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and Dynamic Inefficiency

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Endogenous Population Growth, Social Security, and Dynamic Inefficiency¹

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Abstract.

Population growth rate matters for determining the optimal amount of capital in the context of dynamic inefficiency. From the aspect of social security, if there are public pension schemes, individuals have no motive for having a child as income source for retirement periods. Thus, it is important to take into account an interaction between social security and population growth rate when the optimal level of social security is considered. This paper finds the optimal social security level in an 80-period overlapping generations model with endogenous population growth. It is shown that the optimal payroll tax rate is 0% in the benchmark case, and the optimal payroll tax rate is lower than that in the exogenous case in the economy with dynamic inefficiency at the case there is no social security. This is due to the effect of borrowing constraints and some properties with children, such as requiring parents to make a certain amount of payment for many periods and giving no chance to be free from rearing children, that dominates the effect of efficiency gains. These effects lower the fertility rate, which will lower the population growth rate, and hence lower the optimal social security level.

JEL Classification: E62, H24, J13

Keywords; Endogenous Population Growth, Borrowing Constraint, Social Security, Dynamic Inefficiency.

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1 Introduction

Recently most developed countries are facing the crisis of social security finance mainly because of their unexpected decline in fertility rate. Therefore, it is necessary to reform the social security system in order to cope with the declining fertility. On the other hand, from a theoretical point of view, it has been pointed out and analyzed that the social security system is an important determinant of the fertility rate since individuals do not need to have children for the periods after retirement if they have generous social security system⁴. The purpose of this paper is to study how much the optimal payroll tax is in a growth model with endogenous fertility and borrowing constraints⁵.

There are several research related to this study. Hubbard and Judd (1987) used an applied general equilibrium model to derive the optimal social security level. They concluded that introducing a social security scheme improves welfare level when borrowing constraints are not considered. The effect is not significant, however, when the borrowing constraints are taken into account. İmrohoroğlu *et al.* (1995) introduced a public unemployment insurance scheme to derive the optimal social security level in a model with uncertainty on employment. They concluded that the optimal replacement rate in the benchmark case is 30% and that it is better to have a social security scheme with a more realistic replacement rate 60% than having no scheme. In the sensitivity analysis, they concluded that the optimal replacement rate is 0% when the discount factor is 0.98 instead of 1.011⁶.

Based on the model of İmrohoroğlu *et al.* (1995), we derive the optimal social security level with the population growth rate endogenized. This pop-

⁴Cigno (1991) discusses, however, the possibility that our having social security may cause the lower fertility rate.

⁵Barro and Becker (1989), for example, integrated population growth into the growth model, assuming each generation has bequest motive and parents can inherit their bequests to their children as human capital. Groezen *et al.* (2003) derived the optimal ratio of child benefit to social security scheme in a two-period overlapping generations model with log-utility function. There are research such as Galor and Weil (1996), Hansen and Prescott (2002), and Greenwood *et al.* (2005), which endogenize the population growth for explaining the historical fertility decline in the real economy. In order to derive the optimal social security level, we endogenize population growth into the model where people face borrowing constraints.

⁶Fuster *et al.* (2007) studied a model with two-sided altruism considering bequest motives and concluded that eliminating the public pension is better even without compensating old generation.

ulation growth factor is important because the optimal social security level is affected by the population growth rate in a pay-as-you-go scheme. Social security benefits in this model are financed by the employed paying a payroll tax. Young individuals decide not only the amount of asset holdings but also the number of children they have. Such a model allows us to study an interaction between social security system and population growth rate.

It is necessary to know several aspects of children as a special market good. There are many different properties with children from ordinary market goods. To begin with, once individuals have children, they can obtain utility from children for many periods. This property seems similar to durables, except for few points as following. These differences add to the distinct properties of children as a special market good⁷. Namely, first, parents need to make a certain amount of payment for rearing children for a long time. The expense on children in each period is relatively small, but they need to keep spending the same amount for rearing them, which is different from durables that may require considerable down payment. Related to the previous point, there is no secondary markets for children, so parents cannot give up rearing children even when they face an income shock, or they cannot have “used” child reared by other parents for several years. In a developing economy, children are considered as “investment goods,” so parents may give children to others as workers when they face an income shock and they cannot afford to rear them any more. In a developed economy, however, they usually obtain utility from having their own children as “consumption goods.” In an environment with income shocks, individuals tend to have less risky asset so that they can avoid risk. As compared to other market goods, children are risky in such a case since individuals need to keep children even when they do not want to continue doing so. If individuals are facing income shocks and borrowing constraints and if there is no secondary markets, they would avoid to have many children due to the above properties.

If fertility decision making is endogenous, the population growth rate may change through optimization to an increase in the tax rate in the following logics; in the model with endogenous population growth, an increase in the payroll tax reduces liquidity and may make individuals have smaller number of children. Consequently, an increase in the tax rate may lessen their saving and per capita capital, and this may trigger a rise of the interest rate,

⁷There are several studies on durables, refer to Luengo-Prado(2006), Suzuki(2009) for example.

which would make social security less attractive to savings in terms of return. On the other hand, when there is dynamic inefficiency in the economy, an increase in tax rate would improve welfare because individuals enjoy more consumption and children. Considering these effects, it is not clear whether the optimal tax rate in the endogenous case is higher than that in the exogenous case, and thus we need to perform a numerical analysis to confirm which effect is dominant.

In this study, we will show that the optimal social security level becomes smaller when the fertility rate is endogenized than when it is exogenously given⁸. Put differently, our result suggests that, if the policy maker does not consider the change in the fertility rate, the payroll tax rate would be too high. The result of the study may also give an insight into individuals behavior following a certain policy. For example, when the government imposes higher payroll tax, although individuals may want to have more children due to the now higher consumable income, it is difficult to do so since spending for children will be prolonged far into the future and their liquidity is reduced.

In the following, the basic model is constructed in Section 2, calibration and optimality evaluation of the model are described in Section 3. The main results and analyses are presented in section 4. Section 5 concludes the paper.

2 Model Economy

Individuals live 100 periods at longest in this model⁹. For the first 20 periods, they spend time as children, and do not appear in the economy. They enter the market at 20 years old as age $j = 1$ generation and start working until 64 years old¹⁰. They may or may not get job with a certain probability which follows a Markov process. After 65 they retire and live until 99.

Let $\mathbb{J} = \{1, 2, \dots, J\}$ be a set of ages with $J = 80$. We denote the survival rate of each age $j \in \mathbb{J}$ by ψ_j with which age $j - 1$ individuals survive in the following period. It is assumed that a continuum of ex-ante

⁸In this study, the utility level of a representative individual is used for evaluation as in Michel and Wigniolle (2007). About evaluation in endogenous population growth models, refer to Michel and Wigniolle (2007) and Golosov *et al.* (2007).

⁹As shown in the below, this is 80 periods model, which is longer than 65 periods in İmrohoroğlu *et al.* (1995) and shorter than 90 periods in Hubbard and Judd (1987).

¹⁰Thus a generation whose actual age is j corresponds to the generation age $j - 19$ in this model.

identical individuals with total measure one lives in the model. The weight of population of age j is μ_j , and the sum of them is always set equal to one.

Young generations face uncertainty over their job opportunity. Consider two states; employed and unemployed. Denote each state by e and u , respectively. Let $\mathbb{S} \equiv \{e, u\}$. The probability that the individuals get employed or unemployed is only dependent on the state in the previous period, and the process is governed by a Markov matrix

$$\Pi(s', s) = \begin{pmatrix} p_e & 1 - p_e \\ 1 - p_u & p_u \end{pmatrix} \quad (1)$$

where $p_e \in [0, 1]$ is a probability that agents are employed in the following period if they are employed in this period, and $p_u \in [0, 1]$ is the probability of being unemployed in the following period if they are unemployed in this period. In this study we assume $p_u = 1 - p_e$. Individuals retire when they are at age $j = j^* \equiv 46$ with probability one. After retirement, they never come back to the labor market and start receiving social security benefits from the government.

Agents maximize their expected utility $E[\sum_{j=1}^J \beta^{j-1} \Psi_j u(c_j, n_j)]$ by choosing c_j for all j and n_j for $j = j^{**}$ properly under the budget constraint where c_j and n_j are the consumption level and the child number of age j agents, $\Psi_1 = 1$ and $\Psi_j = \prod_{t=2}^j \psi_t$.

In this paper, we assume the following utility function

$$u(c_j, n_j) = \frac{c_j^{(1-\sigma_c)}}{1-\sigma_c} + \chi \frac{n_j^{(1-\sigma_n)}}{1-\sigma_n}. \quad (2)$$

It is assumed that individuals can have children only when they are at age $j = j^{**} \equiv 11$, and raise them with a cost q exogenously given for 20 years. The other generation cannot have children, which means that we assume $n_j = n_{j^{**}}$ for $j = j^{**} + 1, \dots, j^{**} + 19$ and $n_j = 0$ for other j . Their children enter the market twenty periods later as age $j = 1$.

We adopt several assumptions on market failures as in İmrohoroğlu *et al.* (1995). First, individuals cannot borrow against their future income, which is referred to as the borrowing constraints. Second, we do not have any annuity market with which individuals can insure their longevity risk. In order to mitigate these market failures, individuals save more for the risk of getting no job and living long. When they die young, they leave their savings as unintentional bequest. We assume that those will be distributed equally to the rest of the people in the economy as lump-sum transfer.

While the market is not complete, we have unemployment insurance scheme and social security scheme arranged by the government to insure against those risks. An unemployment insurance scheme provides unemployment benefit to the unemployed, and social security scheme provides social security benefit to the retired, both of which are financed by the tax collected from the employed in the same period.

We consider a representative firm whose production technology is represented by the Cobb-Douglas function $Q = F(K, L) = BK^\alpha L^{(1-\alpha)}$, where K is the amount of aggregate capital input, L is the amount of aggregate labor input, and $\alpha \in [0, 1]$ is the share of capital income. Under the perfect competition, the following first order conditions must be satisfied;

$$r = F_K(K, L) - \delta = B\alpha(L/K)^{(1-\alpha)} - \delta, \quad \text{and} \quad (3)$$

$$w = F_L(K, L) = B(1 - \alpha)(K/L)^\alpha \quad (4)$$

where we denote a capital depreciation rate by δ , and B represents the TFP and grows at κ which is exogenously given.

Individuals always work all their time available in the period if employed. We normalize the available time and set to 1. The labor cannot be divided and individuals only face whether they are employed or unemployed. All the taxes are collected from the employed so the disposable income is given by:

$$g_j = \begin{cases} w\epsilon_j(1 - \tau_b - \tau_p) & \text{if employed, for } j = 1, \dots, j^* - 1 \\ b & \text{if unemployed, for } j = 1, \dots, j^* - 1 \\ p & \text{for } j = j^*, \dots, J \end{cases} \quad (5)$$

where τ_b and τ_p are the tax rates for unemployment insurance and social security, respectively, b is unemployment benefit, and p is social security benefit. The parameter ϵ_j is an efficiency index for age j , which represents the labor productivity for each age j . The weighted average of them is normalized to one.

Denote a tuple of an asset level, a child number, and an employment status by (a, n, s) and that of the following period by (a', n', s') . At each age, a value function V_j is dependent on (a, n, s) :

$$V_j(a, n, s) \equiv \max_{c+qn+a' \leq g+(1+r)a+T} u(c, n) + \beta\psi_{j+1}E_s'V_{j+1}(a', n', s') \quad (6)$$

where we can choose n only at age j^{**} and $n' = n$ for the following 19 periods.

For a computational reason, we assume that agents can pick an asset level only from a grid set $\{a_1, a_2, \dots, a_D\} \equiv \mathbb{A}$ and a child number from a grid set $\{n_1, n_2, \dots, n_Z\} \equiv \mathbb{M}$. Denote the policy function for consumption, child number, and savings for the next period by $C_j : \mathbb{A} \times \mathbb{M} \times \mathbb{S} \rightarrow \mathbb{R}_+$, $N_j : \mathbb{A} \times \mathbb{S} \rightarrow \mathbb{M}$, and $A_j : \mathbb{A} \times \mathbb{M} \times \mathbb{S} \rightarrow \mathbb{A}$, respectively.

We can derive the set of functions in exactly the same way as İmrohoroğlu *et al.* (1995) except for the process where the population growth is determined. Using the distribution λ of age $j = j^{**} = 11$ who decide how many children they have, we obtain the gross population growth level as

$$N = \sum_a \sum_s \lambda_{j^{**}}(a, s) N_{j^{**}}(a, s) \quad (7)$$

where the gross population growth is the population ratio of new born babies to generation j^{**} , and the distribution $\lambda_j(a, s)$ is the share of those whose asset level is a and the employment status is s in generation j . We can get the annual population growth rate from the following operation;

$$\rho = N^{\frac{1}{j^{**}+20}} \quad (8)$$

since their babies will enter the market 20 periods later.

Individuals choose their child number, so after $j = j^{**}$ they have one more state for child number, namely their distribution is represented as $\lambda_j(a, n, s)$. For other generations $j = 1, \dots, j^{**}$ and $j = j^{**} + 21, \dots, J$, that is not the state variable and it is represented as $\lambda_j(a, n, s) = \lambda_j(a, s)$.

It is only age j^{**} when individuals can choose their child number, and in the following 19 periods they cannot change. Thus we can define their number of children as;

$$N_j(a, n, s) = \begin{cases} N_{j^{**}}(a, s) & \text{for } j = j^{**} \\ n & \text{for } j = j^{**} + 1, \dots, j^{**} + 19 \end{cases} \quad (9)$$

A stationary equilibrium is defined as follows.

Definition 1 *Given a set of taxes for policy arrangements (τ_b, τ_p) , a stationary equilibrium is a set of value functions $V_j(a, n, s)$, policy functions for consumption $C_j(a, n, s)$, the number of children $N_j(a, n, s)$ and asset level $A_j(a, n, s)$, age-dependent (but time-invariant) distributions $\lambda_j(a, n, s)$, a population ratio μ_j for each $j \in \mathbb{J}$, a wage rate w , an interest rate r , a population growth rate ρ , a set of benefits of the policy arrangements (b, p) and a lump-sum transfer T which satisfies the following conditions;*

1. *individual and aggregate behavior are consistent;*

$$K = \sum_j \sum_a \sum_n \sum_s \mu_j \lambda_j(a, n, s) A_j(a, n, s) \quad (10)$$

$$L = \sum_{j=1}^{j^*-1} \sum_a \sum_n \mu_j \lambda_j(a, n, e) \epsilon_j h \quad (11)$$

$$N = \sum_j \sum_a \sum_n \sum_s \mu_j \lambda_j(a, n, s) N_j(a, n, s), \quad (12)$$

2. *a set of population ratios μ_j is consistent with the annual population growth rate which is derived from individual's problem and satisfies $\rho = N^{1/(j^{**}+20)}$;*

$$\mu_j = \mu_{j+1} \rho / \psi_{j+1} \quad (13)$$

$$\sum_j \mu_j = 1, \quad (14)$$

3. *relative prices satisfy a firm's profit maximization problem;*

$$r = F_K(K, L) - \delta, \quad w = F_L(K, L), \quad (15)$$

4. *given a pair of relative prices (w, r) and a set of policy arrangements (τ_b, τ_p, b, p) , a set of policy functions solves individual's maximization problem,*

5. *the resource constraint is satisfied;*

$$\begin{aligned} & \sum_j \sum_a \sum_n \sum_s \mu_j \lambda_j(a, n, s) (C_j(a, n, s) + A_j(a, n, s)) \\ & \quad + q \sum_{j=j^{**}}^{j^{**}+19} \sum_a \sum_n \sum_s \mu_j \lambda_j(a, n, s) n \\ & = Q + (1 - \delta) \sum_j \sum_a \sum_n \sum_s \mu_j \lambda_j(a, n, s) A_{j-1}(a, n, s), \end{aligned} \quad (16)$$

6. *asset-state distributions satisfy the Markov matrix;*

$$\lambda_{j+1}(a', n', s') = \sum_s \sum_{a: a' \in A_j(a, n, s)} \Pi(s', s) \lambda_j(a, n, s) \text{ for } j = 1, \dots, J, \quad (17)$$

7. each of the government policy arrangements is budget-balanced;

- unemployment insurance benefits program:

$$b\left(\sum_{j=1}^{j^*-1} \sum_a \sum_n \mu_j \lambda_j(a, n, u)\right) = \tau_b w \left(\sum_{j=1}^{j^*-1} \mu_j \epsilon_j \sum_a \sum_n \lambda_j(a, n, e)\right), \quad (18)$$

- social security system:

$$p\left(\sum_{j=j^*}^J \mu_j\right) = \tau_p w \left(\sum_{j=1}^{j^*-1} \mu_j \epsilon_j \sum_a \sum_n \lambda_j(a, n, e)\right), \quad (19)$$

8. accidental bequests are distributed;

$$T = \sum_j \sum_a \sum_n \sum_s \mu_{j-1} \lambda_{j-1}(a, n, s) (1 - \psi_j) A_{j-1}(a, n, s) (1 + r). \quad (20)$$

3 Calibration

In this section we choose the parameters used in the simulation. We calibrate our model by assuming one period as one year based on the newest data.

The survival rates ψ_j are taken from the Life-expectancy table (*The 19th seimeihyo*, male) edited by the Ministry of Health, labor and Wealth in 2002. Although it is true that some survive over 100-year-old in Japan from the statistics, it is negligible and we will not consider those. The efficiency indices are made from the wage profile equation estimated from the average wage tables in the Wage structure survey (*Chingin kouzo kihon tokei chosa*, male) also edited by the Ministry of Health, labor and Wealth in 2004¹¹. Unemployment rate is set to 0.04 for both cases employed and unemployed

¹¹Using OLS regressing wage level on t^4, t^3, t^2, t , and constant, the equation is estimated as wage level of age j $w_t = 0.0001t^4 - 0.0226t^3 + 1.7972t^2 - 44.062t + 527.92$ where $t \equiv j + 19$. In the Hubbard and Judd (1987), the wage equation is $w_j = -0.00706233t^4 + 1.34816t^3 - 101.878t^2 + 3520.22t + 36999.4$ which is originally derived by Davies (1981). We estimate the wage equation from the statistics of original wage structure. The difference of the two wage schedules is trivial from the two figures. The reason we use cross-section data is that there is no more realistic data set.

since unemployment rate is 4% in 2008, which means we have the following Markov matrix;

$$\Pi(s', s) = \begin{pmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{pmatrix} \quad (21)$$

We assume that age $j = 1$ individuals are unemployed with probability 0.04 when they enter the market. Employment insurance tax rate is set $\tau_b = 0.006$ from the fact that the employed pay the amount for the insurance in Japan.

σ_c is the risk-aversion parameter, the reciprocal of an intertemporal elasticity of substitution for consumption¹². In this model we assume $\sigma_c = 2$ following the preceding research for the baseline case¹³. We normalize the coefficient of production function and set $B = 1$.

For the technology parameters, we need to calibrate α and δ . We will estimate these parameters in an exogenous model where population growth rate is fixed at the beginning. From SNA in 2006, we have data of Produced assets (1272.4 trillion yen) and Tangible non-produced assets (1229.2 trillion yen) for capital stock in the economy, Consumption of fixed capital (106.0 trillion yen) for capital depreciation, and GDP (502.5 trillion yen). If we use these numbers for calculating the capital depreciation rate and capital income ratio, they are 0.042 and 4.977, respectively. These numbers do not seem valid since the depreciation rate is too low and the capital income ratio is too high. It is from the fact that the Tangible non-produced assets includes households land (762.3 trillion yen) and the above capital stock was assumed too high. Subtracting this households land, we have 0.061 for capital depreciation rate and 3.461 for capital income ratio as target, respectively. We can directly set $\delta = 0.061$ from this result. We will calibrate β by targeting this capital income ratio after obtaining other necessary parameters in the following.

The above targets were obtained from the Stock integrated account of SNA. For calibrating α , we need to use the Closing balance sheet account of SNA. It is necessary to have Compensation of employees (262.6 trillion yen) and Mixed income (17.4 trillion yen), which are considered to represent labor income, and Operating surplus (76.1 trillion yen) and Consumption of fixed capital (106.0 trillion yen) which are considered as capital income. Thus the labor income ratio is 0.394 and we will set $\alpha = 0.394$ in this study.

¹²It has been implicitly assumed that σ_c should be a maximum of 10 in the preceding studies. See Mehra and Prescott (1985) and Kocherlakota (1996).

¹³Imrohoroglu *et al.* (1995) used 2 for the benchmark case.

In Japan, the contribution rate for social security is 14.996% in 2008, half of which is paid by employee and the other half paid by employer. In fact, employees also pay the contribution paid by employer 7.4998%, their actual contribution rate is calculated as 13.95%. At this rate, we will get the discount factor for future consumption $\beta = 0.98$.

Next, we will set parameters related to childcaring. q is the cost for having a child. We can set the childcaring cost by q as exogenously given. There are a number of studies measuring the cost of caring a child. For example, Deaton and Muellbauer (1986) used the model of the equivalence scale to estimate the cost of caring a child. They showed that parents need additional 30-40% for having a child with maintaining their living standard. From the fact, we assume that the cost of caring a child is 30% of their consumption. Then, $q = 0.8882$ is obtained from 60% of average consumptions of those who take care of children.

We will calibrate the preference parameters for children χ . For the endogenous case, given the cost of caring a child by $q = 0.8882$, we need to calibrate the same population growth rate assumed in the exogenous case. We assume $\chi = 1.05$ with which the population growth rate in the endogenous case is $\rho = 0.997$ when the payroll tax rate is 13.95%. This population growth rate is calculated from estimated data by National Institute of Population and Social Security Research. They estimate the near future population in Japan and the population growth rate is around 0.997%. In 2018, the total population of Japan is 123.9 million in their estimation, while it is 127.6 in 2008. For σ_n which is the parameter for preference on having children, we assume $\sigma_n = 2$ in the baseline case, which is assumed the same number as σ_c .

Table 1: The Parameter Used in the Analysis

parameter	β	σ_c	α	δ	σ_n	χ	q_0	p_e
baseline	0.98	2	0.394	0.061	2	1.05	0.8882	0.94
decomposed	1.011	2	0.394	0.061	2	9	0.8882	0.94

At the end of calibration, we will briefly mention the computational method. We need to have two sets of grid points for asset level and number of children for computation. For the set of grid points of asset holdings, we take the number of grid points as $D = 301$ for the capacity constraint of the

computer used in this research. If the largest asset level is chosen too high, the computer capacity would be wasted since we would never have a chance to use such a point. By trying the computation many times, we can set a proper grid range efficiently.

About the number of children, individuals can only have an integer number of children in the real economy. Taking into consideration that there is no sex in our model, it is possible to take a valid set of grid points like $\mathbb{M} \equiv \{0, 0.5, 1, 1.5, \dots, n_z\}$, where n_z is a physical upper limit of the children's number, since couples choose their number of children from a set of integers $\{0, 1, 2, 3, \dots, n_z\}$ in the real world. In this study, we will adopt this integer constraint for child number.

4 Theoretical Analysis and Results

First, the exogenous population growth model is replicated with the parameters used in this analysis to get the benchmark results of this study. Assume that individuals gain utility from their consumption and from having children, but the population growth is fixed as in the preceding research, then we can get an equilibrium corresponding to the previous study. For the benchmark case, $\rho = 0.997$ is assumed.

Since the population growth is given exogenously, the labor supply is constant throughout this setting. The optimal social security level is 0 as shown in Table 2. It means that to have pay-as-you-go social security system is not a good idea. The result is consistent with İmrohoroğlu *et al.* (1995) when $\beta = 0.98$.

Table 2: Exogenous Case (baseline, $\rho = 0.9970$)

τ_p	p	Labor supply	Capital stock	K/Q	Interest rate	Wage rate	Average utility
0	0	0.700	8.536	4.552	0.026	1.623	8.724
0.03	0.081	0.700	7.700	4.266	0.031	1.557	7.699
0.06	0.166	0.700	6.962	4.023	0.037	1.498	6.591
0.09	0.258	0.700	6.384	3.817	0.042	1.448	5.418
0.12	0.355	0.700	5.926	3.648	0.047	1.406	4.209
0.15	0.460	0.700	5.508	3.490	0.052	1.366	2.901

In this setting, we will also check the optimal social security level in the

endogenous model. The result is in Table 3, which shows that the optimal level is the same ($\tau_p = 0$). In both cases, the optimal rate is 0, which is corner solution.

Table 3: Endogenous Case (baseline)

τ_p	p	Labor supply	Capital stock	K/Q	Interest rate	Wage rate	Fertility rate	Average utility
0	0	0.718	8.352	4.422	0.028	1.593	0.9999	-2.64
0.03	0.089	0.718	7.593	4.174	0.033	1.535	0.9999	-5.52
0.06	0.182	0.717	6.650	3.856	0.041	1.457	0.9998	-9.13
0.09	0.281	0.717	6.329	3.744	0.044	1.430	0.9996	-11.87
0.12	0.367	0.706	5.723	3.554	0.050	1.382	0.9980	-15.76
0.15	0.446	0.694	5.293	3.424	0.054	1.349	0.9961	-18.52

From the endogenous case, we can find the population growth rate decreases as the contribution rate increases. The reason is that individuals reduce the number of children together with reducing their consumption. In this case, the optimal payroll tax rate is 0, which means that the true optimal tax rate is negative number. If it is low enough, we cannot find the change even if the optimal rate would rise.

Our theoretical interest is whether to make population growth rate endogenous raises the optimal social security level or not when there is dynamic inefficiency in the economy without social security. In the context of dynamic inefficiency, the population growth rate has been always set constant, although population growth rate matters crucially in the optimality condition. It is interesting to investigate how endogenizing population growth rate may affect the optimal social security level when there is dynamic inefficiency. Since it is shown that the optimal social security is greater than 0 when β is high in İmrohoroğlu *et al.* (1995), we will use a discount factor high enough to realize a positive rate of optimal social security level. In this study we will set $\beta = 1.011$ following the previous studies¹⁴. We also set high population growth so that the return from social security is higher than that from saving. We will set $\rho = 1.009$ at the benchmark social security level $\tau_p = 13.95\%$ ¹⁵. A positive optimal social security level ($\tau_p = 0.04$) is obtained in the exogenous case shown in Table 4.

¹⁴In Hurd(1989), İmrohoroğlu *et al.* (1995) and Ríos-Rull (1996) use $\beta = 1.011$.

¹⁵In order to obtain this population growth rate, we choose $\chi = 9$.

Table 4: Exogenous Case (decomposed, $\rho = 1.009$)

τ_p	p	Labor supply	Capital stock	K/Q	Interest rate	Wage rate	Average utility
0	0	0.770	13.613	5.703	0.008	1.880	25.22
0.01	0.039	0.770	12.934	5.529	0.010	1.842	25.49
0.02	0.080	0.770	12.381	5.384	0.012	1.811	25.62
0.03	0.121	0.770	11.762	5.220	0.015	1.774	25.72
0.04	0.163	0.770	11.410	5.124	0.016	1.753	25.76
0.05	0.206	0.770	10.814	4.960	0.018	1.717	25.66
0.06	0.249	0.770	10.455	4.860	0.020	1.694	25.58
0.07	0.294	0.770	10.163	4.777	0.022	1.675	25.48

Using this setting, we endogenize the population growth rate to investigate how the optimal social security level may change. It is shown that the optimal population growth rate is 0 (Table 5). We may interpret the result in the following way: when population growth rate is endogenous, individuals can adjust their number of children. Below the optimal social security level ($\tau_p = 0.04$), individuals increase their consumption level when the tax rate is raised as shown in exogenous case. At this time, they would have more children if the number of children has exactly the same property as consumption, namely normal goods. However, since they have to choose the number from integer, they may not increase it since an additional child is very expensive. In addition, individuals may tend to face borrowing constraints as the contribution rate is raised, so they have to reduce the number of children in that case. This can be understood in the following way; individuals need to make a certain amount of payment for 20 periods once they decide to have an additional child. But they may get unemployed and face income shocks in the following 19 period, so they still need to save for those shocks. If the payroll tax is raised, it may affect on their liquidity and they may lessen their number of children. As a result, the welfare level is worsened since the population growth rate is declined and the return from social security is lower than before.

There is another effect in the endogenous population growth setting. When the population growth rate drops, the labor input is getting scarce relatively, which leads to a rise in interest rate. This may make social security less attractive to savings in terms of return. Individuals would prefer sav-

Table 5: Endogenous Case (decomposed)

τ_p	p	Labor supply	Capital stock	K/Q	Interest rate	Wage rate	Fertility rate	Average utility
0	0	0.790	12.38	5.300	0.013	1.792	1.0131	56.4
0.01	0.045	0.790	11.89	5.172	0.015	1.764	1.0130	53.7
0.02	0.091	0.790	11.36	5.030	0.017	1.732	1.0130	50.9
0.03	0.138	0.790	10.74	4.865	0.020	1.695	1.0129	46.8
0.04	0.186	0.790	10.42	4.776	0.022	1.675	1.0129	43.0
0.05	0.235	0.790	10.36	4.760	0.022	1.671	1.0129	40.0
0.06	0.280	0.787	10.17	4.716	0.023	1.661	1.0123	36.3
0.07	0.328	0.786	9.96	4.661	0.024	1.649	1.0121	31.9

ings to social security as the relative return from social security is worsened, which means that raising social security lowers their utility level.

Taking into account the whole effects, it is possible to conclude that in the endogenous population setting, there is a possibility that (i) borrowing constraints and absence of secondary markets may restrict individuals choice seriously and that worsens welfare level as social security is enlarged (ii) the interest rate goes up as the population growth rate drops, which may make social security less attractive to savings in terms of return. Thus, even in the case when there is dynamic inefficiency, the optimal social security level is lower than in exogenous case with endogenous population growth.

5 Conclusion

In this study, we investigate how the optimal social security level may be altered by endogenizing the population growth rate. As a result, it is shown that the rate in the endogenous case is lower than that of the exogenous case. The possible reasons are as following; individuals may face borrowing constraints and income shocks not only when they decide the number of children but also when they rear children in the following 20 periods. Thus, they have smaller number of children for the case that they face an income shock, and the interest rate rises as population growth rate drops, so the relative attractiveness with social security is lessened as social security is enlarged.

This study deals with steady state analysis, and we cannot tell the optimal

payroll tax rate in a transitional economy from the result. It is important to study a transition path to the steady state since in the real economy we may not achieve the steady state directly. However, it gives us a hint for considering how taking endogenous population into account would change the optimal payroll tax rate. This study finds that borrowing constraints and the absence of secondary markets for children may affect the result and lower the optimal tax rate in the endogenous case.

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