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# Computing Additive Chained Volume Measures of GDP Subaggregates

Jesus C. Dumagan\*

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## Abstract

This paper derives formulas for additive “chained volume measures” (CVMs) of GDP subaggregates depending on the underlying GDP quantity index. In turn, this paper explains why the formulas used in current practice yield non-additive CVMs. This paper’s additive formulas have significant practical implications given that non-additivity prevails in all countries that have adopted the CVM framework for GDP and considering that more countries will be adopting this framework.

*Keywords:* Chained indexes; chained volume measure; additivity

*JEL classification:* C43

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## 1. Introduction

This paper is concerned with the issue of additivity or non-additivity of a “chained volume measure” (CVM). In this paper, a CVM generally means (real) “value in chained prices” like GDP in chained prices. However, since additivity or non-additivity pertains only *below* the level of aggregate GDP, CVMs of GDP subaggregates mean “GDP subaggregates in chained prices” that will be referred to henceforth in this paper as “CVMs” for brevity.

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This paper was inspired by the “first” additivity solution—against the prevailing “mathematical impossibility” consensus of the profession—proposed by Balk and Reich (2008) by a special Paasche price deflation procedure that this paper generalized to an additive deflation procedure employing the Laspeyres, Paasche, or Fisher price index starting from the properties of the “dual” quantity index underlying GDP in chained prices. The author is very grateful to Bert Balk at Rotterdam School of Management and Statistics Netherlands for reviewing this paper and for agreeing with the results subject to the usual caveat that any errors he may not have detected are solely the author’s responsibility. Not the least, he is also very grateful to PIDS for extending his “visiting” status and to his colleagues for providing him with the intellectual atmosphere conducive to serious academic research very rare in a government institution.

In practice, GDP in chained prices is the economy-wide result of following the recommendation by the 1993 UN System of National Accounts for adoption of CVM.<sup>1</sup> Some countries that have adopted CVM chose the Laspeyres quantity index framework (e.g., Australia, Japan, Hong Kong, UK, France, Germany, the Netherlands, and several other OECD and EU countries) while others chose the Fisher quantity index framework (e.g., US and Canada).<sup>2</sup> Notably, some less developed and developing countries have adopted CVM [e.g., Azerbaijan, Guatemala, and Kazakhstan among others (Magtulis, 2010)]. The Philippines is also on the move to adopt CVM.<sup>3</sup>

Presently, non-additivity prevails in GDP accounts of all countries that have adopted CVM and appears a discouraging factor for further adoption. This paper shows that non-additivity is a resolvable procedural issue and, therefore, the present situation need not persist. This is the motivation for this paper. The contents of the rest of this paper are described below.

Section 2 presents the framework for GDP in chained prices (i.e., the aggregate CVM) and describes the non-additivity problem surrounding CVMs in current practice. It is shown that there are two equivalent methods for computing GDP in chained prices—*inflation* of reference-year GDP in current prices (a scalar) by a chained GDP quantity index or *deflation* of GDP in current prices by a chained GDP price index. There are three frameworks where

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<sup>1</sup> The application of chained indexes to the national accounts was earlier shown by Al, P., *et al.* (1986).

<sup>2</sup> For examples of country practices in Laspeyres quantity index framework, see Aspden (2000) for Australia, Robjohns (2007) for the UK, and Maruyama (2005) for Japan, and Census and Statistics Department, Government of the Hong Kong Administrative Region (2007). See also Schreyer (2004) and European Union (2007) for other OECD and EU countries using the above framework. For those using the Fisher quantity index framework, see Landefeld and Parker (1997), Seskin and Parker (1998), and Moulton and Seskin (1999) for the US and Chevalier (2003) for Canada.

<sup>3</sup> Virola (2008) noted that the Philippines started “migration” to CVM by “pilot adoption” in 1997 through technical assistance by the Asian Development Bank and the Philippine-Australian Government Facility Project, 2001-2003. Presently—through the concerted efforts of the Bangko Sentral ng Pilipinas (BSP), Statistical Research and Training Center, National Statistics Office, Bureau of Agricultural Statistics, and National Statistical Coordination Board—there is a hopeful push towards implementation of CVM in the Philippines. Significant steps moving forward were the *Workshops on the Operationalization of Chain-Type GDP and Price Indexes* hosted by BSP on June 2 and July 8-9, 2010. Follow-up implementation workshops—i.e., actual computations at the statistical agencies—are also being planned.

each framework utilizes a pair of quantity and price indexes that yields the same GDP in chained prices. These are the Laspeyres quantity index (inflation method) with the Paasche price index (deflation method). The other index pairs are Paasche quantity with Laspeyres price and Fisher quantity with Fisher price. However, each pair in the three frameworks may yield a different GDP than the others.

Section 3 derives this paper's formulas for additive CVMs in each of the above three frameworks. In turn, this paper's formulas yield the formulas in current practice that produce non-additive CVMs, exposing the implicit assumptions that cause non-additivity.

Section 4 presents empirical illustrations of the preceding analytical results by application to actual GDP data. It is shown that CVMs in current practice are non-additive in all three index frameworks. However, this paper's CVMs—while additive in the Laspeyres quantity-Paasche price and Paasche quantity-Laspeyres price index frameworks regardless of the level of aggregation—are additive in the Fisher index framework only at the *lowest* level subaggregate indexes, at the *starting* quantity relatives when the Fisher quantity index is constructed. That is, this paper's CVMs are additive at the lowest level but non-additive for any upper level subaggregate Fisher quantity indexes. However, it will be shown that the latter are “approximately additive” and, thus, will yield smaller (absolute) residuals than the non-additive CVMs in current practice.

Section 5 concludes this paper with a summary of findings and a recommendation.

## **2. The Framework for GDP in Chained Prices and the Non-Additivity Problem**

To define the framework, consider the time interval  $(0, 1, 2, \dots, T)$  within which two *adjoining* years  $s$  and  $t$ , i.e.,  $t = s + 1$ , and a reference year  $r$  are chosen. Let there be price-quantity data in each year,  $(p_{is}, q_{is})$ ,  $(p_{it}, q_{it})$ , and  $(p_{ir}, q_{ir})$ , for GDP components  $i = 1, 2, \dots, N$ . Also, let  $Y$  denote GDP. Hence, GDP in current prices in  $s$  and  $t$  are  $Y_s$  and  $Y_t$  and reference-year GDP is  $Y_r$ . By definition,

$$Y_s = \sum_{i=1}^N p_{is} q_{is} \quad ; \quad Y_t = \sum_{i=1}^N p_{it} q_{it} \quad ; \quad Y_r = \sum_{i=1}^N p_{ir} q_{ir} . \quad (1)$$

The above data yield quantity indexes denoted by  $Q$  and price indexes denoted by  $P$  as defined below. These indexes have subscripts  $st$  to indicate the two periods between which quantities or prices are compared. Thus,  $Q_{st}$  is the quantity index comparing quantities while  $P_{st}$  is the price index comparing prices between  $s$  and  $t$ . Moreover, superscripts indicate the specific index formula:  $L$  for Laspeyres,  $P$  for Paasche, and  $F$  for Fisher.

The following chain-type quantity and price indexes underlie GDP in chained prices,

$$Q_{st}^L = \frac{\sum_{i=1}^N p_{is} q_{it}}{\sum_{i=1}^N p_{is} q_{is}} \quad ; \quad Q_{st}^P = \frac{\sum_{i=1}^N p_{it} q_{it}}{\sum_{i=1}^N p_{it} q_{is}} \quad ; \quad Q_{st}^F = (Q_{st}^L Q_{st}^P)^{\frac{1}{2}} . \quad (2)$$

$$P_{st}^L = \frac{\sum_{i=1}^N q_{is} p_{it}}{\sum_{i=1}^N q_{is} p_{is}} \quad ; \quad P_{st}^P = \frac{\sum_{i=1}^N q_{it} p_{it}}{\sum_{i=1}^N q_{it} p_{is}} \quad ; \quad P_{st}^F = (P_{st}^L P_{st}^P)^{\frac{1}{2}} ; \quad (3)$$

In this paper, a *chain-type* index and a *chained* index are distinguished as follows. The indexes in (2) and (3) are chain-type indexes in that quantities or prices are compared between two adjoining periods  $s$  and  $t$  where the period  $s$  moves with  $t$  since  $t = s + 1$ . In turn, a chain-type index forms a chained index when succeeding values of the chain-type index are *multiplied* together. That is, a chain-type index by itself is not a chained index.

Let the chain-type quantity index  $Q_{st}$  represent  $Q_{st}^L$ ,  $Q_{st}^P$ , or  $Q_{st}^F$ . The chained quantity index  $J_t$  generated by  $Q_{st}$  may be given as,

$$J_t = J_s Q_{st} \quad ; \quad J_r = 1 \quad ; \quad r = \text{reference year.} \quad (4)$$

Without loss of generality, let  $r = 1$ . Hence, (4) generates the chained quantity indexes,  $J_1 = 1$ ;  $J_2 = J_1 Q_{12} = Q_{12}$ ; and  $J_3 = J_2 Q_{23} = Q_{12} \times Q_{23}$ . In general,

$$J_t = J_s Q_{st} = Q_{12} \times Q_{23} \times \cdots \times Q_{(t-1)t} = \prod_{\tau=1}^{t-1} Q_{\tau,\tau+1} . \quad (5)$$

Similarly, let the chain-type price index  $P_{st}$  represent  $P_{st}^L$ ,  $P_{st}^P$ , or  $P_{st}^F$  and let  $D_t$  be the chained price index generated by  $P_{st}$ . Letting  $D_1 = 1$ , it follows from (4) and (5) that,

$$D_t = D_s P_{st} = P_{12} \times P_{23} \times \cdots \times P_{(t-1)t} = \prod_{\tau=1}^{t-1} P_{\tau, \tau+1}. \quad (6)$$

Let  $Y_s^*$  and  $Y_t^*$  be GDP in chained prices. These are obtained by inflating (multiplying) the GDP in the reference year  $Y_r$  (a scalar) by the chained quantity index or by deflating (dividing) the GDP in current prices by the chained price index. Hence, (4) to (6) yield,

$$Y_s^* = J_s Y_r = \frac{Y_s}{D_s} \quad ; \quad Y_t^* = J_t Y_r = \frac{Y_t}{D_t} \quad ; \quad Y_t^* = Y_s^* Q_{st} = \frac{Y_s}{D_s} Q_{st}. \quad (7)$$

From the last result in (7), GDP in chained prices  $Y_t^*$  may be interpreted as a CVM. In the numerator, the multiplication of GDP in current prices in year  $s$  or  $Y_s$  by the GDP quantity index  $Q_{st}$  yields a “volume” measure in year  $t$  prices. Then the deflation of this volume measure by the GDP chained price index  $D_s$  yields a “chained volume measure” of GDP in reference year  $r$  prices (the year when  $D_r = 1$ ).

Moreover, by combining (1) to (7), the GDP value index  $\frac{Y_t}{Y_s}$  may be decomposed as a product of chain-type quantity and price indexes,

$$\frac{Y_t}{Y_s} = \frac{J_t D_t}{J_s D_s} = Q_{st} P_{st} = Q_{st}^L P_{st}^P = Q_{st}^P P_{st}^L = Q_{st}^F P_{st}^F. \quad (8)$$

The results in (8) imply three compatible pairs of chain-type quantity and price indexes, compatible in the sense that a pair will generate chained indexes that yield the same GDP in chained prices. These are (i) Laspeyres quantity index  $Q_{st}^L$  (for inflation) and Paasche price index  $P_{st}^P$  (for deflation); similarly, (ii) Paasche quantity  $Q_{st}^P$  and Laspeyres price  $P_{st}^L$  indexes; and (iii) Fisher quantity  $Q_{st}^F$  and price  $P_{st}^F$  indexes. As noted earlier (footnote 2), some countries chose the quantity-price index pair in (i) while others chose the pair in (iii) but no country appears to have chosen (ii). However, for completeness, this paper will analyze all three index pairs.

The pairing in (8) is necessary for the inflation and deflation methods in (7) to yield the same GDP in chained prices. As shown analytically and empirically by Dumagan (2010), the

chained indexes from  $(Q_{st}^L, P_{st}^P)$  yield the same GDP. Likewise, chained indexes from  $(Q_{st}^P, P_{st}^L)$  yield the same GDP while those from  $(Q_{st}^F, P_{st}^F)$  also yield the same GDP. However, GDP in chained prices could differ between the three index pairs.

In current practice, CVMs are computed *separately* by applying the same procedure for GDP in (7). The result, however, are non-additive CVMs (Ehemann, Katz, and Moulton, 2002; Whelan, 2002; Balk, 2004b; Balk, 2008; Balk and Reich, 2008).

Formally, the non-additivity problem may be presented as follows. From (1), partition GDP in current prices  $Y_t$  and  $Y_r$  into  $z = 1, 2, \dots, Z$  mutually exclusive subagregates  $Y_t^z$  and  $Y_r^z$ , respectively, defined by the superscript  $z$ . Hence,

$$Y_t = \sum_{z=1}^Z Y_t^z \quad ; \quad Y_r = \sum_{z=1}^Z Y_r^z \quad ; \quad z = 1, 2, \dots, Z. \quad (9)$$

Let the aggregate and subaggregate chained quantity indexes be  $J_t$  and  $J_t^z$  and the aggregate and subaggregate chained price indexes be  $D_t$  and  $D_t^z$ . Non-additivity means that,

$$Y_t^* = J_t Y_r \neq \sum_{z=1}^Z J_t^z Y_r^z \quad ; \quad J_r = J_r^z = 1; \quad (10)$$

$$Y_t^* = \frac{Y_t}{D_t} \neq \sum_{z=1}^Z \frac{Y_t^z}{D_t^z} \quad ; \quad D_r = D_r^z = 1. \quad (11)$$

Non-additivity holds in (10) and (11) from the chained indexes generated by the quantity-price index pairs  $(Q_{st}^L, P_{st}^P)$ ,  $(Q_{st}^P, P_{st}^L)$ , or  $(Q_{st}^F, P_{st}^F)$ . For example, if in (10)  $J_t$  is an aggregate chained Laspeyres quantity index then in current practice each  $J_t^z$  is a subaggregate chained Laspeyres quantity index. In this case,  $D_t$  is an aggregate chained Paasche price index and each  $D_t^z$  is a subaggregate chained Paasche price index. The result is that (10) and (11) will yield equal aggregate GDP in chained prices  $Y_t^*$  and likewise the corresponding GDP subagregates in chained prices are equal but their sum will not equal  $Y_t^*$ . That is, these subagregates are not additive in current practice.

Hence, to resolve non-additivity, the task of this paper is to determine the inflation

formula for each  $Y_r^z$  or, equivalently, the deflation formula for each  $Y_t^z$ ,  $z = 1, 2, \dots, Z$  to yield CVMs that equalize the two sides of (10) or (11). Moreover, this equality should be established in each of the three  $(Q_{st}^L, P_{st}^P)$ ,  $(Q_{st}^P, P_{st}^L)$ , and  $(Q_{st}^F, P_{st}^F)$  index frameworks.

### 3. Computing CVMs

This paper's formulas for additive CVMs are derived for each of the above three quantity-price index frameworks for GDP in chained prices.

#### 3.1. CVMs in the Laspeyres Quantity-Paasche Price Index Framework

GDP in chained prices  $(Y_s^{*L}, Y_t^{*L})$  computed by the inflation method based on chained Laspeyres quantity indexes  $(J_s^L, J_t^L)$  equal GDP in chained prices computed by the deflation method based on chained Paasche price indexes  $(D_s^P, D_t^P)$ . The results are,

$$Y_s^{*L} = J_s^L Y_r = \frac{Y_s}{D_s^P} \quad ; \quad Y_t^{*L} = J_t^L Y_r = \frac{Y_t}{D_t^P} \quad ; \quad Y_t^{*L} = Y_s^{*L} Q_{st}^L = \frac{Y_s}{D_s^P} Q_{st}^L . \quad (12)$$

In (12), the superscript  $L$  in  $Y_s^{*L}$  and  $Y_t^{*L}$  denote GDP in chained prices based on the Laspeyres quantity index.

For simplicity but without loss of generality, it is sufficient for analytical purposes to break up  $Q_{st}^L$  in (12) into two mutually exclusive subaggregate indexes while satisfying additivity. Accordingly, partition  $Y_s$  into two mutually exclusive subaggregates  $Y_s^A$  and  $Y_s^B$ . In this case, borrowing the notation from (9),  $z = (A, B)$ . By definition,

$$Y_s = Y_s^A + Y_s^B \quad ; \quad Y_s = \sum_{i=1}^N p_{is} q_{is} \quad ; \quad N = N^A + N^B ; \quad (13)$$

$$Y_s^A = \sum_{j=1}^{N^A} p_{js}^A q_{js}^A \quad ; \quad Y_s^B = \sum_{k=1}^{N^B} p_{ks}^B q_{ks}^B \quad ; \quad i = (j, k) \quad ; \quad j \neq k . \quad (14)$$

Denote the Laspeyres subaggregate shares as  $w_s^{LA}$  and  $w_s^{LB}$  and the corresponding subaggregate quantity indexes as  $Q_{st}^{LA}$  and  $Q_{st}^{LB}$ . By definition, these are given by,



$$w_s^{LA} = \frac{Y_s^A}{Y_s} ; \quad w_s^{LB} = \frac{Y_s^B}{Y_s} ; \quad Q_{st}^{LA} = \frac{\sum_{j=1}^{N^A} p_{js}^A q_{jt}^A}{\sum_{j=1}^{N^A} p_{js}^A q_{js}^A} ; \quad Q_{st}^{LB} = \frac{\sum_{k=1}^{N^B} p_{ks}^B q_{kt}^B}{\sum_{k=1}^{N^B} p_{ks}^B q_{ks}^B} . \quad (15)$$

It can be verified from the definition of  $Q_{st}^L$  in (2) and from (13) to (15) that,

$$Q_{st}^L = \frac{\sum_{i=1}^N p_{is} q_{it}}{\sum_{i=1}^N p_{is} q_{is}} = \sum_{i=1}^N w_{is}^L \left( \frac{q_{it}}{q_{is}} \right) = w_s^{LA} Q_{st}^{LA} + w_s^{LB} Q_{st}^{LB} ; \quad (16)$$

$$w_{is}^L = \frac{p_{is} q_{is}}{\sum_{i=1}^N p_{is} q_{is}} ; \quad \sum_{i=1}^N w_{is}^L = w_s^{LA} + w_s^{LB} = 1 . \quad (17)$$

To proceed, combine (12) and (17) to obtain,

$$\frac{Y_t}{D_t^P} = \frac{Y_s}{D_s^P} (w_s^{LA} Q_{st}^{LA} + w_s^{LB} Q_{st}^{LB}) . \quad (18)$$

The result in (18) can be expressed as the sum of  $Y_t^{LA}$  and  $Y_t^{LB}$  given by,

$$\frac{Y_t}{D_t^P} = Y_t^{LA} + Y_t^{LB} ; \quad Y_t^{LA} = \frac{Y_s w_s^{LA} Q_{st}^{LA}}{D_s^P} ; \quad Y_t^{LB} = \frac{Y_s w_s^{LB} Q_{st}^{LB}}{D_s^P} . \quad (19)$$

Moreover, by substituting from (15), the expressions in (19) simplify to,

$$Y_t^{LA} = \frac{Y_s^A Q_{st}^{LA}}{D_s^P} ; \quad Y_t^{LB} = \frac{Y_s^B Q_{st}^{LB}}{D_s^P} . \quad (20)$$

Finally, GDP value index decomposition in (8) applies as well to subaggregates. Therefore,

$$\frac{Y_t^A}{Y_s^A} = Q_{st}^{LA} P_{st}^{PA} ; \quad \frac{Y_t^B}{Y_s^B} = Q_{st}^{LB} P_{st}^{PB} . \quad (21)$$

Combining (19) to (21) yields this paper's formula for additive CVMs in the Laspeyres quantity-Paasche price index framework,

$$\frac{Y_t}{D_t^P} = \frac{Y_t^A}{D_s^P P_{st}^{PA}} + \frac{Y_t^B}{D_s^P P_{st}^{PB}} ; \quad \frac{Y_t}{D_t^P} = \sum_{z=1}^Z \frac{Y_t^z}{D_s^P P_{st}^{Pz}} . \quad (22)$$

In (22), the number of subaggregates,  $z = 1, 2, \dots, Z$ , can be expanded while maintaining additivity, given the same aggregate GDP in current prices  $Y_t$  and aggregate chained Paasche price indexes  $D_t^P$  and  $D_s^P$ . This result follows from the ‘‘consistency in aggregation’’ property of the Laspeyres quantity index that at the start generated (22). Hence, all the changes are in  $Y_t^A, Y_t^B, P_{st}^{PA}$ , and  $P_{st}^{PB}$  to accommodate the new subaggregates.

To explain (22), deflating the subaggregates in current prices,  $Y_t^A$  and  $Y_t^B$ , by their corresponding subaggregate Paasche price indexes,  $P_{st}^{PA}$  and  $P_{st}^{PB}$ , adjusts them for price changes within each subaggregate and, thus, converts them to “volume” measures in year  $t$ . Moreover, deflating them by a common aggregate chained Paasche price index  $D_s^P$  adjusts them for changes in the overall price level and converts them into the same unit of measure in terms of reference year  $r$  prices (i.e., the year in which  $D_r^P = 1$ ), thus, making them CVMs.

It is important to recognize at this juncture that this paper’s additive CVMs in (22) are the same as those obtained by Balk and Reich (2008)—albeit starting from different premises—by a double deflation procedure employing the combinations of an aggregate chained Paasche price index and a subaggregate Paasche price index given by  $D_s^P P_{st}^{PA}$  and  $D_s^P P_{st}^{PB}$ . However, Balk and Reich’s deflation framework was limited to Paasche price indexes and, thus, Balk and Reich did not derive any more results comparable to those in the rest of this paper.

In current practice, the inflation method or deflation method for GDP in chained prices are simply replicated for subaggregates. For deflation, corresponding to the aggregate chained Paasche price indexes in (12), the subaggregate chained Paasche indexes are,

$$D_t^{PA} = D_s^{PA} P_{st}^{PA} \quad ; \quad D_t^{PB} = D_s^{PB} P_{st}^{PB} . \quad (23)$$

Hence, by deflating the subaggregates in current prices  $Y_t^A$  and  $Y_t^B$  by (23), the CVMs in current practice are,

$$\frac{Y_t^A}{D_t^{PA}} = \frac{Y_t^A}{D_s^{PA} P_{st}^{PA}} \quad ; \quad \frac{Y_t^B}{D_t^{PB}} = \frac{Y_t^B}{D_s^{PB} P_{st}^{PB}} . \quad (24)$$

Comparing this paper’s CVMs in (22) to those in current practice in (24), it can be seen that the latter are not additive,

$$\frac{Y_t}{D_t^P} \neq \frac{Y_t^A}{D_t^{PA}} + \frac{Y_t^B}{D_t^{PB}} . \quad (25)$$

It appears that the reason for non-additivity is that the CVMs in (24) do not employ the aggregate chained Paasche price index  $D_s^P$  and, thus, do not make adjustments for changes in

the overall price level that are necessary, in light of this paper's CVMs in (22). However, aggregate and subaggregate chained Paasche price indexes are not necessarily equal. That is,

$$D_s^P \neq D_s^{PA} \quad ; \quad D_s^P \neq D_s^{PB} . \quad (26)$$

Therefore, being not deflated by  $D_s^P$ , the inequalities in (26) imply that the CVMs in current practice in (24) do not have the same units of measure in terms of reference year prices. That is, these CVMs are like the proverbial “apples” and “oranges” that should not even be added. But if they are added, as done in current practice, their sum will not equal the aggregate CVM or GDP in chained prices as shown by (25).

### 3.2. CVMs in the Paasche Quantity-Laspeyres Price Index Framework

GDP in chained prices ( $Y_s^{*P}$ ,  $Y_t^{*P}$ ) computed by the inflation method based on chained Paasche quantity indexes ( $J_s^P$ ,  $J_t^P$ ) equal GDP in chained prices computed by the deflation method based on chained Laspeyres price indexes ( $D_s^L$ ,  $D_t^L$ ). Hence,

$$Y_s^{*P} = J_s^P Y_r = \frac{Y_s}{D_s^L} \quad ; \quad Y_t^{*P} = J_t^P Y_r = \frac{Y_t}{D_t^L} \quad ; \quad Y_t^{*P} = Y_s^{*P} Q_{st}^P = \frac{Y_s}{D_s^L} Q_{st}^P . \quad (27)$$

In (27), the superscript  $P$  in  $Y_s^{*P}$  and  $Y_t^{*P}$  denote GDP in chained prices based on the Paasche quantity index.

By procedures similar to those in the preceding section, the Paasche subaggregate shares ( $w_s^{PA}$ ,  $w_s^{PB}$ ) and quantity indexes ( $Q_{st}^{PA}$ ,  $Q_{st}^{PB}$ ) are given by,

$$w_s^{PA} = \frac{\sum_{j=1}^{N^A} p_{jt}^A q_{js}^A}{\sum_{i=1}^N p_{it} q_{is}} \quad ; \quad w_s^{PB} = \frac{\sum_{k=1}^{N^B} p_{kt}^B q_{ks}^B}{\sum_{i=1}^N p_{it} q_{is}} \quad ; \quad (28)$$

$$Q_{st}^{PA} = \frac{\sum_{j=1}^{N^A} p_{jt}^A q_{jt}^A}{\sum_{j=1}^{N^A} p_{jt}^A q_{js}^A} \quad ; \quad Q_{st}^{PB} = \frac{\sum_{k=1}^{N^B} p_{kt}^B q_{kt}^B}{\sum_{k=1}^{N^B} p_{kt}^B q_{ks}^B} . \quad (29)$$

Combining the definition of  $Q_{st}^P$  in (2) with (28) and (29), it can be verified that,

$$Q_{st}^P = \frac{\sum_{i=1}^N p_{it} q_{it}}{\sum_{i=1}^N p_{it} q_{is}} = \sum_{i=1}^N w_{is}^P \left( \frac{q_{it}}{q_{is}} \right) = w_s^{PA} Q_{st}^{PA} + w_s^{PB} Q_{st}^{PB} ; \quad (30)$$

$$w_{is}^P = \frac{p_{it} q_{is}}{\sum_{i=1}^N p_{it} q_{is}} \quad ; \quad \sum_{i=1}^N w_{is}^P = w_s^{PA} + w_s^{PB} = 1. \quad (31)$$

Putting together (27) and (30) yields,

$$\frac{Y_t}{D_t^L} = \frac{Y_s}{D_s^L} (w_s^{PA} Q_{st}^{PA} + w_s^{PB} Q_{st}^{PB}). \quad (32)$$

In turn, (32) can be expressed as a sum of  $Y_t^{PA}$  and  $Y_t^{PB}$  given by,

$$\frac{Y_t}{D_t^L} = Y_t^{PA} + Y_t^{PB} \quad ; \quad Y_t^{PA} = \frac{Y_s w_s^{PA} Q_{st}^{PA}}{D_s^L} \quad ; \quad Y_t^{PB} = \frac{Y_s w_s^{PB} Q_{st}^{PB}}{D_s^L}. \quad (33)$$

In the above case, value index decomposition implies,

$$\frac{Y_t^A}{Y_s^A} = Q_{st}^{PA} P_{st}^{LA} \quad ; \quad \frac{Y_t^B}{Y_s^B} = Q_{st}^{PB} P_{st}^{LB}. \quad (34)$$

Combining (33) and (34) yields this paper's formulas for additive CVMs in the Paasche quantity-Laspeyres price index framework,

$$\frac{Y_t}{D_t^L} = \frac{Y_t^A}{D_s^L P_{st}^{LA} (Y_s^A / Y_s w_s^{PA})} + \frac{Y_t^B}{D_s^L P_{st}^{LB} (Y_s^B / Y_s w_s^{PB})} \quad ; \quad \frac{Y_t}{D_t^L} = \sum_{z=1}^Z \frac{Y_t^z}{D_s^L P_{st}^{Lz} (Y_s^z / Y_s w_s^{Pz})}. \quad (35)$$

By similar explanation applied earlier to (22)—i.e., like the Laspeyres quantity index, the Paasche quantity index that generated (35) is also “consistent in aggregation”—the number of subaggregates,  $z = 1, 2, \dots, Z$ , in (35) can be expanded while maintaining additivity, given the same aggregate GDP in current prices  $Y_t$  and aggregate chained Laspeyres price indexes  $D_t^L$  and  $D_s^L$ . Moreover, by the same explanation for the CVMs in (22), the values in (35) are also CVMs.

In contrast, the formulas in current practice are,

$$\frac{Y_t^A}{D_t^{LA}} = \frac{Y_t^A}{D_s^{LA} P_{st}^{LA}} \quad ; \quad \frac{Y_t^B}{D_t^{LB}} = \frac{Y_t^B}{D_s^{LB} P_{st}^{LB}}. \quad (36)$$

Comparing (35) and (36), it can be seen that CVMs in current practice are not additive,

$$\frac{Y_t}{D_t^L} \neq \frac{Y_t^A}{D_t^{LA}} + \frac{Y_t^B}{D_t^{LB}}. \quad (37)$$

There are two reasons for non-additivity. One is that the CVMs in (36) do not deflate by  $D_s^L$  and, thus, do not make adjustments for changes in the overall price level that are necessary, in light of this paper's additive CVMs in (35). However, aggregate and subaggregate chained Laspeyres price indexes may be different, i.e.,

$$D_s^L \neq D_s^{LA} \quad ; \quad D_s^L \neq D_s^{LB} . \quad (38)$$

Therefore, the inequalities in (38) imply that the CVMs in current practice in (36)—being not deflated by  $D_s^L$ —are not in the same units of measure in terms of reference year prices and, hence, should not even be added.

Moreover, the apportionment of aggregate GDP in current prices based on subaggregate Paasche shares may not equal the GDP subaggregates in current prices. That is,

$$Y_s w_s^{PA} \neq Y_s^A \quad ; \quad Y_s w_s^{PB} \neq Y_s^B . \quad (39)$$

However, it is true that,

$$Y_s w_s^{PA} + Y_s w_s^{PB} = Y_s = Y_s^A + Y_s^B \quad ; \quad w_s^{PA} + w_s^{PB} = 1 . \quad (40)$$

In light of this paper's additive CVMs in (35), the CVMs in current practice in (36) implicitly assume that equalities hold in (39). This assumption, being contrary to (39), is another reason for non-additivity in (37).

### **3.3. CVMs in the Fisher Quantity-Fisher Price Index Framework**

As noted above, CVMs in the Laspeyres quantity-Paasche price index framework given by (22) and in the Paasche quantity-Laspeyres price index framework given by (35) are additive regardless of the number of subaggregates or level of aggregation. This result follows from the “consistency in aggregation” property of the Laspeyres and Paasche quantity indexes, i.e., they can be expressed as the weighted arithmetic sum of an arbitrary number of subaggregate quantity indexes.

The Fisher quantity index, however, does not have the above property but it has the “additive decomposition” property of the Laspeyres and Paasche indexes and, thus, it can be

expressed as a weighted arithmetic sum of each quantity relative  $\left(\frac{q_{it}}{q_{is}}\right)$ . As a result, CVMs are additive at the lowest level subaggregate index—at the level of the *starting* quantity relatives when the Fisher index is constructed—but are not additive for any upper level subaggregate index.

In light of the above, the Fisher index framework needs more expanded analysis by deriving the formula for additive CVMs at the lowest quantity relative and the formula for non-additive CVMs for any upper level subaggregate quantity index. This expanded analysis also attains more importance as it relates directly to the Fisher index framework of US GDP in chained prices (dollars). It will be shown that this paper’s formula for additive CVMs is the additive *level* contributions formula that mathematically corresponds to the additive *growth* contributions formula in the official US GDP framework based on the Fisher index. That is, the above level contribution and growth contribution formulas mathematically imply each other. However, the US does not implement the above additive level contributions formula. Instead, US GDP subaggregates in chained prices are computed in similar manner to what this paper has been referring to as non-additive CVMs in current practice. However, it will be shown that this paper’s non-additive CVMs will yield smaller (absolute) non-additivity residuals than those of the US in current practice.

### **3.3.1. Additive CVMs in the Fisher Quantity-Fisher Price Index Framework**

The Fisher quantity index can be expressed as a weighted arithmetic sum of quantity relatives (van IJzeren, 1952; Dumagan, 2002; Balk, 2004a).<sup>4</sup> Using the quantity and price indexes in (2) and (3), Dumagan (2002) showed that the Fisher quantity index  $Q_{st}^F$  can be expressed as,

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<sup>4</sup> Balk (2004a) pointed out that van IJzeren (1952) was the first to derive a satisfactory *additive* decomposition, “unfortunately in an article in a rather obscure publication series of what is now called Statistics Netherlands.” As a result, van IJzeren’s additive decomposition escaped wider attention in the statistical community and, thus, Balk noted that “Dumagan (2002) independently rediscovered” this decomposition.

$$Q_{st}^F = (Q_{st}^L Q_{st}^P)^{\frac{1}{2}} = \sum_{i=1}^N w_{it}^F \left( \frac{q_{it}}{q_{is}} \right) \quad ; \quad w_{it}^F = \left( \frac{P_{st}^F}{P_{st}^L + P_{st}^F} \right) w_{is}^L + \left( \frac{P_{st}^L}{P_{st}^L + P_{st}^F} \right) w_{is}^P \quad ; \quad (41)$$

$$w_{is}^L = \frac{p_{is} q_{is}}{\sum_{i=1}^N p_{is} q_{is}} \quad ; \quad w_{is}^P = \frac{p_{it} q_{is}}{\sum_{i=1}^N p_{it} q_{is}} \quad ; \quad \sum_{i=1}^N w_{it}^F = \sum_{i=1}^N w_{is}^L = \sum_{i=1}^N w_{is}^P = 1. \quad (42)$$

In (42),  $w_{is}^L$ ,  $w_{is}^P$ , and  $w_{it}^L$  are the weights of the quantity relatives  $\left( \frac{q_{it}}{q_{is}} \right)$  in the Laspeyres, Paasche, and Fisher quantity indexes.

GDP in chained prices ( $Y_s^{*F}$ ,  $Y_t^{*F}$ ) computed by the inflation method based on chained Fisher quantity indexes ( $J_s^F$ ,  $J_t^F$ ) equal GDP in chained prices computed by the deflation method based on chained Fisher price indexes ( $D_s^F$ ,  $D_t^F$ ). Hence,

$$Y_s^{*F} = J_s^F Y_r = \frac{Y_s}{D_s^F} \quad ; \quad Y_t^{*F} = J_t^F Y_r = \frac{Y_t}{D_t^F} \quad ; \quad Y_t^{*F} = Y_s^{*F} Q_{st}^F = \frac{Y_s}{D_s^F} Q_{st}^F. \quad (43)$$

In (43), the superscript  $F$  in  $Y_s^{*F}$  and  $Y_t^{*F}$  denote GDP in chained prices based on the Fisher quantity index.

Now, combine (41) and (43) to obtain,

$$Y_t^{*F} = \frac{Y_t}{D_t^F} = (D_s^F)^{-1} \sum_{i=1}^N w_{it}^F Y_s \left( \frac{q_{it}}{q_{is}} \right) = \sum_{i=1}^N y_{it}^F \quad ; \quad y_{it}^F = w_{it}^F \left( \frac{Y_s}{D_s^F} \right) \left( \frac{q_{it}}{q_{is}} \right). \quad (44)$$

In (44),  $y_{it}^F$  is the additive contribution of a component  $i$  to the *level* of GDP in chained prices. The term  $w_{it}^F Y_s$  apportions  $Y_s$  according to each component's Fisher weight  $w_{it}^F$ . Since  $w_{it}^F Y_s$  is in current prices in  $s$ , it is inflated by the component's relative quantity growth  $\left( \frac{q_{it}}{q_{is}} \right)$  to convert it to a "volume" measure in  $t$ , which is  $w_{it}^F Y_s \left( \frac{q_{it}}{q_{is}} \right)$ . Moreover, the latter is deflated by the aggregate chained Fisher price index  $D_s^F$  to convert it to reference year prices.

In light of the above,  $y_{it}^F = w_{it}^F \left( \frac{Y_s}{D_s^F} \right) \left( \frac{q_{it}}{q_{is}} \right)$  is a CVM.

In turn, (41) and (43) also yield the contribution of the same component to *growth*,

$$\frac{Y_t^{*F}}{Y_s^{*F}} - 1 = \sum_{i=1}^N w_{it}^F \left( \frac{q_{it}}{q_{is}} - 1 \right) = \sum_{i=1}^N g_{it}^F \quad ; \quad g_{it}^F = w_{it}^F \left( \frac{q_{it}}{q_{is}} - 1 \right). \quad (45)$$

It is notable in (45) that  $g_{it}^F$  is equivalent to the present official US formula for additive

contributions to *growth* of US GDP in chained dollars based on the Fisher index (Seskin and Parker, 1998).<sup>5</sup> However, the US does not implement (44) while it implements (45), although these two formulas mathematically imply each other.

Notice in (44) above that the category of each  $y_{it}^F$  is known from the construction of the Fisher GDP quantity index  $Q_{st}^F$ . Therefore, subaggregates can be computed by grouping  $y_{it}^F$  according to definitions in GDP accounts (e.g., *Consumption, Investment, Net Exports, and Government Expenditures* in the expenditure side of GDP or *Agriculture, Industry, and Services* in the product side). Since each  $y_{it}^F$  is an additive CVM, the subaggregates when added together will equal the aggregate CVM or GDP in chained prices  $Y_t^{*F} = Y_t/D_t^F$  in (44).

It may be noted that Dumagan (2008a, 2008b, and 2010) has implemented both (44) and (45) in converting GDP from constant to chained prices in selected ASEAN countries employing the Fisher index framework. The implementation showed that the prevailing non-additivity of CVMs in current practice is a resolvable procedural issue even when employing the more complicated Fisher index. It should be clear that for each component  $i$ , this paper's additive Fisher formulas for *level* contributions in (44)—where each is a CVM—and for *growth* contributions in (45) apply as well to the Laspeyres quantity and Paasche quantity indexes simply by replacing the Fisher weights correspondingly.

### 3.3.2. Non-Additive CVMs in the Fisher Quantity-Fisher Price Index Framework

Let the mutually exclusive subaggregates be the same as before. Hence, the Fisher subaggregate shares  $w_t^{FA}$  and  $w_t^{FB}$  are mutually exclusive partitions of total shares. That is,

$$\sum_{i=i}^N w_{it}^F = w_t^{FA} + w_t^{FB} = 1 \quad ; \quad N = N^A + N^B \quad ; \quad (46)$$

$$w_t^{FA} = \sum_{j=1}^{N^A} w_{jt}^F \quad ; \quad w_t^{FB} = \sum_{k=1}^{N^B} w_{kt}^F \quad ; \quad i = (j, k) \quad ; \quad j \neq k \quad . \quad (47)$$

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<sup>5</sup> The US formula (Seskin and Parker, 1998; Moulton and Seskin, 1999) for the growth rate contribution of a component looks different from  $g_{it}^F$  above but the two have been shown to be equivalent (Dumagan, 2000).



Therefore, the subaggregate Fisher quantity indexes are, by definition,

$$Q_{st}^{FA} = (Q_{st}^{LA} Q_{st}^{PA})^{\frac{1}{2}} \quad ; \quad Q_{st}^{FB} = (Q_{st}^{LB} Q_{st}^{PB})^{\frac{1}{2}} . \quad (48)$$

A property of the Fisher index is that it is only “approximately” consistent in aggregation (Diewert, 1978). In the above instance, this property implies that,

$$Q_{st}^F \approx w_t^{FA} Q_{st}^{FA} + w_t^{FB} Q_{st}^{FB} . \quad (49)$$

Therefore, by combining (43) and (49),

$$\frac{Y_t}{D_t^F} \approx \frac{Y_s}{D_s^F} (w_t^{FA} Q_{st}^{FA} + w_t^{FB} Q_{st}^{FB}) . \quad (50)$$

For further analysis, define the Fisher subaggregates in (50) as,

$$Y_t^{FA} = \frac{Y_s w_s^{FA} Q_{st}^{FA}}{D_s^F} \quad ; \quad Y_t^{FB} = \frac{Y_s w_s^{FB} Q_{st}^{FB}}{D_s^F} . \quad (51)$$

As before, value index decomposition also applies in this case. That is,

$$\frac{Y_t^A}{Y_s^A} = Q_{st}^{FA} P_{st}^{FA} \quad ; \quad \frac{Y_t^B}{Y_s^B} = Q_{st}^{FB} P_{st}^{FB} . \quad (52)$$

Finally, combining (50) to (52) yields this paper’s non-additive Fisher CVMs,

$$\frac{Y_t}{D_t^F} \approx \frac{Y_t^A}{D_s^F P_{st}^{FA} (Y_s^A / Y_s w_s^{FA})} + \frac{Y_t^B}{D_s^F P_{st}^{FB} (Y_s^B / Y_s w_s^{FB})} \quad ; \quad \frac{Y_t}{D_t^F} \approx \sum_{z=1}^Z \frac{Y_t^z}{D_s^F P_{st}^{Fz} (Y_s^z / Y_s w_s^{Fz})} . \quad (53)$$

In (53), the number of subaggregates,  $z = 1, 2, \dots, Z$ , can be expanded and, thereby, affect the non-additivity residual given the same  $Y_t, D_s^F$ , and  $D_t^F$ . However, as the number of subaggregates increases, each subaggregate approaches the lowest quantity relative. At the limit, (53) becomes (44) and the residual is zero.

In contrast, the Fisher CVMs in current practice are,

$$\frac{Y_t^A}{D_t^{FA}} = \frac{1}{D_s^{FA}} \left( \frac{Y_t^A}{P_{st}^{FA}} \right) \quad ; \quad \frac{Y_t^B}{D_t^{FB}} = \frac{1}{D_s^{FB}} \left( \frac{Y_t^B}{P_{st}^{FB}} \right) . \quad (54)$$

In light of (53), the CVMs in (54) must be non-additive,

$$\frac{Y_t}{D_t^F} \neq \frac{Y_t^A}{D_t^{FA}} + \frac{Y_t^B}{D_t^{FB}} . \quad (55)$$

In this case, there are two reasons for non-additivity. One is that the CVMs in (54) do not deflate by  $D_s^F$  and, thus, do not make adjustments for changes in the overall price level that are necessary, in light of this paper's CVMs in (53). However, aggregate and subaggregate chained Fisher price indexes could differ from each other, i.e.,

$$D_s^F \neq D_s^{FA} \quad ; \quad D_s^F \neq D_s^{FB} . \quad (56)$$

Therefore, the inequalities in (56) imply that the CVMs in current practice in (54)—being not deflated by  $D_s^F$ —are not even in the same units of measure in terms of reference year prices and, hence, are not additive in principle.

Moreover, the apportionment of aggregate GDP in current prices based on subaggregate Fisher shares may not equal the GDP subaggregates in current prices. It can be verified that,

$$Y_s w_s^{FA} \neq Y_s^A \quad ; \quad Y_s w_s^{FB} \neq Y_s^B . \quad (57)$$

However, it is true that,

$$Y_s w_s^{FA} + Y_s w_s^{FB} = Y_s = Y_s^A + Y_s^B \quad ; \quad w_s^{FA} + w_s^{FB} = 1 . \quad (58)$$

In view of this paper's CVMs in (53), CVMs in current practice in (54) implicitly assume that equalities hold in (57). This contrary assumption is another reason for non-additivity in (55).

However, given that the inequalities in (56) and (57) will hold, the non-additivity in current practice would be more severe. That is, this paper's CVMs in (53) will yield smaller (absolute) non-additivity residuals than the current practice CVMs in (54).

#### **4. Empirical Comparison Between This Paper's CVMs and Those in Current Practice**

The analytical differences between this paper's CVMs and those in current practice may be illustrated empirically by applying them to Philippine GDP data in Table 1 and Table 2.

To facilitate computation of the chain-type price and quantity indexes in (2) and (3), the data are treated like those at the commodity level for *illustration* purposes. That is, data on GDP components in current prices in Table 1 are interpreted as  $(p_{is}q_{is}, p_{it}q_{it})$  and those in constant prices in Table 2 are interpreted as  $(p_{ib}q_{is}, p_{ib}q_{it})$ .

**Table 1. Philippine GDP in Current Prices (Billion Pesos)**

	2002	2003	2004	2005	2006	2007	2008
<b>Production Sources</b>							
Agriculture and Fishery	595.6	629.7	730.7	774.1	851.1	932.3	1,099.2
Forestry	1.8	2.3	3.5	4.3	4.3	4.1	4.4
Mining and Quarrying	33.5	43.6	52.9	63.6	75.6	108.2	109.0
Manufacturing	915.2	1,004.0	1,122.9	1,264.7	1,381.2	1,463.8	1,671.4
Construction	185.7	194.1	212.8	210.2	235.2	304.6	352.8
Electricity Gas and Water	124.1	137.2	155.8	196.7	216.1	230.8	238.0
Transport Communication and Storage	276.9	313.2	367.4	413.9	446.2	478.4	506.6
Trade	556.3	602.8	681.7	776.9	877.5	981.1	1,124.3
Finance	170.5	186.0	215.7	263.4	312.0	362.0	414.5
Ownership of Dwellings and Real Estate	252.9	270.1	292.2	320.4	350.7	374.0	416.8
Private Services	484.9	556.5	653.3	742.0	830.2	936.9	1,045.4
Government Services	362.3	377.1	382.7	413.9	452.6	472.8	515.4
<b>GDP in Current Prices</b>	<b>3,959.6</b>	<b>4,316.4</b>	<b>4,871.6</b>	<b>5,444.0</b>	<b>6,032.6</b>	<b>6,648.8</b>	<b>7,497.5</b>

Source: National Statistical Coordination Board.

**Table 2. Philippine GDP in Constant 1985 Prices (Billion Pesos)**

	2002	2003	2004	2005	2006	2007	2008
<b>Production Sources</b>							
Agriculture and Fishery	206.5	214.4	225.1	229.6	238.5	250.0	257.9
Forestry	0.7	0.9	1.3	1.4	1.3	1.3	1.4
Mining and Quarrying	15.3	17.9	18.3	20.0	18.8	23.7	23.8
Manufacturing	252.6	263.3	278.6	293.3	306.8	317.2	330.9
Construction	46.7	47.1	48.7	45.9	49.2	61.9	66.9
Electricity Gas and Water	34.2	35.3	36.8	37.7	40.1	42.7	46.1
Transport Communication and Storage	80.8	87.7	97.6	104.8	111.4	120.7	125.2
Trade	170.8	180.5	192.7	203.6	216.0	233.8	244.8
Finance	48.9	51.8	56.9	64.6	72.0	81.3	85.3
Ownership of Dwellings and Real Estate	48.9	51.0	53.7	56.5	59.7	63.2	67.7
Private Services	78.0	84.4	93.4	100.4	107.9	116.4	123.0
Government Services	49.6	51.0	51.2	53.8	55.2	56.5	59.1
<b>GDP in Constant 1985 Prices</b>	<b>1,033.0</b>	<b>1,085.1</b>	<b>1,154.3</b>	<b>1,211.5</b>	<b>1,277.0</b>	<b>1,368.6</b>	<b>1,432.1</b>

Source: National Statistical Coordination Board.

Computing the chain-type price and quantity indexes require cross-products of prices and quantities ( $p_{is}q_{it}$ ,  $p_{it}q_{is}$ ) that can be obtained from Table 1 and Table 2 as follows,

$$\frac{p_{is}q_{is}}{p_{ib}q_{is}} = \frac{p_{is}}{p_{ib}} ; \frac{p_{is}}{p_{ib}} p_{ib}q_{it} = p_{is}q_{it} ; \frac{p_{it}q_{it}}{p_{ib}q_{it}} = \frac{p_{it}}{p_{ib}} ; \frac{p_{it}}{p_{ib}} p_{ib}q_{is} = p_{it}q_{is} . \quad (59)$$

Moreover, the same data also yield the price and quantity ratios,

$$\frac{p_{it}q_{it}/p_{is}q_{is}}{p_{ib}q_{it}/p_{ib}q_{is}} = \frac{p_{it}}{p_{is}} ; \frac{p_{ib}q_{it}}{p_{ib}q_{is}} = \frac{q_{it}}{q_{is}} . \quad (60)$$

Overall GDP and sectoral GDP chain-type and chained quantity and price indexes—with 2005 as the reference year—were computed. The sectoral indexes are obtained by combining production sources of GDP into **Agriculture** (agriculture, fishery, and forestry), **Industry**

(mining, quarrying, manufacturing, construction, electricity, gas, and water), and **Services** (transport, communication, storage, trade, finance, ownership of dwellings, real estate, private services, and government services).

Comparisons are presented for each of the three quantity-price index pairs—as noted in the titles of the following tables—and are reflected by the residuals, the difference between GDP in CVM and the sum of the sectoral CVMs.

In Table 3, the additivity of this paper’s CVMs is shown by the residuals that can be verified to be necessarily zero in each year. In contrast, the CVMs in current practice are non-additive as shown by the non-zero residuals, *except* in the reference year 2005 and in the year *immediately after* in 2006. The residual of the CVMs in current practice in the reference year 2005 equals zero necessarily because in this year the chained quantity and price indexes equal 1. However, Schreyer (2004) noted and Dumagan (2010) showed analytically that the residual is necessarily zero also in the year *immediately after* the reference year in the Laspeyres quantity-Paasche price index framework.

**Table 3. Overall and Sectoral Philippine GDP in Chained 2005 Prices Implementing Laspeyres Quantity-Paasche Price Index Framework**

	2002	2003	2004	2005	2006	2007	2008
<b>GDP in CVM (Billion Pesos)</b>							
(Chained 2005 Prices)	4,637.48	4,869.81	5,179.63	5,444.04	5,745.11	6,152.39	6,448.56
<b>Agriculture CVM</b>							
This Paper (additive)	699.49	727.06	749.76	796.22	808.34	853.54	894.16
Current Practice (non-additive)	698.36	725.68	763.10	778.37	808.34	846.90	873.94
<b>Industry CVM</b>							
This Paper (additive)	1,453.58	1,532.66	1,637.08	1,701.08	1,817.46	1,951.40	2,048.85
Current Practice (non-additive)	1,530.60	1,591.60	1,674.90	1,735.15	1,817.46	1,951.85	2,050.85
<b>Services CVM</b>							
This Paper (additive)	2,484.41	2,610.09	2,792.79	2,946.74	3,119.32	3,347.44	3,505.55
Current Practice (non-additive)	2,410.64	2,553.71	2,741.84	2,930.52	3,119.32	3,353.82	3,524.33
<b>Residual</b>							
This Paper (additive)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Current Practice (non-additive)	-2.12419	-1.17223	-0.21154	0.00000	0.00000	-0.17376	-0.55295

**Source:** Author's calculations based on this paper's additive CVMs in (22) and non-additive CVMs in current practice in (24).

In Table 4, this paper’s CVMs are additive as shown by necessarily zero residuals while CVMs in current practice are non-additive *except* in the reference year 2005 and in the year

immediately *before* in 2004 when the residuals are also necessarily zero. These exceptions can be verified analytically in the Paasche quantity-Laspeyres price index framework.

**Table 4. Overall and Sectoral Philippine GDP in Chained 2005 Prices Implementing Paasche Quantity-Laspeyres Price Index Framework**

	2002	2003	2004	2005	2006	2007	2008
<b>GDP in CVM (Billion Pesos)</b>							
(Chained 2005 Prices)	4,635.93	4,869.12	5,179.30	5,444.04	5,743.32	6,152.03	6,446.64
<b>Agriculture CVM</b>							
This Paper (additive)	699.46	712.89	780.55	778.37	814.43	866.44	948.84
Current Practice (non-additive)	698.46	725.78	763.09	778.37	808.34	846.91	873.96
<b>Industry CVM</b>							
This Paper (additive)	1,473.43	1,555.44	1,641.91	1,735.15	1,816.48	1,949.83	2,038.81
Current Practice (non-additive)	1,530.23	1,591.60	1,674.69	1,735.15	1,815.97	1,951.94	2,050.41
<b>Services CVM</b>							
This Paper (additive)	2,463.05	2,600.79	2,756.84	2,930.52	3,112.41	3,335.76	3,458.99
Current Practice (non-additive)	2,408.39	2,552.02	2,741.52	2,930.52	3,119.17	3,353.87	3,524.36
<b>Residual</b>							
This Paper (additive)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Current Practice (non-additive)	-1.15138	-0.28613	0.00000	0.00000	-0.16040	-0.68469	-2.08687

**Source:** Author's calculations based on this paper's additive CVMs in (35) and non-additive CVMs in current practice in (36).

Finally, Table 5 shows this paper's additive CVMs that are obtained by aggregation starting from the lowest quantity relative up to the sector level. Their residuals are necessarily zero. However, starting at the sector level, this paper's CVMs are non-additive like the CVMs in current practice.

**Table 5. Overall and Sectoral Philippine GDP in Chained 2005 Prices Implementing Fisher Quantity-Fisher Price Index Framework**

	2002	2003	2004	2005	2006	2007	2008
<b>GDP in CVM (Billion Pesos)</b>							
(Chained 2005 Prices)	4636.70	4869.47	5179.46	5444.04	5744.22	6152.21	6447.60
<b>Agriculture CVM</b>							
This Paper (additive)	699.47	719.98	765.15	787.30	811.38	859.99	921.51
This Paper (non-additive)	699.47	719.98	765.16	787.30	811.38	859.99	921.51
Current Practice (non-additive)	698.41	725.73	763.09	778.37	808.34	846.91	873.95
<b>Industry CVM</b>							
This Paper (additive)	1,463.50	1,544.05	1,639.50	1,718.11	1,816.97	1,950.62	2,043.83
This Paper (non-additive)	1,463.51	1,544.05	1,639.50	1,718.11	1,816.97	1,950.62	2,043.83
Current Practice (non-additive)	1,530.42	1,591.60	1,674.80	1,735.15	1,816.71	1,951.89	2,050.63
<b>Services CVM</b>							
This Paper (additive)	2,473.73	2,605.44	2,774.81	2,938.63	3,115.86	3,341.60	3,482.27
This Paper (non-additive)	2,473.73	2,605.44	2,774.82	2,938.63	3,115.86	3,341.60	3,482.27
Current Practice (non-additive)	2,409.51	2,552.86	2,741.68	2,930.52	3,119.24	3,353.84	3,524.34
<b>Residual</b>							
This Paper (additive)	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
This Paper (non-additive)	-0.00864	0.00079	-0.00631	0.00053	0.00006	-0.00054	0.00072
Current Practice (non-additive)	-1.63757	-0.72905	-0.10577	0.00000	-0.08012	-0.42923	-1.31997

**Source:** Author's calculations based on this paper's additive CVMs in (44), non-additive CVMs in (53) and non-additive CVMs in current practice in (54).

However, this paper's residuals are smaller in absolute size than the current practice residuals, except in the reference year when CVMs in current practice necessarily have zero residuals because chained indexes equal 1. Chained indexes equal 1 in the reference year also in this paper's CVMs but it does not suffice to yield a zero residual in Table 5 as can be seen in (53) that generated this paper's sectoral CVMs. Notice that setting  $D_s^F = D_r^F = 1$  in (53) will not necessarily yield a zero residual because the approximation rules in this case. However, this paper's residuals become zero when round off to one decimal place, implying that this paper's CVM are "approximately additive."

Except in the reference year, the finding above stands that this paper's CVMs have smaller absolute residuals than those in current practice. This finding has special relevance to the US because US GDP subaggregates in chained dollars are now calculated in the same way as the CVMs in current practice in Table 5. So, the US has two better alternatives which are either this paper's additive or approximately additive CVMs in Table 5.

## **5. Conclusion**

This paper has provided the analytical bases and empirical illustrations for additive CVMs in all three quantity-price index frameworks for GDP in chained prices. This paper's CVMs necessarily have zero residuals in the Laspeyres quantity-Paasche price and Paasche quantity-Laspeyres price index frameworks and, thus, are superior to CVMs in current practice that may have non-zero residuals in these frameworks except in two years around the reference year. For the Fisher quantity-Fisher price index framework, this paper's CVMs are either additive or non-additive but even in the latter case, they are approximately additive and, thus, superior to the non-additive CVMs in current practice.

In light of the above analytic and empirical findings, this paper's CVMs should replace those in current practice.

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