Bertrand competition - a ‘Schumpeter-type’ result. The cost of technology needs to be reduced even further to very low level ( $r < \underline{r}^L$ ) to induce both firms to adopt the technology under Bertrand competition also. Figure 1 depicts the equilibrium choice of firms regarding technology adoption corresponding to different levels of the cost of technology under alternative modes of competition. Clearly, the industry becomes more cost-efficient under Bertrand competition than under Cournot competition, if the cost of technology is high. Otherwise, level of cost-efficiency of the industry under Bertrand competition is same as that under Cournot competition.<sup>6</sup>

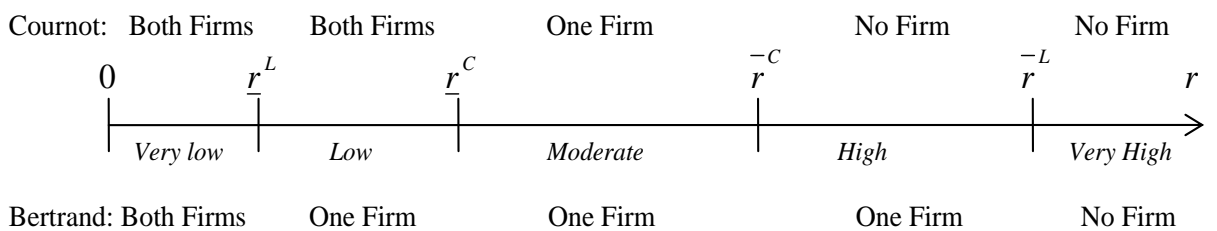


Figure 1: Cost of technology and technology adoption

Next, we turn to the scenario in which the marginal cost reduction ( $x$ )

<sup>6</sup>If the cost of technology is low, only one firm adopts the technology under Bertrand competition, but the non-adopting firm is driven out of the market.



through the technology adoption is relatively less, that is,  $0 < x < \frac{(a-c)(2-\gamma-\gamma^2)}{\gamma}$ . In this scenario, as discussed in Section 2, both firms remain active in the market irrespective of the mode of product market competition and the cost of technology. Therefore, under Bertrand competition, a firm's gain from technology adoption when the other firm does not adopt the technology is lower than that in the previous scenario. Also, under Bertrand competition, when one firm adopts the technology, the other firm will also find it profitable to adopt the technology provided that the cost of technology is less than what is required in the previous scenario. It implies that,  $\underline{r}^B < \underline{r}^L$  and  $\bar{r}^B < \bar{r}^L$ . Nonetheless, comparing critical values of the cost of technology, as given in Lemma 1 and Lemma 2, we get similar rankings as in (12):  $0 < \underline{r}^B < \underline{r}^C < \bar{r}^C < \bar{r}^B$ , except when  $x < \frac{2(1-\gamma)(a-c)}{\gamma}$ . If  $x < \frac{2(1-\gamma)(a-c)}{\gamma}$ , only the relative position of  $\bar{r}^B$  and  $\bar{r}^C$  changes, i.e., we get  $0 < \underline{r}^B < \underline{r}^C < \bar{r}^B < \bar{r}^C$ . Clearly, if  $\frac{2(1-\gamma)(a-c)}{\gamma} < x < \frac{(a-c)(2-\gamma-\gamma^2)}{\gamma}$ , the comparison of Cournot and Bertrand equilibria remains same as before. However, since both firms remain active in equilibrium irrespective of the mode of product market competition, the level of cost-efficiency of the industry is higher (lower) under Cournot competition than under Bertrand competition, if the cost of technology is low (high). But, when  $x < \frac{2(1-\gamma)(a-c)}{\gamma}$ , the comparison is as follows. (a) If  $\bar{r}^B < r < \bar{r}^C$ , no firm adopts the technology under Bertrand competition whereas one firm adopts under Cournot competition. (b) If  $\underline{r}^C < r < \bar{r}^B$ , one firm adopts the technology irrespective of the mode of product market competition. (c) If  $\underline{r}^B < r < \underline{r}^C$ , both firms adopt the technology under Cournot competition whereas only one firm adopts the technology under Bertrand competition. (d) If  $0 < r < \underline{r}^B$  ( $r > \bar{r}^C$ ), both firms (no firm) adopt(s) the technology irrespective the mode of product market competition. Therefore, if the cost reducing effect of the technol-

ogy is low ( $x < \frac{2(1-\gamma)(a-c)}{\gamma}$ ), the level of cost efficiency of the industry is same under both Bertrand and Cournot competition provided that the cost of technology is moderate ( $\underline{r}^C < r < \bar{r}^B$ ); otherwise, the industry becomes more cost-efficient under Cournot competition.

**Proposition 1:** (a) *When the marginal cost of production reduces, due to technology adoption, by more than a critical level ( $x > \frac{(a-c)(2-\gamma-\gamma^2)}{\gamma} = x_1$ , say), the industry becomes more cost-efficient under Bertrand competition than under Cournot competition if the cost of technology is high ( $r > \bar{r}^C$ ); otherwise, both Bertrand and Cournot competition leads to the same level of cost-efficiency of the industry.*

(b) *If the amount of marginal cost reduction ( $x$ ), due to technology adoption, is less than  $x_1$  but greater than  $x_2 (= \frac{2(1-\gamma)(a-c)}{\gamma})$ , Cournot competition leads to lower (higher) level of cost-efficiency of the industry than Bertrand competition provided that the cost of technology is high (low:  $r < \underline{r}^C$ ). However, if the technology reduces the marginal cost by less than the amount  $x_2$ , Cournot competition leads to more cost-efficiency than the Bertrand competition unless the cost of technology is moderate ( $\underline{r}^C < r < \bar{r}^B$ ).*

Clearly, the cost of technology has differential impact on cost-efficiency of the industry under alternative modes of product market competition. This result has implications to ‘technology subsidy’ policies. Also, note that, if  $\gamma$  is closer to one,  $x_1$  is closer to zero. That is, if the degree of substitutability between products is very high, the possibility of  $x > x_1$  is high. Therefore, we can say that, Bertrand competition leads to higher cost-efficiency of the industry than Cournot competition when products are close substitutes provided that the cost of technology is high. On the other hand, if products are highly differentiated ( $\gamma$  is close to zero),  $x_2$  is very large, i.e., the possibility

of  $x < x_2$  is high and, thus, Cournot competition leads to more cost-efficiency of the industry than the Bertrand competition unless the cost of technology is moderate. These results are similar to that in Bester and Petrakis (1993), which argues that the incentive to innovate is higher (lower) under Cournot competition than under Bertrand competition if the degree of substitutability is low (high). However, note that the results of Bester and Petrakis (1993) holds only for a selected range(s) of the cost of technology. Therefore, in our set up, the results of Bester and Petrakis (1993) emerges as a special case.

Proposition 1 also suggests that, the relation between intensity of competition and technology adoption is not necessarily monotonic, which is in line with Boone (2001). For example, given the degree of substitutability, if the cost of technology is low ( $r < \underline{r}^C$ ), Cournot competition leads to higher cost-efficiency of the industry than Bertrand competition when  $x < x_1$ , but the level of cost-efficiency does not vary with the mode of product market competition when  $x > x_1$ .

## 4 Conclusion

In this paper we have compared technology adoption in a differentiated duopoly under two alternative modes of product market competition, Cournot and Bertrand. We have analysed how the cost of technology affects this comparisons in a more general setup by enlarging the parameter space so as to relax the commonly subscribed assumption of positive primary outputs. We have shown that the cost of technology has differential impact on technology adoption under alternatives modes of competition in the product market. The possibility of *ex post* cost asymmetry is higher under Bertrand competi-

tion than under Cournot competition. A comparison of *ex post* cost-efficiency of the industry reveals that, when the cost reducing effect of the technology is high, Bertrand competition leads to higher cost efficiency than Cournot competition if the cost of technology is high; otherwise, cost-efficiency of the industry is invariant to the mode of product market competition. On the other hand, unless the cost of technology is moderate, cost-efficiency of the industry is higher under Cournot competition than under Bertrand competition when the technology reduces the marginal cost of production by a very low amount. These results have implications to ‘technology subsidy’ policies.

It seems to be interesting to extend the present analysis by considering possible tradeoff between product and process innovation. It might also be interesting to examine the implications of (semi)collusion on technology adoption and profitability in the present context. We leave these issues for future research.

## References

- Aghion, P., Harris, C., and Vickers, J. (1997). Competition and growth with step-by-step innovation: An example. *European Economic Review*, 41:771–782.
- Amir, R. and Jin, J. (2001). Cournot and Bertrand equilibria compared: Substitutability, complementarity and concavity. *International Journal of Industrial Organization*, 19:303–317.
- Arrow, K. (1962). Economic welfare and the allocation of resources for inventions. In Nelson, R., editor, *The rate and direction of inventive activity*. Princeton University Press, USA.

- Bester, H. and Petrakis, E. (1993). The incentives for cost reduction in a differentiated industry. *International Journal of Industrial Organization*, 11(4):519–534.
- Bonnano, G. and Haworth, B. (1998). Intensity of competition and the choice between product and process innovation. *International Journal of Industrial Organization*, 16:495–510.
- Boone, J. (2001). Intensity of competition and the incentive to innovate. *International Journal of Industrial Organization*, 19:705–726.
- Delbono, F. and Denicolo, V. (1990). R&D investment in a symmetric and homogeneous oligopoly. *International Journal of Industrial Organization*, 8:297–313.
- Lin, M. H. and Ogawa, H. (2005). Cost reducing incentives in a mixed duopoly market. *Economics Bulletin*, 12(6):1–6.
- Schumpeter, J. (1943). *Capitalism. socialism and democracy*. Allan and Unwin, London.
- Singh, N. and Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *Rand Journal of Economics*, 15:546–554.
- Zanchettin, P. (2006). Differentiated duopoly with asymmetric costs. *Journal of Economics and Management Strategy*, 15(4):999–1015.