PRI Discussion Paper Series (No.08A-06)

Delay in Fiscal Reform

Kentaro Katayama

Visiting Scholar
Policy Research Institute, Ministry of Finance, Japan

January 2008

Research Department Policy Research Institute, MOF
3-1-1 Kasumigaseki, Chiyoda-ku, Tokyo 100-8940, Japan
TEL 03-3581-4111

Delay in Fiscal Reform

Kentaro Katayama*

University of Warwick
and
Ministry of Finance, Japan

e-mail: k.katayama@warwick.ac.uk

March 24, 2008

Abstract

This paper analyzes the political economy of delayed agreement over fiscal reforms, in a setting where two interest groups can bargain over the allocation of the cost of the stabilization. This contrasts with the classic contribution of Alesina and Drazen, who assume that a group which concedes earlier bears a *fixed* disproportionate share of the burden. The approach of this paper is to study an alternating offers model of bargaining in the economic environment of Alesina and Drazen i.e. where bargaining takes place in continuous time, and there is two-sided uncertainty. This allows a systematic comparison of expected delay in the bargaining game and in the concession game of Alesina and Drazen. When interest groups are sufficiently patient, or when shares in the concession game are very unequal, agreement is reached more quickly on average under bargaining. But, both games have the common feature that delay signals the "toughness" of the interest group.

JEL classification: C78; D72; E62; H39
Key Words: Strategic Delay; Stabilization; Bargaining; Coalition.

*I am deeply grateful to Professor Ben Lockwood for his insightful discussion and suggestion. I also thank Bhasker Dutta, Amrita Dhillon and the seminar participants at University of Warwick for valuable comments. All errors are of course mine. Views and opinions expressed in this paper are those of the author and do not necessarily reflect those of the Ministry of Finance.

1 Introduction

The sustainability problem of government deficits is currently in the spotlight all over the world. It is vital for developing countries seeking to avoid bankruptcy to develop adequate debt management policies, especially in the wake of the 1997 financial crisis. This need for fiscal sustainability is also shared by industrialized countries, even the European Union and the United States. Spending for social programs continues to rise, reflecting the rapidly aging population. In order to keep the fiscal condition sustainable, in general, cutting spending, increasing taxes, or both would be required. On the other hand, countries sometimes pursue economic policies that are widely recognized as unsustainable and costly to all groups. Why such a fiscal reform is delayed? How can we promote the reform?

The current theoretical explanations for the delay are summarized as follows. First, free-rider problem allows every group to have incentives to avoid sharing the burden as in Alesina and Drazen (1991) and Velasco (2000). Even when it was widely recognized that fiscal adjustment were necessary, a social consensus on the sharing of the burden of stabilization programme was difficult to achieve. Secondly, lobbying activities of interest groups may affect the date of stabilization as in Tornell (1998). Expenditure cuts might involve the so called "pork-barrel" problem, and thus also bring on political difficulties. Thirdly, every group is likely to expect that things would get better before such measures being implemented. More practically, politicians tend to avoid tax increase, because this policy change can burden citizens directly. Therefore, given the expectation, the reform is likely to be politically shirked by national rebellion.

Among them, one of the most influential explanations has been taken by Alesina and Drazen (1991). They applied "war-of-attrition" to explain why the reforms are delayed, where they elaborated on earlier ideas by Riley (1980) and Bliss and Nalebuff (1984).[1]

In their model, even if overall benefits are obviously expected to exceed the overall cost of the reform, the reform can be delayed as long as the burden of stabilization is unequally distributed. Stabilization occurs only when one group concedes and bears a disproportionate share of the burden, which is exogenously fixed. The groups have to concede at some point because there is the cost of waiting, which is private information. Thus, as long as participants in the process believe that someone else may have a higher cost of waiting, concede earlier and then accept a larger share of the burden, every group has an incentive to attempt to shift the burden of stabilization onto other groups by waiting for the action of others, as in the free-riding behaviour in provision of public goods. As a result, stabilization does not occur immediately. If every group takes the cooperative strategy, they could obtain the benefit, but this solution can not be obtained on equilibrium path. This is called "a war of attrition" among interest groups. They solve for the expected

[1] Riley (1980) built the biological war-of-attrition model, and Bliss and Nalebuff (1984) focused on the public good model.

time of stabilization in a model of "rational" delay and analyze it relating to several political and economic variables.

This influential paper Alesina and Drazen (1991) has been extended in several directions. In Drazen and Grilli (1993)'s non-monetary model without an explicit inflation rate, monetization is introduced as a distortionary tax before stabilization. Casella and Eichengreen (1996) analyzes the conditions under which a foreign aid can accelerate stabilization. The aid are used to reduce the fiscal burden of the group that concedes first. Spolaore (2004) examines the relative performance of three different government systems in terms of the efficiency of stabilization. Martinelli and Escorza (2007) modified the assumption in Alesina and Drazen (1991) that each group chooses the same expected concession time due to *ex-ante* symmetry which leads to a symmetric Nash equilibrium. Further discussions will be described in the following section.

While these models successfully figure out the mechanism of delayed reforms, most of literatures with the concession framework assume that each group may take a non cooperative behaviour. This concession process can be regarded as a dead locked situation in the divided government with endless debates where interest groups have no ways to compromise. But it can be also considered that in the process of reaching an agreement on stabilization, each group in a coalition government may negotiate or bargain on the share of the cost of stabilization.[2] Hence it is natural to assume that the groups would negotiate over the combination of policies by offering the proposal and that all groups could agree with an allocation of the cost as a consequence of the bargaining.[3]

Stabilization policies then can be made through the bargaining in the legislative process. Bargaining over the share can be a device to adopt the information dynamics into the concession process. In other words, with the legislative bargaining process for incompletely informed groups or parties, the expected delay of reform can be different from in the war-of-attrition setting. Needless to say, both process should be compared in order to make a reform with less delay. Hence the aim of this paper is to describe the model of delayed stabilization in both bargaining and concession framework under the same economic environment in order to compare the expected delay of both processes. This comparison may lead to some policy implications.

Very few papers deal with bargaining of delayed reform. Hsieh (2000) and Sibert and Perraudin (2000) extend Alesina and Drazen (1991)'s war of attrition model

[2]See Persson and Tabellini (2000) chapter 7.2 which describes the legislative bargaining.

[3]In order to see this point practically, consider the case where the interest groups could be the representatives of firms and workers; a group for firms is likely to insist to raise the value added tax or build the proportional tax, while the group for workers tends to propose an increase in the capital tax or more strengthened progressive tax. The final solution would be a mixed policy of these taxes. They might also fight upon the burden of welfare benefits. For concrete example, as in Alesina and Drazen (1991), the components of successful Poincaré stabilization in 1926 were considered as a revised version of his initial proposal of 1924, which had been denied by the other party.

by endogenizing the distribution of the stabilization costs through a bargaining process. But due to their strong assumptions, they have limitation to compare with the result of Alesina and Drazen (1991). Both of them suppose *one-sided* uncertainty. Their models also assume *one-sided* offering.[4] As Alesina and Drazen (1991) considered conflicts among identical interest groups with private information, alternating offer would be rather reasonable assumption. Furthermore, both of them could not indicate the length of delay, as they assume a finite horizon model with discrete time, i.e., two or three stages.

Unlike them, in this paper we will build the two-sided incomplete information and alternating offer model under an infinite horizon with continuous time, regarding Cramton (1992) for describing and characterizing sequential equilibrium. The interest groups use a delay between offers as a strategic variable. Then we could obtain the expected delay in the bargaining game which can be directly compared with the delay in the concession game.

Again while my paper is related to Alesina and Drazen (1991) in the analysis of the delay of macro stabilizations, the contribution of this paper is to set up and solve the bargaining game in the legislative process in addition to the war-of-attrition game as in Alesina and Drazen (1991) under same circumstances; in this paper, the interest groups can communicate and exchange private information through a generalized bargaining system.[5]

Main findings in this paper are summarized as follows. We build a model of delayed agreement on fiscal reforms with microfoundations, in which two interest groups conflict over the share of the cost of the stabilization. On the process of reaching an agreement, the groups can take two types of interaction: *Bargaining* where groups can offer the share, and *Concession* where one of the groups has to bear the fixed disproportionate share. In both processes, interest groups have incentives to shift the burden of the cost on the other by waiting for the other to offer or concede first. Hence the delay can be used as a signal of the toughness of the interest group. Under the same economic setting, we obtain formulas for the expected delay in both the bargaining and concession game, which allow a systematic comparison. We then show that when interest groups are sufficiently patient, agreement is reached more quickly on average under the bargaining game. Instead, when groups are very impatient, the concession game may lead to the earlier stabilization. Furthermore, in more polarized economy where shares in the concession game are very unequal, it is more likely that the bargaining process leads to the shorter delay. Therefore bargaining process can be a tool to smooth the expected delay. We also show that as the expected delay becomes shorter, total welfare increases. Hence our results will shed a light on the selection of political process in a divided government.

The rest of the paper is organized as follows. Related literatures are reviewed in Section 2. The set up of the model is described in Section 3. The equilibrium

[4] Hsieh (2000) applies conflicts among capitalists and labours over wages.

[5] It is said that a large part of the political economy literature are silent on how the relevant agents acquire and aggregate information. See Drazen (2001).

in the bargaining process are presented in Section 4. The War of Attrition game is discussed in Section 5. Section 6 analyze the welfare in the economy. Section 7 compares two results and obtains policy implication. Section 8 concludes.

2 Related Literature

First of all, we will briefly look at Alesina and Drazen (1991). They applied "war-of-attrition" to explain why the reforms are delayed. They solve for the expected time of stabilization in a model of "rational" delay and analyze it relating to several political and economic variables. As long as participants in the process believe that someone else may have a higher cost of waiting, stabilization does not occur immediately. If all groups are identical like a single agent, then one stabilizes immediately as one knows that he will be the stabilizer with probability one. An increase in the cost of waiting will move the expected date of a stabilization forward. If the gain from waiting is larger, each group holds out longer. The difference in the shares of the burden of stabilization could be interpreted as representing the degree of political cohesion in the society. If the difference is larger, the economy is more polarized or less cohesive. As the relative burden of stabilization is unequally distributed, it might be harder to reach agreements on how to allocate tax increases among coalition partners. If the burden of stabilization is shared relatively equally, stabilization occurs more immediately.

In Drazen and Grilli (1993)'s non-monetary model without an explicit inflation rate, monetization is introduced as a distortionary tax before stabilization. They show that highly distortionary finance can be welfare improving. Higher inflation will shorten the delay by raising the cost of living in the economy before a stabilization. There is a trade-off with higher inflation of lowering welfare until a stabilization and inducing an earlier time of agreement on use of nondisortinary financing. This paper also shows by simulation methods that the U-shape relationship between the expected utility and inflation rate, and between the expected concession time and the inflation rate.

During a war of attrition, a change in the environment may lead to a change in the date of concession. For example, Casella and Eichengreen (1996) analyzes the conditions under which foreign aid can accelerate stabilization, where the aid reduces the fiscal burden of the group that concedes first. Incoming aid will reduce future fiscal burden and therefore this should hasten stabilization. But at the same time, there is an incentive for players to postpone concession until arriving closer to the moment of transfer. Due to distributional conflicts to shift the cost onto its rival, the aid announced relatively early in the inflation process can accelerate stabilization. On the other hand, the aid announced or delivered after a considerable delay can have the opposite effect. Thus the effects of aid are contingent and timing of release of information is crucial.

Spolaore (2004) analyzes the relative performance of three different government systems in terms of inefficient delays of stabilization to occur; *cabinet system*, in which one decision maker has full control over adjustment policies; *consensus system*, in which adjustment policies must be agreed upon by all agents; and *checks-and-balances system*, in which one agent decides what policy adjustment should be used, but the remaining agents may veto its use. The result is that checks-and-balance system dominates pure consensus systems, but may or may not outperform cabinet systems. The outcome depends on the degree of political fragmentation and the size of distributions of shocks.

In symmetric Nash equilibrium as in Alesina and Drazen (1991), each group chooses the same expected concession time. Martinelli and Escorza (2007) modified this strong assumption of ex ante symmetry. As the gains from stabilization of each group are drawn from the different distribution, an interest group, which is more exposed to inflation costs, will be likely to give in immediately, leading to earlier stabilization. They show by simulation that, if the expected cost of inflation increases for the more exposed group, then the probability of immediate reform increases. The effect of a reduction in the cost of inflation, benefiting mostly the less exposed group, may be a shorter delay. Intuition is that the more exposed group will prefer to give up at time zero by realizing the cost.

The paper also shows by simulation that, if the distributive outcome of reform become more unequal, the probability of immediate agreement increases as the more exposed group which realize the high cost, prefers to give up at time zero. But at the same time, the expected delay of stabilization increases, since an increase in the distributional outcome raises the willingness to fight against the opponent.

Furthermore, a policymaker may have an incentive to abandon fiscal responsibility and revert to inflation method, as this way is costless rather than other taxation with legislative process. If the public is uncertain about the degree of commitment of the policymaker to fiscal responsibility, success is less likely.

3 The Model

In this paper, we analyze a stripped-down version of Alesina and Drazen (1991) in order to analyze the difference between bargaining results and concession results. Time is continuous and infinite $t \in (0, \infty)$. At $t = 0$, the government deficit is zero and the economy is hit by a shock reducing tax revenue by amount τ. From then until the date of stabilization T, the government deficit τ has to be financed at each period by distortionary taxation.[6]

There are two political groups or parties $i = L, R$.[7] Before stabilization, each

[6] In Alesina and Drazen (1991), deficit is covered by distortionary taxation and governmental bond. But this assumption is not very essential. For example, Martinelli and Escorza (2007) put an assumption of no bond issue, and Drazen and Grilli (1993) considered the monetary version of this model where the deficit is financed by inflation tax without issued bond.

[7] This may be generalized easily to more than two groups in a case where we set the assumption of exogenous fixed shares of cost of stabilization. On the other hand, in the bargaining model,

party pays one half of the distortionary taxation and, in addition, suffers from some welfare loss $\theta_i \tau$, where θ_i is private information to each party i. The parameters θ measures the deadweight loss of tax burden $\frac{\tau}{2}$ suffered by group $i = L, R$. Thus the total loss due to taxation suffered by group i at any time before stabilization is described as $\frac{\tau}{2} + \theta_i$.

The type of group i, θ_i , is independently drawn at $t = 0$ from a common continuous distribution $F(\theta)$ with $\theta \in [\underline{\theta}, \bar{\theta}]$.[8] This θ_i is private information, which is known only to the group itself, while the other only knows the distribution $F(\theta)$ and its positive density function $f(\theta)$.

The distortions disappear with stabilization at $t = T$. In other words, all groups benefit from stabilization because of the existence of distortionary taxes before stabilization. But the two groups can negotiate, or bargain, over the sharing of the burden of stabilization. Let κ_i be the share borne by group i ; $\kappa_L + \kappa_R = 1$. This means that after stabilization at T, group i must make a tax payment to the government of $\kappa_i \tau$ in perpetuity.

We are now ready to write down the utility flows to group i, given a stabilization occurs at T, with shares κ_L and κ_R. For group i, utility at instant t is equal consumption, which in turn is equal to exogenous income (normalized to zero), minus tax payments and the deadweght loss before T,

$$u_i(t) = \begin{cases} (-1/2 - \theta_i)\tau & \text{if} \quad t \le T \\ -\kappa_i \tau & \text{if} \quad t > T. \end{cases} \tag{1}$$

We also assume that groups are infinitely lived and discount the future according to a common rate r, which is known to both groups. Hence total discounted payoff with stabilization at $t = T$ can be written as

$$\begin{aligned} V(T, \kappa_i; \theta_i) &= \int_0^\infty u_i(t) e^{-rt}\, dt \\ &= -\int_0^T \left(\frac{\tau}{2} + \theta_i \tau\right) e^{-rt}\, dt - \int_T^\infty \kappa_i \tau e^{-rt}\, dt \\ &= \frac{\tau}{r} \left[e^{-rT} \left(\theta_i + \frac{1}{2} - \kappa_i\right) + \left(-\frac{1}{2} - \theta_i\right)\right]. \end{aligned} \tag{2}$$

In what follows, note that the only part of V that depends on T and κ_i is $e^{-rT}\left(\theta_i + \frac{1}{2} - \kappa_i\right)$, so we can think of each group as maximizing just

$$U^i(T, \kappa_i; \theta_i) = e^{-rT} \left(\theta_i + \frac{1}{2} - \kappa_i\right). \tag{3}$$

In the bargaining game, to be described below, the players bargain over κ_L and κ_R by making alternating offers. However, it is convenient and without loss of

multiple players case might become very complicated.

[8]In concession setup, we need to set the lower bound of $\underline{\theta} > 0$ to avoid no concession cases.

generality to transform the problem by assuming that they bargain over the object $\alpha = \frac{1}{2} - \kappa_R$, which must lie between $-\frac{1}{2}$ and $\frac{1}{2}$. A positive α means $\kappa_R < \frac{1}{2}$, ie group R has to pay relatively smaller share of the burden, and conversely a negative α means that group L has to pay relatively smaller share of the burden.[9] Then, expected payoffs over agreement on (α, T) can be written as

$$U^L = e^{-rT}(\theta_L - \alpha) \quad \text{and} \quad U^R = e^{-rT}(\theta_R + \alpha). \tag{4}$$

In the concession framework, which will be solved in Section 5, the share of cost of stabilization is *exogenously* determined. A group which concedes earlier than the other has to bear a higher share $\kappa > \frac{1}{2}$, while the rest of this, $1 - \kappa$, is borne by the other group. Each party maximizes its expected lifetime payoff by choosing a time to concede if the other party has not yet conceded. They, therefore, have an incentive to wait till the other takes the initiative of the reform. Note that, in the concession game, we assume $\theta_i + \frac{1}{2} - \kappa_i > 0$ to avoid no concession, meaning that stabilization occurs in finite time with probability one. If this does not hold, groups will always postpone their decisions to concede or offer.

4 Equilibrium Delay in the Bargaining Game

In this section, we analyze the war-of-attrition with the bargaining process by allowing each group to bargain over the share of cost of stabilization, while this parameter in Alesina and Drazen (1991) is exogenous. We adopt Cramton (1992)'s model for this purpose.

4.1 Description of the Game

At $t = 0$, each group gets to know their own private information θ_i of how much exposed to distortionary taxes. Then they start to negotiate over α. In this section, as θ_i can be used to express a value revealed by offering in the bargaining process, true valuations are often denoted by L and R respectively, in order to explicitly distinguish the true value with the revealed value.

If at time t neither group L nor R has yet made an offer of α, either group L or R can make an offer at t. If, for example, group L has made an offer at t, group R either (a) accept, in which the game ends, with payoffs given by equation (2), or (b) after a minimum period t_0, make a counter offer. The minimum period is set at $t^0 = -(1/r)\log\delta$ as in Admati and Perry (1987). δ is the discount factor from one period delay.[10] As they make an offer alternately, they can choose the delay between offers as a signal.

As time passes, the gain from stabilization is discounted by r. Thus both groups prefer agreement on today to the same agreement tomorrow. We now show that

[9] Needless to say, if the cost is equally distributed at $\kappa_R = \kappa_L = 1/2$, $\alpha = 0$.

[10] Later on, we analyze the limiting case such that the minimum time between offers goes to zero ($\delta \to 1$), in order to compare with the concession set up.

delay is more costly for a high θ than for a low θ. More precisely, a high θ type is willing to give a bigger concession in term of α to get a given reduction in the stabilization time. In other words, the utility function satisfies the *single crossing property*, with which utility of interest each group is strictly monotone in α and the slope of indifference curve is strictly monotone in θ. Then the indifference curves of θ and θ' cross only once. Figure 1 shows this property for the interest group L and Figure 2 for the group R respectively.

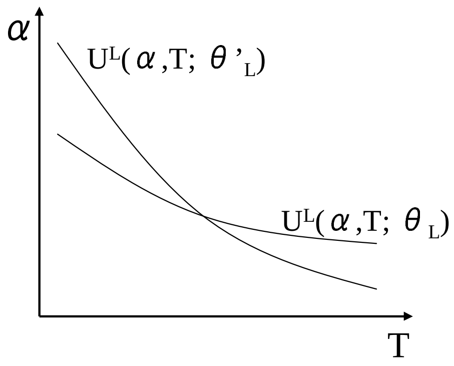

Figure 1: $\theta'_L > \theta_L$

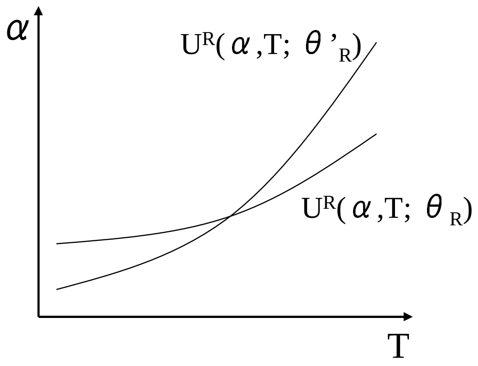

Figure 2: $\theta'_R > \theta_R$

Lemma 1. *The payoffs of each group satisfy the single crossing property.*

Proof. For R, the expected payoff over the bargaining process is $U^R(\alpha, T; \theta_R) = e^{-rT}(\theta_R + \alpha)$. Hence $\partial U^R/\partial\alpha = e^{-rT} > 0$ and $\partial U^R/\partial T = -re^{-rT}(\theta_R + \alpha) < 0$. Thus the slope of the indifference curve is $-(\partial U^R/\partial T)/(\partial U^R/\partial\alpha) = r(\theta_R+\alpha)$, which is strictly increasing in θ_R. For L, we identically obtain $-(\partial U^L/\partial T)/(\partial U^L/\partial\alpha) = -r(\theta_L - \alpha)$, which is strictly decreasing in θ_L. The proof for the single crossing property of the original function in Alesina and Drazen (1991) is given in Lemma 2. □

4.2 Equilibrium Strategies

We will build a sequential equilibrium where both groups use a delay as a strategic variable and interact by offering and accepting. Before analyzing the equilibrium path, we define the equilibrium offer, the acceptance or delay decision as a function of beliefs.

Equilibrium offer under no uncertainty After both type are revealed by offering, equilibrium offer after this become the Rubinstein (1982) full information offer and the most patient type accept the offer.

In full information about both type, the alternating offer game with a fixed time between offers has a unique subgame perfect equilibrium where the groups stabilize immediately at the share $\alpha(L, R)$ if L makes the initial offer and $\alpha(R, L)$ if R makes the initial offer.[11]

At Rubinstein offers, each group is indifferent between accepting at the other's offer immediately or stabilizing at his own offer after a one period delay such as

$$R + \alpha(L, R) = \delta(R + \alpha(R, L)) \text{ and } L - \alpha(R, L) = \delta(L - \alpha(L, R)).$$

Hence equilibrium shares are gives as

$$\alpha(L, R) = \frac{\delta L - R}{1 + \delta} \text{ and } \alpha(R, L) = \frac{L - \delta R}{1 + \delta}. \tag{5}$$

Thus offering group receives a truncated payoff of $\delta(R + L)/(1 + \delta)$ and the other receives $\delta^2(R + L)/(1 + \delta)$. After revealing value θ_R and θ_L, along the equilibrium path, L makes offer $\alpha(\theta_L, \theta_R)$, and R accepts this offer, since rejecting and making counter offer $\alpha(\theta_R, \theta_L)$ tomorrow yields the same payoff as accepting $\alpha(\theta_L, \theta_R)$ today.

Acceptance/Delay decision under one-sided uncertainty Suppose that L reveals θ_L, but R's value is still private information. In this case, a less patient R accepts the offer, while a more patient R rejects the offer.

Define $\tilde{\theta_R}(\theta_L, \alpha)$ as the type of R that is indifferent between accepting or rejecting the offer α with revealing θ_L. Hence $\tilde{\theta_R}(\theta_L, \alpha)$ is indifferent between $\alpha(\theta_L, \tilde{\theta_R})$ today and $\alpha(\tilde{\theta_R}, \theta_L)$ tomorrow.

$$\tilde{\theta_R} + \alpha(\theta_L, \tilde{\theta_R}) = \delta(\tilde{\theta_R} + \alpha(\tilde{\theta_R}, \theta_L)) = \delta(\tilde{\theta_R} + \frac{\theta_L - \delta\tilde{\theta_R}}{1 + \delta}) = \frac{\delta}{1 + \delta}(\tilde{\theta_R} + \theta_L)$$

Solving $\tilde{\theta_R}$ and the analogous $\tilde{\theta_L}$, we obtain

$$\tilde{\theta_R} = -(1 + \delta)\alpha(\theta_L) + \delta\theta_L \text{ and } \tilde{\theta_L} = (1 + \delta)\alpha(\theta_R) + \delta\theta_R. \tag{6}$$

A group $R \geq \tilde{\theta_R}(\theta_L, \alpha)$ accepts the offer $\alpha(\theta_L, \tilde{\theta_R})$, while a group $R < \tilde{\theta_R}(\theta_L, \alpha)$ prefers to delay before making the revealing offer $\alpha(\theta_R, \theta_L)$. L infers that $R = \theta_R(\Delta \mid \theta_L, \tilde{\theta_R})$ if R delays Δ before making the offer $\alpha(\theta_R, \theta_L)$ and that $R \geq \tilde{\theta_R}$ would have accepted L's offer. The optimal length of delay $\Gamma(R \mid \theta_L, \tilde{\theta_R})$ before offering $\alpha(R, \theta_L)$ is given by the incentive constraint

$$e^{-r\Gamma}\left[R + \alpha(\theta_R(\Gamma), \theta_L)\right] = \max_{\Delta} e^{-r\Delta}[R + \alpha(\theta_R(\Delta), \theta_L)].$$

First order condition with regards to Δ can be derived as

$$\frac{\partial U^R}{\partial \Delta} = -re^{-r\Delta}[R + \alpha(\theta_R, \theta_L)] + e^{-r\Delta}\frac{\partial \alpha}{\partial \theta_R}\frac{\partial \theta_R}{\partial \Delta} = 0$$

[11] See Rubinstein (1982).

$$\Leftrightarrow -r\left(R + \frac{L-\delta R}{1+\delta}\right) + \frac{-\delta}{1+\delta}\frac{\partial \theta_R}{\partial \Delta} = 0.$$

Then this yields the separable first order differential equation

$$\frac{\partial \theta_R}{\partial \Delta} = -\frac{r}{\delta}(\theta_R + \theta_L).$$

The optimal delay is then obtained by integration of $\partial \Delta$ as

$$\begin{aligned} \Gamma(R\,|\,\theta_L, \tilde{\theta_R}) &= \int_{\tilde{\theta_R}}^{R} -\frac{\delta}{r}\frac{d\theta_R}{\theta_R+\theta_L} = -\frac{\delta}{r}\int_{\tilde{\theta_R}}^{R}(\theta_R+\theta_L)^{-1}d\theta_R \\ &= -\frac{\delta}{r}\Big[\log(\theta_R+\theta_L)\Big]_{\tilde{\theta_R}}^{R} = -\frac{\delta}{r}\log\frac{R+\theta_L}{\tilde{\theta_R}+\theta_L}. \end{aligned} \tag{7}$$

The inverse $\theta_R(\Delta\,|\,\theta_L, \tilde{\theta_R})$ is given as

$$-\frac{r\Delta}{\delta} = \log\frac{R+\theta_L}{\tilde{\theta_R}+\theta_L}$$

$$\Leftrightarrow\ \theta_R(\Delta\,|\,\theta_L, \tilde{\theta_R}) = (\tilde{\theta_R}+\theta_L)e^{-\frac{r\Delta}{\delta}} - \theta_L. \tag{8}$$

By the single-crossing property, since Γ and θ_R are strictly decreasing, this is necessary and sufficient for the optimization problem. The analogous functions for group L are derived as

$$\Gamma(L\,|\,\theta_R, \tilde{\theta_L}) = -\frac{\delta}{r}\log\frac{\theta_R+L}{\theta_R+\tilde{\theta_L}} \tag{9}$$

$$\theta_L(\Delta\,|\,\theta_R, \tilde{\theta_L}) = (\theta_R+\tilde{\theta_L})e^{-\frac{r\Delta}{\delta}} - \theta_R. \tag{10}$$

4.3 Equilibrium Path

Initially, both groups delay negotiations by refusing to make an initial offer. As time goes on, both groups become anxious about the gains from stabilization, since if the gains are larger, the other group would have made an offer. Define $\theta_R(\Delta)$ as a value of R that makes an initial offer after Δ, and $\Gamma(R) = \theta_R^{-1}(R)$ as the delay until R makes an initial offer if L does not make one until then. $\theta_L(\Delta)$ and $\Gamma(L) = \theta_L^{-1}(L)$ are analogous. Less patient group which can obtain higher gains from stabilization is likely to stabilize earlier. Hence $\theta_R(\Delta)$, $\theta_L(\Delta)$, $\Gamma(L)$ and $\Gamma(R)$ are decreasing functions.

After a delay of Δ without an offer, R believes that the value of L is no less than $\theta_L(\Delta)$, that is, L is in $[0, \theta_L(\Delta)]$, as a less patient group $L > \theta_L(\Delta)$ would have made an offer before Δ. L's belief on R's valuation conditional on the history is the truncated prior:

$$F_L(R\,|\,\Delta) = \frac{F(R)}{F(\theta_R(\Delta))} \ \text{ for } \ 0 \le R \le \theta_R(\Delta).$$

Similarly, after a delay of Δ, L believes that the value of R is in $[0, \theta_R(\Delta)]$.

Then there are three possible cases in the equilibrium path, depending on the valuations L and R:

1. L is less patient than R. L makes the initial offer $\alpha(\theta_L, \tilde{\theta_R})$ after a delay of $\Gamma(L)$, where $\tilde{\theta_R}$ is the most patient R to accept the offer. If $R \geq \tilde{\theta_R}$, R accepts the offer without delay. Otherwise, R rejects the offer and reveals his type R by delaying $\Gamma(R)$ plus the minimum delay t^0 before making the counter offer $\alpha(R, L)$. L accepts the offer without delay.

2. R is less patient than L. R makes the initial offer $\alpha(\theta_R, \tilde{\theta_L})$ after a delay of $\Gamma(R)$, where $\tilde{\theta_L}$ is the most patient L to accept the offer. If $L \geq \tilde{\theta_L}$, L accepts the offer without delay. Otherwise, L rejects the offer and reveals his type L by delaying $\Gamma(L)$ plus the minimum delay t^0 before making the counter offer $\alpha(L, R)$. R accepts the offer without delay.

3. R is equal to L. If the groups happen to make initial offers at the same timing, a coin is flopped to determine which group will be the initial one. Then one's offer is accepted without delay.

Negotiations end in finite time after at most two offers, but the equilibrium path still depends on the group's option to make alternating offers.

The strategies depend only on the current beliefs and the recent offer. The posterior beliefs following an offer depend only on the prior belief and the amount of delay before the offer.

4.4 Equilibrium Delay

We will state the strategies and beliefs in the three phases of the game; *Phase 0*, in which no offers have been made; *Phase 1*, in which one offer has been made; and *Phase 2*, in which two or more offers have been made. Suppose L makes an initial offer and then R responds with acceptance or a counteroffer. The other possible path, in which R makes the first offer, are symmetric. Then we solve equilibrium in backward way such as dynamic programming.

For simplicity, we assume that the group never make offers which are more attractive than their revealed value.

Phase 2 Suppose that the previous offers reveal valuations θ_R and θ_L, and that R just made an offer $\alpha(\theta_R)$. Let's $\theta_L^0 = \min\{\theta_L, \tilde{\theta_L}(\theta_R, \alpha)\}$.

If L counter offers after delay of Δ in response to R's offer $\alpha(\theta_R, \theta_L)$, then R infers that L's valuation is $\theta_L(\Delta \,|\, \theta_R, \theta_L^0)$. L's response to R's offer $\alpha(\theta_R, \theta_L)$ is:
(i) if $L \geq \theta_L^0$, accept α without delay, given $L - \alpha(\theta_R, \theta_L) \geq \delta[L - \alpha(\theta_L^0, \theta_R)]$, otherwise, counter $\alpha(\theta_L^0, \theta_R)$ without delay. Then R accepts the offer with probability one.

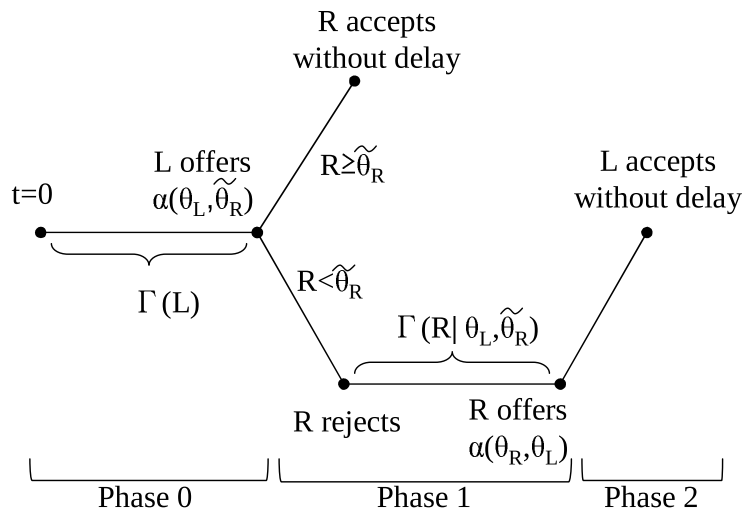

Figure 3: Timing of the game

(ii) if $L < \theta_L^0$, counter $\alpha(L, \theta_R)$ after delay $\Gamma(L \,|\, \theta_R, \theta_L^0)$. Then R accepts the offer with probability one.

Proposition 1. *In the subgame after both group have revealed their private information θ_R and θ_L, the belief and strategies form an equilibrium. In the equilibrium path, stabilization occurs without delay at a share of cost $\alpha(\theta_L, \theta_R)$ if L make an offer or $\alpha(\theta_R, \theta_L)$ if R make an offer.*

Proof. See Appendix. □

As Cramton (1992) argued, this equilibrium path satisfies the "spirit" of the Cho-Kreps intuitive criterion, since beliefs are revised following the signal of strength.[12] Thus beliefs may change by the actions that groups take. In other words, delay provides a group with a mean to convince the other of the truth.

On the other hand, if groups threaten with beliefs, the Rubinstein solution can not be sustained, while beliefs in Rubinstein model are fixed due to the common knowledge of valuation. Wheres fixing beliefs violates the Cho-Kreps criterion.

[12]Cho-Kreps intuitive criterion provide a refinement of sequential equilibrium, while not every sequential equilibrium satisfies this criterion. In a signaling game with a sequential equilibrium, an action that will not reach in equilibrium is said to violate the Cho-Kreps Intuitive Criterion if: there exists some out-of-equilibrium action so that one type can gain by deviating to this action, when the receiver interprets her type correctly, while every other type cannot gain by deviating to this action even if the receiver interprets her truly.

Hence, the separating equilibrium satisfying Cho-Kreps intuitive criterion is more 'robust' than other sequential equilibrium or Perfect Bayesian equilibrium. See more details in Cho and Kreps (1987).

Then, in order to guarantee the Rubinstein outcome in this subgame, we assume beliefs stay fixed at the revealed value, unless an offer is delayed or an offer that should have been accepted is mistakenly rejected. This discussion is for off equilibrium path, where mistakes can be taken.

Phase 1 Suppose that L revealed θ_L with an offer $\alpha(\theta_L)$, and that R just made an offer α. Let's $\theta_R^0 = \min\{\theta_R, \tilde{\theta_R}(\theta_L, \alpha)\}$.

If R counter offers after delay of Δ in response to L's offer, then L infers that R's valuation is $\theta_R(\Delta \,|\, \theta_L, \theta_R^0)$ with probability one. R's response to L's offer α is:
(i) if $R \geq \theta_R^0$, accept α without delay, given $R + \alpha(\theta_L) \geq \delta[R + \alpha(\theta_R^0, \theta_L)]$, otherwise, counter $\alpha(\theta_R^0, \theta_L)$ without delay. Then R accepts the offer with probability one.
(ii) if $R < \theta_R^0$, counter $\alpha(R, \theta_L)$ after delay $\Gamma(R \,|\, \theta_L, \theta_R^0)$. Then follow phase 2.

Proposition 2. *In the subgame after L has revealed θ_L with an offer $\alpha(\theta_L)$, the beliefs and strategies form an equilibrium. In the equilibrium path, R accepts $\alpha(\theta_L)$ without delay if $R \geq \tilde{\theta_R}(\theta_L, \alpha)$, and otherwise counter the offer $\alpha(R, \theta_L)$ after a delay $\Gamma(R)$. This offer is accepted without delay by L.*

Proof. See the Appendix. □

Phase 0 Suppose that no offers have been made. In this initial subgame, if R does not make any offer before a delay Δ, L infers that R's valuation is $[0, \theta_R(\Delta)]$. If R makes the initial offer α after Δ, L infers that R's valuation is $\theta_R(\Delta)$ with probability one. Then follows phase 1. If R doe not make an offer, L makes an initial offer $\alpha(L, \tilde{\theta_R}(L))$ after $\Gamma(L)$. Then follow Phase 1.

The groups determine when they make an offer as a function of their valuations. As the timing of offer is a monotone function of one's valuation, a separate equilibrium may exist. The followings are focused on L, but the analysis for R is analogous. First, we determine L's optimal offer at time Δ, provided that R infers that the valuation of L is θ_L with probability one.

Proposition 3. *If L makes an initial offer after a delay of Δ and L believes that $R \in [0, \theta_R(\Delta)]$, then in the equilibrium path, L makes an initial offer of $\alpha(\theta_L, \tilde{\theta_R})$ where $\tilde{\theta_R}$ uniquely satisfies*

$$F(\theta_R) - F(\tilde{\theta_R}) - (1 - \delta^2)(\tilde{\theta_R} + \theta_L) = \delta^3 \int_0^{\tilde{\theta_R}} \left(\frac{R + \theta_L}{\tilde{\theta_R} + \theta_L} \right)^{1+\delta} dF(R). \tag{11}$$

Proof. See the Appendix. □

Then we determine the time of delay $\Gamma(L)$ and valuation $\theta_L(\Delta)$. Due to the incentive constraint, a less patient group would imitate a more patient group by delaying longer $\Delta > \Gamma(L)$, in order to convince the other that her valuation is more patient, that is, $L < \theta_L(\Delta)$. Hence, to make $\theta_L(\Delta)$ be a part of an equilibrium, the guarantee that such a deviation is not incentive compatible should be imposed. The best response to a delay Δ should be a offer after $\Gamma(L)$.

Proposition 4. *If the time between offers is sufficiently small, then the strategies and beliefs form an equilibrium. The initial delay for L is*

$$\Gamma(L) = \int_0^L \frac{q(\theta_L) + k(\theta_L) - \delta c(\theta_L)}{r(\tilde{\theta_R} + \theta_L)c(\theta_L)} d\theta_L$$

where $c(\theta_L) = F(\theta_L) - F(\tilde{\theta_R}) - (1-\delta)(\tilde{\theta_R} + \theta_L) f(\tilde{\theta_R})$, $q(\theta_L) = \delta^2(\delta - 1) \int \left(\frac{R+\theta_L}{\tilde{\theta_R}+\theta_L}\right)^\delta dF(R)$ *and* $k(\theta_L) = \frac{\delta}{1+\delta} f(\theta_L) (2\tilde{\theta_R} - 2\delta\theta_L)$*.* $\tilde{\theta_R}(\theta_L)$ *uniquely satisfies (8).*

Proof. See the Appendix. □

In order to derive explicit solutions to compare with war-of-attrition case, consider the case where the valuation of both groups are on uniform distribution.

Proposition 5. *Suppose that the valuation of both groups are uniformly distributed. Then L delays* $\Gamma(L)$ *before making an initial offer.* $\theta_L(\Delta)$ *makes an initial offer* $\alpha(\theta_L(\Delta))$ *after* Δ*. Then* $R \geq \tilde{\theta_R}(\Delta)$ *immediately accepts the offer, where*

$$\Gamma(L) = \frac{\delta(4\gamma - 2\delta - \gamma\delta - 2)}{2r(1+\delta)\gamma(\gamma\delta - 2\gamma + 1)} \log L \quad \textit{and} \quad \gamma = \frac{2+\delta}{4 + 2\delta - 2\delta^2}.$$

Proof. See the Appendix. □

By using the result of Proposition 5, the equilibrium strategies in this case can be derived as

$$\theta_L(\Delta) = e^{\frac{2r(1+\delta)\gamma(\gamma\delta - 2\gamma+1)}{\delta(4\gamma - 2\delta - \gamma\delta - 2)}\Delta} \quad \text{and} \quad \tilde{\theta_R}(\Delta) = (2\gamma - 1)\theta_L(\Delta).$$

Thus, the minimum expected delay in the bargaining is $\Gamma(L)$, when L makes an initial offer and it is accepted immediately. On the other hand, on equilibrium path, there may be a sequential equilibrium, where whether the initial offer is accepted or rejected depends on the valuation of the other. If the initial offer is rejected, R makes a counter offer after $\Gamma(R \mid \theta_L, \tilde{\theta_R})$ and it is accepted by L. In this case, the expected delay of stabilization is $\Gamma(L) + \Gamma(R)$. Hence the expected delay of stabilization can be described as

$$ED_B = \begin{cases} \Gamma(L) & \text{if} \quad L \geq R \geq \tilde{\theta_R} \\ \Gamma(L) + \Gamma(R \mid \theta_L, \tilde{\theta_R}). & \text{if} \quad 0 < R < \tilde{\theta_R} \end{cases}$$

In bargaining framework, a group become more impatient if their expected gain from stabilization is larger. Then the group with larger expectations of gains makes initial offer, hence makes concession earlier on. Initial offer is only accepted if the gain are sufficiently large. Thus the larger are gains, stabilization may occur sooner. Also the larger is discount rate, the time of delay is more shortened.

5 Equilibrium Delay in the War-of-Attrition Game

In order to compare the results in both concession and bargaining setup, in this section, we discuss the equilibrium delay in the war-of-attrition case, where the share of cost of stabilization is exogenously determined as in Alesina and Drazen (1991) and most of extensions. In the concession framework, the problem of each party is to maximize its expected lifetime payoff by choosing a time to concede if the other party has not yet conceded.

Due to concession, one of the two groups (the *loser*) has to agree to bear a higher fraction of the new non distortionary taxes, while the rest of this is borne by the other group (the *winner*). The share imposed on a group which concedes earlier is $\kappa_i^L = \kappa > 1/2$, which is exogenous parameter.

Note that in Alesina and Drazen (1991), κ is treated as the measures of the divergence between the distributional implications of the reform plan, or 'degree of polarization' of society.

If a group concedes earlier, then one receives the flow payoff $u_i^L = -\kappa\tau$ and the other receives $u_i^W = -(1-\kappa)\tau$. Thus the expected payoff as of time $t = 0$ as a function of one's chosen concession time T_i is the sum of $V^i(\theta_i, T, \kappa_i^L)$ multiplied by the probability of the other not having conceded by T_i and $V^i(\theta_i, T, \kappa_i^W)$ multiplied by the probability of the other group conceding at t for all $t \leq T_i$. Define $H(T)$ as the distribution of the opponent's optimal concession time and $h(T)$ as the associated density function. Then the expected payoff as a function of T_i can be written as

$$\begin{aligned} EV^i(T_i) &= [1 - H(T_i)] \left[\int_0^{T_i} \underline{u}_i(t) e^{-rt} dt + \int_{T_i}^{\infty} u_i^L(t) e^{-rt} dt \right] \\ &\quad + \int_{t=0}^{t=T_i} \left[\int_0^t \underline{u}_i(x) e^{-rx} dx + \int_t^{\infty} u_i^W(x) e^{-rx} dx \right] h(t) dt. \end{aligned}$$

Lemma 2. *Concession time is monotonically decreasing in θ_i such as $T_i'(\theta_i) < 0$.*

Proof. See the Appendix. □

This shows that the higher is the cost to distortion, the earlier group concedes.

In order to find the optimal concession time $T(\theta)$ for a group of type θ, we will consider a symmetric Nash equilibrium.[13] In this equilibrium, if each group's concession behaviour is described by the same function $T(\theta)$, it is optimal for a group to behave according to $T(\theta)$. Given concession time as a function of θ, the expected delay of stabilization in the concession ED_C is then the expected $\min\{T(\theta_L), T(\theta_R)\}$ with the expectation taken over $F(\theta)$.

Proposition 6. *The expected concession time in a symmetric Nash equilibrium is chosen to maximize the expected payoff as*

$$\left[-\frac{f(\theta)}{F(\theta)} \frac{1}{T'(\theta)} \right] \frac{2\kappa - 1}{r} = \theta + \frac{1}{2} - \kappa$$

[13]This derivation mainly follows Alesina and Drazen (1991).

and the boundary condition $T(\bar{\theta}) = 0$ holds.

Proof. See the Appendix. □

The right hand side is the cost of waiting another instant to concede, that is, the difference between the loss due to distortion and the increase in tax burden by the stabilization to the group who concedes. The left hand side is the expected gain from waiting another instant to concede, which is the product of the conditional probability that another concedes multiplied by the gain if the other concedes. Concession occurs when the cost of waiting is equal to the expected gain from waiting.

If the group is characterized by the maximum possible cost of distortion, it will concede immediately and there will be no war of attrition. As long as all groups in the process initially believe that someone else may have a higher θ, stabilization does not occur immediately.

Then, we will consider the case where $F(\theta)$ is uniform over $[\underline{\theta}, \bar{\theta}]$.[14] In this case, as $F(\theta) = (\theta - \underline{\theta})/(\bar{\theta} - \underline{\theta})$, then $-f(\theta)/F(\theta) = 1/(\underline{\theta} - \theta)$. Under this assumption, $T'(\theta)$ is therefore given by

$$\begin{aligned} T'(\theta) &= -\frac{f(\theta)}{F(\theta)} \frac{2\kappa - 1}{r} \frac{1}{\theta + 1/2 - \kappa} \\ &= \frac{(2\kappa - 1)/r}{(\underline{\theta} - \theta)(\theta + 1/2 - \kappa)}. \end{aligned}$$

Using the method of partial fractions and integrating, the optimal time of concession of a group of type θ can be obtained by

$$\begin{aligned} T(\theta) &= \int_{\underline{\theta}}^{\theta} \frac{-(2\kappa - 1)/r}{(\theta - \underline{\theta})(\theta + 1/2 - \kappa)} d\theta \\ &= \int_{\underline{\theta}}^{\theta} \frac{-(2\kappa - 1)/r}{(\underline{\theta} + 1/2 - \kappa)(\theta - \underline{\theta})} d\theta + \int_{\underline{\theta}}^{\theta} \frac{-(2\kappa - 1)/r}{-(\underline{\theta} + 1/2 - \kappa)(\theta + 1/2 - \kappa)} d\theta \\ &= \frac{(1 - 2\kappa)/r}{\underline{\theta} + 1/2 - \kappa} \left(\log(\theta - \underline{\theta}) - \log\left(\theta + 1/2 - \kappa\right)\right) + C^0 \\ &= \frac{(1 - 2\kappa)/r}{\underline{\theta} + 1/2 - \kappa} \left(\log \frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}} + \log \frac{\bar{\theta} + 1/2 - \kappa}{\theta + 1/2 - \kappa} \right) \\ &\text{where} \quad C^0 = \frac{(1 - 2\kappa)/r}{\underline{\theta} + 1/2 - \kappa} \left(-\log(\bar{\theta} - \underline{\theta}) + \log(\bar{\theta} + 1/2 - \kappa) \right) \end{aligned}$$

We assume that C^0 is set to assure $T(\bar{\theta}) = 0$. Again, given concession time as a function of θ, the expected delay of stabilization in the concession ED_C is then $\min\{T(\theta_L), T(\theta_R)\}$ with the expectation taken over $F(\theta)$.

[14] As in Section 2, we assume $F(\theta) \in [0.1]$. On the other hand, we also assume $\theta_i + 1/2 - \kappa_i > 0$ to assure the concession to occur. Hence in the concession setup, the lower bound $\underline{\theta}$ exists to satisfy this assumption. Below this threshold, no concession occurs, that is, the expected delay goes to infinity. Note that $\underline{\theta}$ may vary, depending on κ_i.

The uncertainty about the cost to waiting of other groups is important to delay stabilizations. As long as groups in the process believe that someone else may have a higher θ and then give up first, stabilization does not occur immediately. If all groups are identical, we could interpret this as a single agent. In this case, he knows with probability 1 that he will be the stabilizer. Thus he stabilize immediately. Meanwhile the higher θ lead to shorter concession time. An increase in the cost, for unchanged distribution θ, will move the expected date of a stabilization forward.

Higher κ leads to later stabilization. The gain from waiting in that one's opponent will concede is larger. Hence each group holds out longer. It might be hard to reach agreements on how to allocate tax increases among interest groups.

6 Welfare Analysis

In this section, we will analyze the welfare in this economy. In the bargaining framework, the stabilization occurs when a group accepts the offer by the other, while in the concession framework, the stabilization occurs when a group concedes. With the interactions that two parties take, how does the total welfare change in the process?

Proposition 7. *The welfare increases as the time of stabilization becomes earlier.*

Proof. In this economy, there are two political groups which maximizes each utility function (1). Hence total welfare W can be described as

$$\begin{aligned} W = V^L + V^R &= \frac{\tau}{r}\left[e^{-rT}\left(\theta_L + \theta_R + 1 - (\kappa_L + \kappa_R)\right) + (-1 - \theta_L - \theta_R)\right] \\ &= \frac{\tau}{r}[e^{-rT}(\theta_L + \theta_R) + (-1 - \theta_L - \theta_R)]. \end{aligned}$$

We can see how W is affected when the timing of stabilization changes by differentiating the above such as

$$\frac{\partial W}{\partial T} = -\tau e^{-rT}(\theta_L + \theta_R) \leq 0.$$

□

This result can be interpreted intuitively as follows. There is a distortion in the economy before stabilization. Therefore, given that the total cost of the reform does not change irrespective of its distribution, the earlier the reform which terminates the distortion takes place, the larger total welfare becomes.

As in the previous sections, the only solutions that we could obtain in this paper are the expected time of delay due to two-sided uncertainty about the cost to waiting of the other group. In the concession process, the expected delay for a group ED_C^i is the optimal timing for the group to concede given the other group has not conceded yet. Then the actual delay T_C is $\min\{ED_C^L, ED_C^R\}$. In the bargaining process, the expected delay for a group ED^i_B is the optimal timing for the group to offer the share given the other group is more patient and then has not offered yet. Then the actual delay T_B is $\min\{ED^L_B, ED^R_B\}$.

When a group makes a decision, stabilization occurs. Therefore for a group i, ED^i can be considered as T. Hence it can be concluded that every group should recognize the process in which the expected delay is shorter would be more desirable in terms of welfare improving.

7 Comparative Simulation

In this section, by using numerical method, we will analyze the equilibrium delay in both the bargaining and war-of-attrition setup, which has been obtained in the previous sections under the assumption of uniform distribution of the type.

Hereafter, we consider the limiting case of bargaining model that the minimum time between offers goes to zero as $(\delta \to 1)$, in order to compare properly with the concession setup, where the payoff is discounted only by r. Hence, as in section 3, given L makes an initial offer, the expected delay in the bargaining can be obtained as

$$\begin{aligned} ED_B &= \begin{cases} \Gamma(L) & \text{if} \quad L \geq R \geq \tilde{\theta_R} \\ \Gamma(L) + \Gamma(R \mid \theta_L, \tilde{\theta_R}) & \text{if} \quad 0 < R < \tilde{\theta_R} \end{cases} \\ &= \begin{cases} -\frac{7}{3r}\log L & \text{if} \quad L \geq R \geq \frac{1}{2}\theta_L \\ -\frac{7}{3r}\log L - \frac{1}{r}\log\frac{2(\theta_R+\theta_L)}{3\theta_L}. & \text{if} \quad \frac{1}{2}\theta_L > R > 0 \end{cases} \end{aligned}$$

Note that the cost share does not affect the time of delay as the bargaining process endogenously determine the share. On the equilibrium path, groups take strategies to form a sequential equilibrium, where separating equilibrium may occur depending on the valuation of the other.

7.1 Expected Delay in Each Process

Before direct comparison, analysis for the delay in each process would be useful to examine how groups interact in general.

Figure 4 shows the expected delay in the bargaining case. To simplify, in Figure 2, we examine the the minimum delay $\Gamma(L)$.[15] A group become more impatient if their expected gain from stabilization is larger. The group with larger expectations of gains makes initial offer, hence makes concession earlier on. Thus the larger are gains, stabilization occurs sooner. Also the larger is discount rate, the time of delay is more shorten.

Figure 5 shows the time of delay in the war-of-attrition case. In concession game, the share of cost of stabilization τ is exogenously fixed. Thus the gain from

[15]One can easily show that even if we plot $\Gamma(L) + \Gamma(R)$, each slope will not change.

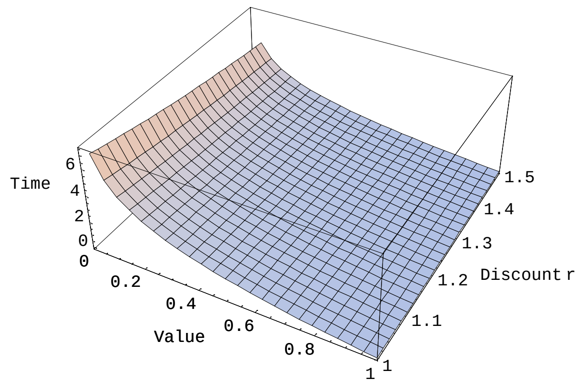

Figure 4: Expected delay in the Bargaining case

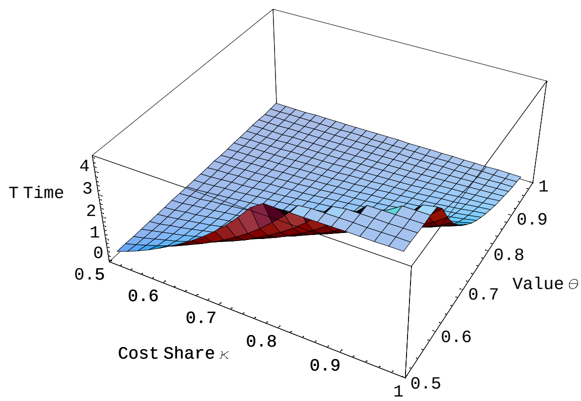

Figure 5: Expected delay in the war-of-attrition case

stabilization, which is independently drawn from distribution F, and the share can affect the time of delay. A group more exposed to distortinary taxation before stabilization, that is, which may have larger gains from stabilization, will be likely to give in immediately, leading to immediate stabilization. If the cost share for stabilization becomes more polarized, the expected gains or losses from waiting increases. Therefore, concession time will be prolonged as groups become more patient. Stabilizations are delayed as long as groups believe someone else may have a higher cost for distortion. On the contrary, if a group recognizes his cost as the highest, no delay may occur.

7.2 Comparison of Expected Delay

Figure 6 and 7 compare the two setups in terms of the expected time of delay. To simplify, we assume that discount rate r is normalized to one. In concession case, we need to put further assumption of $\theta_i + 1/2 - \kappa > 0$, in order to make sure that groups concede at some point, otherwise, no one concedes due to less profit from stabilization. Thus we consider the case of $\theta_i \in [0.3, 1]$, that is, $\underline{\theta} = 0.3$ for the concession. In the bargaining process, however, as the cost share is endogenously determined, the time of delay is still independent of the value of κ. Thus, even in the range of $0 < \theta < 0.3$, the bargaining process does have an equilibrium path, while the war of attrition model may have no concession in the range.

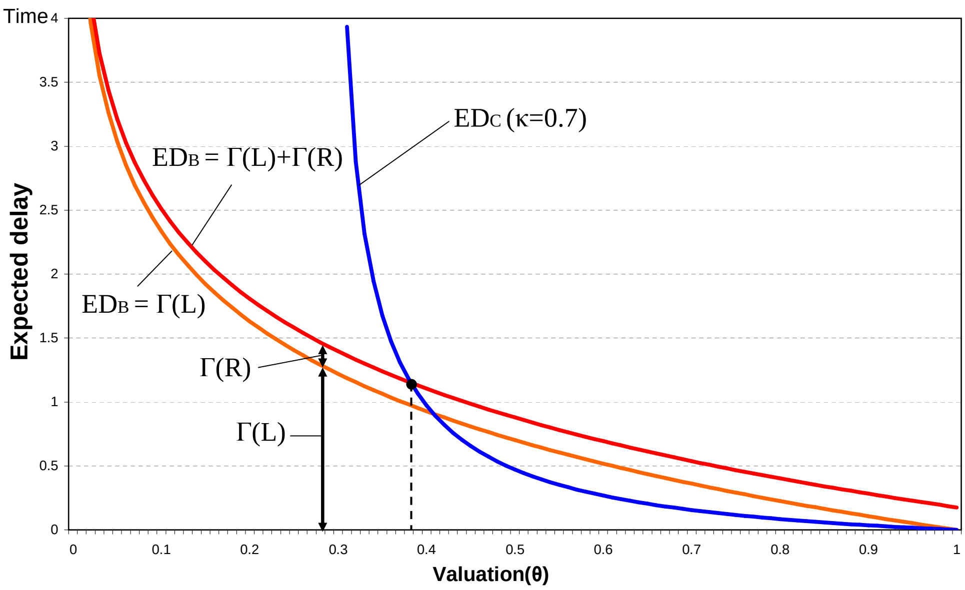

Figure 6: Comparison at $\kappa = 0.7$

Figure 6 implies that the bargaining process may take more time to reach an agreement. At the range of higher valuation, in which a group is more impatient, the delay in the bargaining case is longer than in the concession case. Even if a group is likely to concede sooner, due to uncertainty at the initial period, groups have to wait in order to make an offer which can be indifferent between acceptance and reject.

On the other hand, at the range of lower valuation, in which a group is more patient, however, the bargaining process may hasten the stabilization rather than in the concession. This may be interpreted as an intuitive way that due to the bargaining process of exchanging and signaling the information, groups could make an offer to reach agreement sooner.

Thus when interest groups are unlikely to concede, the bargaining process may hasten the stabilization. Meanwhile, when groups are willing to settle early, the bargaining may rather take more time to reach agreement on the stabilization due to negotiation procedure. One can conclude that the introduction of bargaining process has a smoothing effect on reaching an agreement of fiscal reform.

In Figure 7, we examine the sensitivity analysis of a change in κ. As stated above, the expected delay in the bargaining is independent of κ.

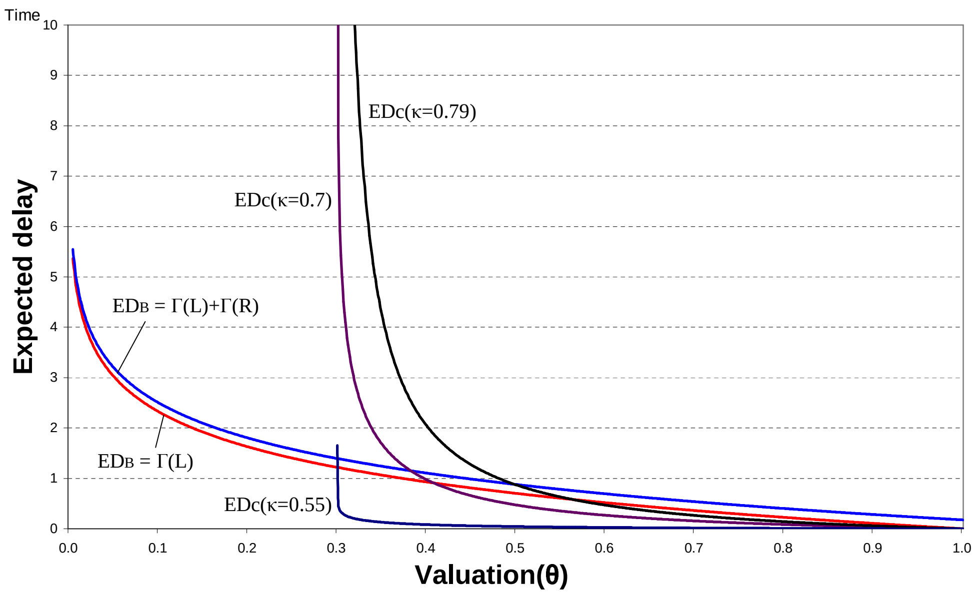

Figure 7: Sensitivity analysis of κ

If the share of cost for stabilization is more equally distributed in concession model such as $\kappa = 0.55$, the difference between the gain from waiting and losses from concession become smaller. Then groups concede earlier, so that the bargaining process may delay the reform in most cases.

On the other hand, when the share of cost for stabilization is more polarized such as $\kappa = 0.79$, the gain from waiting become larger. Then groups tend to wait the other concede longer. In this case, the bargaining process may be more useful on reaching agreement earlier on rather than equally distributed case.

Thus, in more polarized economy, the bargaining process may work for shorter delay more effectively.

7.3 Policy implication

This situation with conflicts among two groups can be considered as a divided government or parliament, where two interest parties need to agree at some point in order to make the policy decision, but the party which supports the president in a country is different from the party which keeps the majority of the congress. From historical perspectives for distributive conflicts, the right party tends to insist the proportional income tax and indirect tax while the left prefers capital tax and more progressive tax.

In the concession process, both groups can not compromise until reaching their upper limit of cost of waiting. Such a lack of compromise may lead to a deadlock situation of complete fiscal inaction with endless debate. In the bargaining process, both groups can discuss the share of cost by offering each other. This is going to be a coalition government such as in 1923 German where Great Coalition of right and left cut through legislative deadlock.

As seen in the previous sections, when a group realizes own cost of waiting, one can know which process can be welfare improving and then insist to choose a process. Intuitively, the bargaining procedure seems to hasten the legislative process in any case, but the simulation in this section shows that under certain condition where groups are willing to concede early due to higher cost of waiting, the bargaining may take more time to reach an agreement. Then in terms of the length of delay, there is a case where it would be more efficient to keep a dead locked situation without calling on the other to start bargaining process.

As an extension of this paper, strategic behaviour based on each payoff can be considered. In that case, the political cost of process decision might be an important element to be examined.

8 Concluding Remarks

Stabilization is often delayed even though every group benefits from earlier stabilizations. One of the reason is that interest groups conflicts over the distribution of the cost for stabilization. We analyze a model of delayed agreement on fiscal reforms with microfoundations, in which two interest groups conflict over the share of the cost of the stabilization. On the process of reaching an agreement, the groups can take two types of interaction: *Bargaining* where groups can offer the share, and *Concession* where one of the groups has to bear the fixed disproportionate share. In both processes, interest groups have incentives to shift the burden of the cost on the other by waiting for the other to offer or concede first. Hence the delay can be used as a signal of the toughness of the interest group. Under the same economic setting, we obtain formulas for the expected delay in both the bargaining and concession game, which allow a systematic comparison.

In war-of-attrition framework, a group more exposed to distortinary taxation before stabilization, that is, which may have larger gains from stabilization, will be likely to give in immediately, leading to immediate stabilization. If the cost share for stabilization becomes more polarized, the expected gains or losses from waiting increases. Therefore, concession time will be prolonged as groups become more patient. Stabilizations are delayed as long as groups believe someone else may have a higher cost for distortion. On the contrary, if a group recognizes his cost as the highest, no delay may occur.

In bargaining framework, a group become more impatient if their expected gain from stabilization is larger. Then the group with larger expectations of gains makes initial offer, hence makes concession earlier on. Initial offer is only accepted if the gain are sufficiently large. Thus the larger are gains, stabilization occurs sooner. Also the larger is discount rate, the time of delay is more shortened.

Under the same economic setting, we obtain formulas for the expected delay in both the bargaining and concession game, which allow a systematic comparison. We then show that when interest groups are sufficiently patient, agreement is reached more quickly on average under the bargaining game. Instead, when groups are very impatient, the concession game may lead to the earlier stabilization. Furthermore, in more polarized economy where shares in the concession game are very unequal, it is more likely that the bargaining process leads to the shorter delay. Therefore bargaining process can be a tool to smooth the expected delay.

We also show that as the expected delay becomes shorter, total welfare increases. Hence our results will shed a light on the selection of political process in a divided government. Intuitively, the bargaining procedure seems to hasten the legislative process in any case, but this paper shows that under certain condition where groups are willing to concede early due to higher cost of waiting, the bargaining may take more time to reach an agreement. Then in terms of the length of delay, there is a case where it would be more efficient to keep a dead locked situation without calling on the other to start bargaining process.

A Appendix

Lemma 3. *The payoff function in Alesina and Drazen (1991) satisfies the single crossing property.*

Proof. In Alesina and Drazen (1991), payoff function is defined as

$$\begin{aligned} U(\theta;T,\alpha) &= \int_0^T u_i^D(x)e^{-rx}\,dx + e^{-rT}V^j(T) \\ &= \int_0^T \left[-\frac{\tau(x)}{2} - \theta_i\tau(x)\right] e^{-rx}\,dx + e^{-rT}\frac{-\alpha_j\tau(T)}{r} \\ &= \int_0^T \left[-\left(\frac{1}{2}+\theta_i\right)\gamma r\bar{b}e^{(1-\gamma)rx}\right] e^{-rx}\,dx + e^{-rT}\frac{-\alpha_j}{r}r\bar{b}e^{(1-\gamma)rx} \\ &= \int_0^T -\left(\frac{1}{2}+\theta_i\right)\gamma r\bar{b}e^{-\gamma rx}\,dx - \alpha_j\bar{b}e^{-\gamma rx}. \end{aligned}$$

Hence, we obtain

$$\begin{aligned} \frac{dU}{dT} &= -\left(\frac{1}{2}+\theta_i\right)\gamma r\bar{b}e^{-\gamma rx} + \gamma r\alpha_j\bar{b}e^{-\gamma rx} \\ &= \left(\alpha_j - \frac{1}{2} - \theta_i\right)\gamma r\bar{b}e^{-\gamma rx} \;<\; 0 \\ \frac{dU}{d\alpha} &= -\bar{b}e^{-\gamma rx} \;<\; 0. \end{aligned}$$

Therefore, the slope of indifference curve $[T,\alpha]$ becomes

$$\begin{aligned} -\frac{dU}{dT}\Big/\frac{dU}{d\alpha} &= -\frac{\left(\alpha_j - \frac{1}{2} - \theta_i\right)\gamma r\bar{b}e^{-\gamma rx}}{-\bar{b}e^{-\gamma rx}} \\ &= \left(\alpha_j - \frac{1}{2} - \theta_i\right)\gamma r. \end{aligned}$$

This suggests that this indifference curve is strictly decreasing in θ. □

Proof of Proposition 1 We need to show that L's strategy is optimal given the belief and strategies of R when L believes $R = \theta_R$.

Suppose $L \geq \theta_L^0$. An immediate counter offer by L implies $L = \theta_L^0$. The counter offer $\alpha(\theta_L^0, \theta_R)$ is accepted by R with probability one. Then L accepts the offer α if $L - \alpha \geq \delta(L - \alpha(\theta_L^0, \theta_R))$, given that offering $\alpha(\theta_L^0, \theta_R)$ without delay is the optimal.

If L offer a α' after a delay Δ, R believes $\theta_L = \theta_L(\Delta \,|\, \theta_R, \theta_L^0)$. The deviation from offering $\alpha(\theta_L, \theta_R)$ makes losses; offering $\alpha' > \alpha(\theta_L, \theta_R)$ yields losses, as R accepts this with probability one. offering $\alpha' < \alpha(\theta_L, \theta_R)$ also yields losses, as R counter offer $\alpha(\theta_R, \theta_L) > \alpha(\theta_L, \theta_R)$. Then L's optimal offer is $\alpha(\theta_L, \theta_R)$ with optimal delay Δ.

L's expected payoff from offering $\alpha(\theta_L, \theta_R)$ after Δ is $U^L(\Delta) = e^{-r\Delta}(L -$

$\alpha(\theta_L, \theta_R))$. Recalling (2) and (7),

$$\begin{aligned}
\frac{\partial U^L}{\partial \Delta} &= -r\, e^{-r\Delta}(L - \alpha(\theta_L)) + e^{-r\Delta}\left(-\frac{\partial \alpha}{\partial \theta_L}\frac{\partial \theta_L}{\partial \Delta}\right) \\
&= -r\, e^{-r\Delta}\left(L - \frac{\delta L - R}{1+\delta} - \frac{(\theta_R + \theta_L^0)e^{-\frac{r\Delta}{\delta}}}{1+\delta}\right) \\
&= -r\, e^{-r\Delta}\left(\theta_R - (\theta_R + \theta_L^0)e^{-\frac{r\Delta}{\delta}} + L\right) < 0
\end{aligned}$$

Then the optimal delay Δ is zero.

Suppose $L < \theta_L^0$. As above, if $L \geq \theta_L$, optimal offer is $\alpha(\theta_L)$ and R infers $L = \theta_L$. As $\theta_L(\Delta) = (\theta_R + \theta_L^0)e^{-\frac{r\Gamma}{\delta}} - \theta_R$ holds, $\partial U^L/\partial\Delta = 0$ if $\Delta = \Gamma(L)$. Hence for $L < \theta_L$, L's optimal behavior is by rejecting $\alpha(\theta_R)$ and offering $\alpha(L, \theta_R)$ after delay $\Gamma(L \,|\, \theta_R, \theta_L^0)$.

Along the equilibrium path $L = \theta_L$ and $R = \theta_R$, stabilization occurs with probability one at each share.

Proof of Proposition 2 Suppose $R \geq \theta_R^0$. if R makes a counter offer after Δ, L infers $R = \theta_R(\Delta \mid \theta_L, \theta_R^0)$. But, Proposition 1 suggests R's optimal counter offer is $\alpha(\theta_R^0, \theta_L)$ without delay. Thus R accepts $\alpha(\theta_L)$ without delay if $R + \alpha \geq \delta(R + \alpha(\theta_R^0, \theta_L))$.

Suppose $R < \theta_R^0$. counter-offer after Δ reveals $R = \theta_R(\Delta \mid \theta_L, \theta_R^0)$. But then we proceed to the subgame with complete information in Proposition 1. In this subgame, R's optimal response is to offer $\alpha(R, \theta_L)$ after delay of $\Gamma(R \mid \theta_L, \theta_R^0)$, which is accepted without delay by L.

Along equilibrium path, $\theta_R^0 = \tilde{\theta_R}$ and $R < \theta_R(\Delta)$. Thus, if $R \geq \tilde{\theta_R}(\theta_L, \alpha)$, acceptance of α is optimal, as $R \geq \tilde{\theta_R}(\theta_L, \alpha)$ holds if and only if $R + \alpha \geq \delta(R + \alpha(\tilde{\theta_R}, \theta_L))$.

Proof of Proposition 3 Suppose L makes a revealing offer $\alpha(\theta_L, \tilde{\theta_R})$. The offer is accepted if $R \in [\tilde{\theta_R}, \theta_R]$, otherwise, after a delay Γ, L accepts R's counter offer $\alpha(R, \theta_L)$, where the payoff is discounted by

$$\delta e^{-r\Gamma(R|\theta_L, \tilde{\theta_R})} = \delta\left(\frac{R + \theta_L}{\tilde{\theta_R} + \theta_L}\right)^{\delta} \quad \text{as} \quad \Gamma = -\frac{\delta}{r}\log\frac{R + \theta_L}{\tilde{\theta_R} + \theta_L}$$

Then L's expected flow payoff from offering $\alpha(\theta_L, \tilde{\theta_R})$ is given by $\tilde{U}^L$, which is continuous on $[0, \theta_R]$, such as

$$\tilde{U}^L(L, \tilde{\theta_R} \,|\, \theta_L, \theta_R) = [F(\theta_R) - F(\tilde{\theta_R})][L - \alpha(\theta_L, \tilde{\theta_R})] + \int_0^{\tilde{\theta_R}} \delta(L - \alpha(R, \theta_L))\left(\frac{R + \theta_L}{\tilde{\theta_R} + \theta_L}\right)^{\delta} dF(R).$$

L could choose $\tilde{\theta_R}$ to maximize her utility as α and $\tilde{\theta_R}$ are the correspondence. In equilibrium path at $L = \theta_L$, L's marginal utility from $\tilde{\theta_R}$ is as follows;

$$\begin{aligned}
\frac{\partial \tilde{U}^L(\theta_L)}{\partial \tilde{\theta_R}} &= -F(\theta_R)\frac{\partial \alpha(\theta_L, \tilde{\theta_R})}{\partial \tilde{\theta_R}} - f(\tilde{\theta_R})L + f(\tilde{\theta_R})\alpha + F(\tilde{\theta_R})\frac{\partial \alpha(\theta_L, \tilde{\theta_R})}{\partial \tilde{\theta_R}} \\
&\quad + \frac{\partial}{\partial \tilde{\theta_R}} \int_0^{\tilde{\theta_R}} \delta(L - \alpha(R, \theta_L)) \left(\frac{R + \theta_L}{\tilde{\theta_R} + \theta_L}\right)^\delta dF(R) \\
&= \frac{1}{1+\delta}\left(F(\theta_R) - F(\tilde{\theta_R}) - (\tilde{\theta_R} + \theta_L) f(\tilde{\theta_R})\right) \\
&\quad + \frac{\delta^2}{1+\delta}\left((\tilde{\theta_R} + \theta_L) f(\tilde{\theta_R}) - \delta \int_0^{\tilde{\theta_R}} \left(\frac{R + \theta_L}{\tilde{\theta_R} + \theta_L}\right)^{1+\delta} dF(R)\right)
\end{aligned}$$

Thus, if $\tilde{U}^L$ is maximized,

$$F(\theta_R) - F(\tilde{\theta_R}) - (1-\delta^2)(\tilde{\theta_R} + \theta_L) = \delta^3 \int_0^{\tilde{\theta_R}} \left(\frac{R + \theta_L}{\tilde{\theta_R} + \theta_L}\right)^{1+\delta} dF(R)$$

holds. Furthermore, as $\tilde{U}' < 0$ at $\tilde{\theta_R} = \theta_R$ and $\tilde{U}' > 0$ at $\tilde{\theta_R} = 0$, at some point of $\tilde{\theta_R} \in [0, \theta_R]$, the maximum occurs. Then $\tilde{U}''$ is given by

$$\begin{aligned}
(1+\delta)\frac{\partial \tilde{U}'}{\partial \tilde{\theta_R}} &= (\delta^2 - 2) f(\tilde{\theta_R}) - (1-\delta^2)(\tilde{\theta_R} + \theta_L) f'(\tilde{\theta_R}) - \delta^3 \frac{\partial}{\partial \tilde{\theta_R}} \int_0^{\tilde{\theta_R}} \left(\frac{R + \theta_L}{\tilde{\theta_R} + \theta_L}\right)^{1+\delta} dF(R) \\
&= (-\delta^3 + \delta^2 - 2) f(\tilde{\theta_R}) - (1-\delta^2)(\tilde{\theta_R} + \theta_L) f'(\tilde{\theta_R}) \\
&\quad + \delta^3(1+\delta) \int_0^{\tilde{\theta_R}} \frac{(R + \theta_L)^{1+\delta}}{(\tilde{\theta_R} + \theta_L)^{2+\delta}} dF(R).
\end{aligned}$$

Thus, in order to have a unique solution, we assume the distribution F satisfies

$$(-\delta^3 + \delta^2 - 2) f(\tilde{\theta_R}) - (1-\delta^2)(\tilde{\theta_R} + \theta_L) f'(\tilde{\theta_R}) + \delta^3(1+\delta) \int_0^{\tilde{\theta_R}} \frac{(R + \theta_L)^{1+\delta}}{(\tilde{\theta_R} + \theta_L)^{2+\delta}} dF(R) < 0, \tag{12}$$

as $\tilde{u}'$ is strictly decreasing over $\tilde{\theta_R}$ if this holds.

Proof of Proposition 4 We assume (9) holds to specify $\tilde{\theta_R}(\theta_L)$. L makes a revealing offer after delay $\Delta \geq \Gamma(L)$, given that R's strategy is $\theta_R(\Delta)$ and that R believes L makes $\theta_L(\Delta)$.

Case 1; $R \in [0, \tilde{\theta_R}(\theta_L(\Delta))]$. L reveals first after Δ, and R reject the offer. Then R counter $\alpha(R, \theta_L)$ after delay Γ. The offer is accepted immediately. Thus the expected payoff of L is

$$e^{-r\Delta}\, e^{-r\Gamma}\, \delta(L - \alpha(R, \theta_L)) = \frac{1}{1+\delta}\delta\, e^{-r\Delta} \left(\frac{R + \theta_L}{\tilde{\theta_R} + \theta_L}\right)^\delta ((1+\delta)L + \delta R - \theta_L).$$

Case 2; $R \in [\tilde{\theta_R}(\theta_L(\Delta)), \theta_R(\Delta)]$. L offer $\alpha(\theta_L, \tilde{\theta_R})$ after Δ and R accepts immediately. The expected payoff is

$$e^{-r\Delta}\,(L - \alpha(\theta_L, \tilde{\theta_R})) = \frac{1}{1+\delta}\,e^{-r\Delta}\,\left((1+\delta)L + \tilde{\theta_R} - \delta\theta_L\right).$$

Case 3; $R \in [\theta_R(\Delta), \theta_R(\Gamma(\theta_L))]$. R reveals first with $\alpha(R, \tilde{\theta_L})$ after $\Gamma(R)$, and L accepts immediately as $L > \tilde{\theta_L}(\theta_L \Gamma(R))$. Then the expected payoff is

$$e^{-r\Gamma(R)}\,(L - \alpha(R, \tilde{\theta_L})) = \frac{1}{1+\delta}\,e^{-r\Gamma(R)}\,\left((1+\delta)L + \delta R - \tilde{\theta_L}\right).$$

The total expected payoff of L is calculated by integrating the above payoffs for each range such as

$$\begin{aligned}(1+\delta)U^L(L,\Delta) \;=\; & \int_0^{\tilde{\theta_R}(\theta_L)} e^{-r\Delta}\,\left((1+\delta)L + \tilde{\theta_R} - \delta\theta_L\right) f(R)dR \\ & + [F(\theta_R(\Delta)) - F(\tilde{\theta_R}(\theta_L))]e^{-r\Delta}\,\left((1+\delta)L + \tilde{\theta_R} - \delta\theta_L\right) \\ & + \int_{\theta_R(\Delta)}^{\theta_R(\Gamma(L))} e^{-r\Gamma(R)}\,\left((1+\delta)L + \delta R - \tilde{\theta_L}\right) f(R)dR.\end{aligned}$$

A necessary condition for $\Gamma(L)$ to be a best response is that the marginal utility of delay is zero at $L = \theta_L(\Delta)$. Taking the derivative of the above with respect to Δ and substituting $L = \theta_L(\Delta)$ yields

$$\begin{aligned}(1+\delta)\frac{\partial U^L}{\partial \Delta} \;=\; & \delta e^{-r\Delta}\left(\frac{R+\theta_L}{\tilde{\theta_R}+\theta_L}\right)^{\delta}\left[(\delta^2-1)\theta_L' - \delta(R+\theta_L)\left(r + \delta\,\frac{\tilde{\theta_R}'+\theta_L}{\tilde{\theta_R}+\theta_L}\right)\right] \\ & + \tilde{\theta_R}' e^{-r\Delta}\delta^2(\tilde{\theta_R}+\theta_L)f(\tilde{\theta_R}) \\ & + e^{-r\Delta}(F(\theta_R) - F(\tilde{\theta_R})) \times \\ & \quad [(\tilde{\theta_R}' - \delta\theta_L' - r(\tilde{\theta_R}+\theta_L)) + (\tilde{\theta_R}+\theta_L)(f(\theta_R)\,\theta_R' - f(\tilde{\theta_R})\,\tilde{\theta_R}')] \\ & - e^{-r\Delta}\,\theta_R'\,(\delta\theta_R - \tilde{\theta_L} + (1+\delta)\theta_L)\,f(\theta_R).\end{aligned}$$

Hence, $\partial U^L/\partial\Delta = 0$ can be written as

$$\begin{aligned}0 \;=\; & [F(\theta_L) - F(\tilde{\theta_R})][\tilde{\theta_R}' - \delta\theta_L' - r(\tilde{\theta_R}+\theta_L)] + \theta_R' f(\theta_R)(\tilde{\theta_R} + \tilde{\theta_L} - \delta(\theta_R+\theta_L)) \\ & - \tilde{\theta_R}' f(\tilde{\theta_R})(1-\delta^2)(\tilde{\theta_R}+\theta_L) + \delta(\delta^2-1)\theta_L' \int_0^{\tilde{\theta_R}} \left(\frac{R+\theta_L}{\tilde{\theta_R}+\theta_L}\right)^{\delta} dF(R) \\ & - \left(\frac{r}{\delta}(\tilde{\theta_R}+\theta_L) + (\tilde{\theta_R}' + \theta_L')\right)\int_0^{\tilde{\theta_R}} \delta^3 \left(\frac{R+\theta_L}{\tilde{\theta_R}+\theta_L}\right)^{1+\delta} dF(R).\end{aligned}$$

By using the first order condition (8) and multiplying δ, we obtain

$$\begin{aligned}
0 \;=\; & [F(\theta_L) - F(\tilde{\theta_R})] \times \\
& \quad [-\delta(1+\delta)\theta_L' - r(1+\delta)(\tilde{\theta_R} + \theta_L)) + \delta\theta_R' f(\theta_R)(\tilde{\theta_R} + \tilde{\theta_L} - \delta(\theta_R + \theta_L)) \\
& + r(1-\delta^2)(\tilde{\theta_R} + \theta_L)^2 f(\tilde{\theta_R}) + \delta f(\tilde{\theta_R})(\tilde{\theta_R} + \theta_L)(1-\delta^2)\theta_L' \\
& + \delta^2(\delta^2 - 1)\theta_L' \int_0^{\tilde{\theta_R}} \left(\frac{R+\theta_L}{\tilde{\theta_R}+\theta_L}\right)^{\delta} dF(R).
\end{aligned}$$

As $\theta_L(\Delta) = \theta_R(\Delta)$ and then $\tilde{\theta_L}(\theta_R(\Delta)) = \tilde{\theta_R}(\theta_L(\Delta))$, substituting these and dividing by $1+\delta$ yields

$$\begin{aligned}
0 \;=\; & -(\delta\theta_L' + r(\tilde{\theta_R} + \theta_L)[F(\theta_L) - F(\tilde{\theta_R}) - (1-\delta)(\tilde{\theta_R} + \theta_L) f(\tilde{\theta_R})] \\
& + \delta^2(\delta - 1)\theta_L' \int_0^{\tilde{\theta_R}} \left(\frac{R+\theta_L}{\tilde{\theta_R}+\theta_L}\right)^{\delta} dF(R) + \frac{\delta}{1+\delta}\theta_L' f(\theta_L)(2\tilde{\theta_R} - 2\delta\theta_L).
\end{aligned}$$

Assuming that $c(\theta_L) = F(\theta_L) - F(\tilde{\theta_R}) - (1-\delta)(\tilde{\theta_R} + \theta_L) f(\tilde{\theta_R})$, $q(\theta_L) = \delta^2(\delta - 1)\int \left(\frac{R+\theta_L}{\tilde{\theta_R}+\theta_L}\right)^{\delta} dF(R)$ and $k(\theta_L) = \frac{\delta}{1+\delta} f(\theta_L)(2\tilde{\theta_R} - 2\delta\theta_L)$, the first order differential equation can be obtained as

$$\begin{aligned}
\theta_L' \;&=\; \frac{\partial \theta_L(\Delta)}{\partial \Delta} = \frac{r(\tilde{\theta_R} + \theta_L) c(\theta_L)}{q(\theta_L) + k(\theta_L) - \delta c(\theta_L)} \\
\Leftrightarrow \quad \Gamma(L) \;&=\; \int_0^L \frac{q(\theta_L) + k(\theta_L) - \delta c(\theta_L)}{r(\tilde{\theta_R} + \theta_L) c(\theta_L)} d\theta_L.
\end{aligned}$$

Proof of Proposition 5 Consider $\tilde{\theta_R}(\theta_L)$ which maximizes $\tilde{U}^L(\theta_L, \tilde{\theta_R} \mid \theta_L, \theta_R)$. Since (9) is satisfied for $\delta > 0$, $\tilde{\theta_R}(\theta_L)$ maximizes the expected payoff by binding (8). As $F(R) = R$ and $L = \theta_L$, (8) becomes;

$$\begin{aligned}
\theta_R - \tilde{\theta_R} - (1-\delta^2)(\tilde{\theta_R} + \theta_L) \;&=\; \frac{\delta^3}{(\tilde{\theta_R} + \theta_L)^{1+\delta}} \int_0^{\tilde{\theta_R}} (R+\theta_L)^{1+\delta} dR = \frac{\delta^3}{2+\delta}(\tilde{\theta_R} + \theta_L) \\
& \Leftrightarrow \quad \theta_R - 2\tilde{\theta_R} - \theta_L + \frac{2\delta^2}{2+\delta}(\tilde{\theta_R} + \theta_L) = 0 \\
& \Leftrightarrow \quad \tilde{\theta_R} = \frac{2+\delta}{4+2\delta-2\delta^2}\theta_R + \frac{2\delta^2 - 2 - \delta}{4+2\delta-2\delta^2}\theta_L.
\end{aligned}$$

Since $\theta_L(\Delta) = \theta_R(\Delta)$, we can transform this as

$$\tilde{\theta_R} = \frac{2\delta^2}{4+2\delta-2\delta^2}\theta_L = (2\gamma - 1)\theta_L \quad \text{where} \quad \gamma = \frac{2+\delta}{4+2\delta-2\delta^2}.$$

Hence, by substituting, we obtain

$$q(\theta_L) = \frac{2\delta^2(\delta-1)\gamma}{1+\delta}\theta_L, \quad k(\theta_L) = \frac{2\delta(2\gamma - \delta - 1)}{1+\delta}\theta_L, \quad \text{and} \quad c(\theta_L) = 2(\delta\gamma - 2\gamma + 1)\theta_L.$$

As $\theta_L' = r(\tilde{\theta_R} + \theta_L)c(\theta_L)/(q(\theta_L) + k(\theta_L) - \delta c(\theta_L))$, θ_L' can be written as

$$\theta_L' = \frac{2r(1+\delta)\gamma(\gamma\delta - 2\gamma + 1)}{\delta(4\gamma - 2\delta - \gamma\delta - 2)}\theta_L.$$

Then, since $\theta_L' = \partial\theta_L/\partial\Delta$,

$$\Gamma(L) = \int_0^L \frac{\delta(4\gamma - 2\delta - \gamma\delta - 2)}{2r(1+\delta)\gamma(\gamma\delta - 2\gamma + 1)} \frac{1}{\theta_L} d\theta_L = \frac{\delta(4\gamma - 2\delta - \gamma\delta - 2)}{2r(1+\delta)\gamma(\gamma\delta - 2\gamma + 1)} \log L.$$

Proof of Lemma 2 Differentiating the expected payoff with respect to T_i, we obtain

$$\begin{aligned}
\frac{\partial EV}{\partial T} &= (1-H)\left(\underline{u}(T)e^{-rT} - re^{-rT}\frac{u^L}{r} + e^{-rT}\frac{\partial(u^L/r)}{\partial T}\right) \\
&\quad - h\left(\int_0^T \underline{u}(t)e^{-rt}dt + e^{-rT}\frac{u^L}{r}\right) + \left(\int_0^T \underline{u}(t)e^{-rt}dt + e^{-rT}\frac{u^W}{r}\right)h \\
&= e^{-rT}\left[h(T)\left(\frac{-u^L + u^W}{r}\right) + (1 - H(T)(\underline{u} - u^L)\right] \\
&= e^{-rT}\left[h(T)\frac{(2\kappa - 1)\tau}{r} + (1 - H(T))\left(-\theta - \frac{1}{2} + \kappa\right)\tau\right].
\end{aligned}$$

Differentiating with respect to θ_i, we obtain

$$\frac{\partial^2 EV}{\partial T \partial\theta} = e^{-rT}\left(-(1 - H(T)\right) < 0.$$

This implies that when others are acting optimally, dEV/dT is decreasing in θ. Therefore concession time T_i is monotonically decreasing in θ_i.

Proof of Proposition 6 Suppose the other interest group acts accoriding to the optimal concession time $T(\theta)$. Choosing T_i would be equal to choosing a value $\hat{\theta}$ and conceding at time $T_i = T(\hat{\theta}_i)$. As T is monotonically decreasing in θ, we can derive the relation between $H(T_i)$ and $F(\theta_i)$, as $1 - H(T(\theta_i)) = F(\theta_i)$. By changing the variables, we obtain

$$\begin{aligned}
EP(\hat{\theta}, \theta) &= F(\hat{\theta})\left[\int_{\hat{\theta}}^{\bar{\theta}} -\underline{u}(x)e^{-rT(x)}T'(x)dx + e^{-rT(\hat{\theta})}\frac{u^L(T(\theta_i))}{r}\right] \\
&\quad + \int_{\hat{\theta}}^{\bar{\theta}}\left[\int_x^{\bar{\theta}} -\underline{u}(x)e^{-rT(z)}T'(z)dz + e^{-rT(x)}\frac{u^W(T(x))}{r}\right]f(x)dx.
\end{aligned}$$

Differentiating with respect to $\hat{\theta}$, we can obtain

$$\begin{aligned}
\frac{\partial EP}{\partial \hat{\theta}} &= f(\hat{\theta})\left[\int_{\hat{\theta}}^{\bar{\theta}} -\underline{u}e^{-rT(x)}T'(x)dx + e^{-rT(\hat{\theta})}\frac{u^L}{r}\right] \\
&\quad + F(\hat{\theta})\left[-\underline{u}e^{-rT(\hat{\theta})}T'(\hat{\theta}) - rT'(\hat{\theta})e^{-rT(\hat{\theta})}\frac{u^L}{r} + e^{-rT}\frac{\partial(u^L/r)}{\partial T}\frac{\partial T}{\partial \hat{\theta}}\right] \\
&\quad + \left(\int_{\hat{\theta}}^{\bar{\theta}} \underline{u}(x)e^{-rT(x)}dx + e^{-rT(\hat{\theta})}\frac{u^W}{r}\right) f(\hat{\theta}) = 0 \\
\Leftrightarrow &\quad -f(\hat{\theta})\left(\frac{u^W - u^L}{r}\right) + F(\hat{\theta})\left(\underline{u}(\theta,\hat{\theta}) - r\frac{u^L}{r}\right)T'(\hat{\theta}) \\
&= -f(\hat{\theta})\left(\frac{2\kappa - 1}{r}\right)\tau + F(\hat{\theta})\left(-\theta - \frac{1}{2} + \kappa\right)\tau T'(\hat{\theta}) = 0.
\end{aligned}$$

As $T(\theta)$ is the optimal time of concession for a group with cost θ, $\hat{\theta} = \theta$ when $\hat{\theta}$ is chosen optimally. Hence first order condition evaluated $\hat{\theta} = \theta$ becomes the equation in this proposition.

As for the boundary condition, for any value of $\theta < \hat{\theta}$, the gain from waiting until other's concession is positive. Thus groups with $\theta < \hat{\theta}$ will not concede immediately. Then a group with $\theta = \hat{\theta}$ will find it optimal to choose $T(\hat{\theta}) = 0$ as the group knows it has the highest possible cost of waiting.

References

Admati, A.R. and M. Perry, "Strategic Delay in Bargaining," *Review of Economic Studies*, 1987, *54* (3), 345–364.

Alesina, A. and A. Drazen, "Why are Stabilizations Delayed?," *American Economic Review*, 1991, *81* (5), 1170–1188.

Bliss, C.J. and B. Nalebuff, "Dragon-slaying and Ballroom Dancing: The Private Supply of a Public Good," *Journal of Public Economics*, 1984, *25*, 1–12.

Casella, A. and B. Eichengreen, "Can Foreign Aid Accelerate Stabilisation," *Economic Journal*, 1996, *106* (436), 605–619.

Cho, I.K. and D.M. Kreps, "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, 1987, *102* (2), 179–222.

Cramton, P.C., "Strategic Delay in Bargaining with Two-Sided Uncertainty," *Review of Economic Studies*, 1992, *59* (1), 205–225.

Drazen, A. and V. Grilli, "The Benefit of Crises for Economic Reforms," *American Economic Review*, 1993, *83* (3), 598–607.

Drazen, Allan, *Political Economy in Macroeconomics*, new ed ed., Princeton Univ Pr, 12 2001.

Hsieh, C.T., "Bargaining over reform," *European Economic Review*, 2000, *44* (9), 1659–76.

Martinelli, C. and R. Escorza, "When are stabilizations delayed? Alesina–Drazen revisited," *European Economic Review*, 2007, *51* (5), 1223–1245.

Persson, T. and G.E. Tabellini, *Political economics*, MIT Press Cambridge, Mass, 2000.

Riley, John G, "Strong evolutionary equilibrium and the war of attrition," *Journal of Theoretical Biology*, 1980, *82* (3), 383–400.

Rubinstein, A., "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 1982, *50* (1), 97–109.

Sibert, A. and W. Perraudin, "Timing of Multilateral Lending," *Economic Journal*, 2000, *110*, 192–211.

Spolaore, E., "Adjustments in Different Government Systems," *Economics and Politics*, 2004, *16* (2), 117–146.

Tornell, A., "Reform from Within," *NBER Working Paper*, 1998, *6497.*

Velasco, A., "Debts and deficits with fragmented fiscal policymaking," *Journal of Public Economics*, 2000, *76* (1), 105–125.