NUS
National University of Singapore

SCAPE

Singapore Centre for Applied and Policy Economics

A Gaussian Test for Cointegration

by

Tilak Abeysinghe and Gulasekaran Rajaguru

Department of Economics
SCAPE Working Paper Series
Paper No. 2009/05 – Dec 2009
http://www.fas.nus.edu.sg/ecs/pub/wp-scape/0905.pdf

A Gaussian Test for Cointegration

Tilak Abeysinghe[a,*] and Gulasekaran Rajaguru[b]

[a] Department of Economics, National University of Singapore
[b] School of Business, Bond University, Australia

December 2009

Abstract

We use a mixed-frequency regression technique to develop a test for cointegration under the null of stationarity of the deviations from a long-run relationship. What is noteworthy about this MA unit root test, based on a variance-difference, is that, instead of having to deal with non-standard distributions, it takes the testing back to the normal distribution and offers a way to increase power without having to increase the sample size substantially. Monte Carlo simulations show minimal size distortions even when the AR root is close to unity and that the test offers substantial gains in power against near-null alternatives in moderate size samples. An empirical exercise illustrates the relative usefulness of the test further.

Key words: Null of stationarity, MA unit root, mixed-frequency regression, variance difference, normal distribution, power.

JEL: C12, C22

----

*Corresponding author at: Department of Economics, National University of Singapore, 1 Arts Link, Singapore 117570. Tel: +65 6516 6116, Fax: +65 6775 2646.
Email: tilakabey@nus.edu.sg . Email for second author: rgulasek@bond.edu.au .

The authors would like to mention gratefully the research and travel grants provided by the Faculty of Arts and Social Sciences and the Singapore Center for Applied and Policy Economics, National University of Singapore. Thanks are also due to Gu Jiaying for her able research assistance, Choy Keen Meng and seminar participants at Singapore Management University, Monash University, LaTrobe University, Griffith University and the Australasian Meeting of the Econometric Society for their valuable comments on an earlier draft of the paper.

1. Introduction

Non-standard distributions are a common feature of many tests for unit-roots and cointegration that are currently available.[1] The main problem with non-standard distributions is that when the true data generating process is unknown, which is the case in general, it is not easy to engage in a specification search because the distribution changes as the specification changes, especially with respect to deterministic components. As Cochrane (1991, p. 202) expressed: "To a humble *macroeconomist* it would seem that an edifice of asymptotic distribution theory that depends crucially on unknown quantities must be pretty useless in practice." Some reprieve to this has been offered by Phillips (1998, 2002) who showed that the limiting forms of autoregressive unit root processes can be expressed entirely in terms of deterministic trend functions. The implication of this finding is that "one might mistakenly 'reject' a unit root model in favour of a trend 'alternative' when in fact the alternative model is nothing other than an alternative representation of the unit root process itself." (Phillips, 2002, p.324). Considering the complexities involved in the specification of deterministic trend models his recommendation is, especially on grounds of parsimony and forecasting, to use pure autoregressions. Nevertheless, economic reasoning may necessitate some deterministic components in the model that will take us back to the same problem of multitude of non-standard distributions.

In this exercise we re-visit the problem with the objective of presenting a test for cointegration based on the null of stationarity of the deviations from a long-run

[1] See Maddala and Kim (1998) for an extensive survey of the unit root literature.

relationship. The test brings the distribution back to the normal distribution and at the same time offers a substantial improvement in power. The importance of tests based on the null of stationarity need not be overemphasized. Although a disproportionate amount of research has gone into I(1) processes, the I(1) characterization of economic time series may be too restrictive in many practical situations. What is of general interest is whether the regression provides stable parameters with stationary residuals regardless of the nature of the non-stationarity of the individual series. For example, two variables which are causally related may have structural breaks in them and the usual unit root tests may perceive them to be I(1) processes. In a regression relationship, however, the structural break may disappear and the regression may deliver stationary residuals.[2] Therefore, forming a null of stationarity will allow us to test it against different alternatives such as autoregressive (AR) unit roots, fractional integration, structural breaks and policy interventions. The relevant alternative has to depend on the particular empirical analysis carried out. In this exercise we consider only the AR unit root alternative and defer the evaluation of other alternatives to future work.

The test presented here focuses on a moving average (MA) unit root. Although the idea of testing for an MA unit root is not new (see Table A.1) the importance of such tests need to be re-emphasized. Being a behavioral outcome an AR unit root could be somewhat illusive (see Hamilton, 1994, Sec. 15.4) whereas an MA unit root can be created by over-differencing a stationary process, therefore, easier to pin down. The basic

[2] There are also cases where economic theory leads to using variables like investment/GDP ratio or the average tax rate in regressions. The meaning of a unit root in these variables is unclear.

idea underlying our test procedure emanated from a mixed-frequency regression presented in Abeysinghe (1998, 2000) and temporal aggregation and dynamic relationships studied in Rajaguru and Abeysinghe (2002) and Rajaguru (2004). The test procedure involves a simple data transformation to obtain a mixed frequency regression and focuses on the difference in error variances of the original model and the transformed model. This method can be exploited to develop even better tests with standard distributions.

2. Power of Existing Unit Root Tests

As can be seen later in a Monte Carlo simulation, our proposed test entails substantial gains in power at near null alternatives. For comparison Table A.1 in the Appendix provides a non-exhaustive summary, extracted from the cited studies, of the power of both AR and MA unit root tests near the null at a sample size 100 (or 200 in a few cases). Panel (a) in the table is for the non-stationary null (AR unit root) and panel (b) for the stationary null (MA unit root or its variants). Panel (a) also includes a representative citation of power under structural breaks. The literature on unit roots under structural breaks has also grown rapidly and we do not digress into this literature. The reference model given in the table involves an over-simplification for some simulation exercises. A general specification of the stationary null is given in models (1) and (2) of the paper.

The summary in Table A.1 highlights the low power of unit root tests in general although some test procedures produce reasonably large power at a sample of size 100.[3] As stated earlier, most of these tests have to deal with non-standard distributions and increasing the power requires increasing the sample size. These are the problems that we try to address by the proposed test procedure.

3. Methodology

Consider the following model that Leybourne and McCabe (1994) extended from Harvey (1989) and Kwiatkowski et al. (1992) to test the null of stationarity against an alternative of difference stationarity:

$$\begin{aligned} \phi(L)y_t &= \alpha_t + \beta t + \varepsilon_t \\ \alpha_t &= \alpha_{t-1} + \eta_t,\ \alpha_0 = \alpha \end{aligned} \tag{1}$$

where $\varepsilon_t \sim iid(0,\sigma_\varepsilon^2)$, $\eta_t \sim iid(0,\sigma_\eta^2)$, both of which are independent of each other, and $\phi(L) = 1-\phi_1 L - ... - \phi_p L^p$ with roots outside the unit circle. This has the following ARIMA(p,1,1) representation:

$$\Delta y_t = \beta + \phi_1 \Delta y_{t-1} + ... + \phi_p \Delta y_{t-p} + u_t - \theta u_{t-1} \tag{2}$$

where $u_t \sim iid(0,\sigma^2)$ with $\sigma^2 = \sigma_\varepsilon^2/\theta$, $\theta = (\lambda - (\lambda^2 + 4\lambda)^{1/2} + 2)/2$ and $\lambda = \sigma_\eta^2/\sigma_\varepsilon^2$ is the signal-to-noise ratio. The so-called hyper-parameter σ_η^2 is a measure of the size of the random walk in (1). If $\sigma_\eta^2 = \lambda = 0$, $\theta = 1$ and model (2) collapses to a stationary AR(p)

[3] It should be noted that Monte Carlo results by Gonzalo and Lee (1996) show that the size and power properties of Dickey-Fuller type unit root tests in many situations are better than the standard t-tests for stationary roots of autoregressive processes.

process. Alternatively, Δy_t in (2) has a non-invertible ARMA(p,1) representation. To test the null of stationarity a number of researchers formulated tests based on $H_0 : \sigma_\eta^2 = 0$ vs $H_1 : \sigma_\eta^2 > 0$. These are in effect tests of an MA unit root and the distributions involved are in general non-standard. As λ increases, θ approaches zero and we get a standard unit root autoregression. In this exercise the ARIMA model in (2) forms the basis of our test.

3.1 Null of Stationarity (MA Unit Root)

As stated earlier our test is based on a mixed frequency regression procedure (Abeysinghe, 1998, 2000) that helps in increasing the power of the test at a given sample size. To illustrate the idea, (2) can be written as

$$u_t = \theta u_{t-1} - \beta + \phi(L)\Delta y_t . \tag{3}$$

If u_t is assumed to be observed at intervals $t = m, 2m, ..., T$, where $m \geq 2$ is a positive integer, and Δy_t is observed at intervals $t = 1, 2, .., T$, the basic idea of the mixed frequency regression is to transform u_{t-1} in (3) to u_{t-m} such that all the observations of Δy_t are retained in the regression. This transformation is easily obtained by multiplying (3) through by the polynomial $\theta(L) = 1 + \theta L + ... + \theta^{m-1}L^{m-1}$. The transformed model can be written as

$$\theta(L)\phi(L)\Delta y_t = \theta(1)\beta + V_t \tag{4}$$

where $V_t = \theta(L)(1-\theta L)u_t = u_t - \theta^m u_{t-m}$.

Now note that under the null $H_0 : \theta = 1$, $Var(V_t) = 2\sigma^2$ and under the alternative $H_1 :| \theta |< 1$, $Var(V_t) = (1+\theta^{2m})\sigma^2 < 2\sigma^2$. Therefore, $\sigma_m^2 - 2\sigma^2$, where $\sigma_m^2 = Var(V_t)$, forms the basis of our test. By transforming the test of θ into a test of $Var(V_t)$ we can arbitrarily increase the distance between the null and the alternative simply by increasing *m* whereby extra gains in power is made possible. For example, a test of $\theta = 1$ when $\theta = 0.9$ translates into comparing $2\sigma^2$ against $Var(V_t) = 1.43\sigma^2$ for *m*=4 and $Var(V_t) = 1.08\sigma^2$ for *m*=12. This transformation allows us to formulate a number of test statistics that follow standard distributions.

Given that we can obtain consistent estimates of the parameters in (2), we can compute $\hat{\sigma}^2$ and $\hat{\sigma}_m^2$ (see below) and then form the test statistic $\sqrt{T}\,(\hat{\sigma}_m^2 - 2\hat{\sigma}^2)$ to test $\theta = 1$ against $| \theta |< 1$. Using the subscript *T* to indicate the dependence on the sample size, the following theorem establishes the asymptotic distribution of the test statistic.

Theorem

Given that $u_t \sim iid(0,\sigma^2)$ and assuming $E(u_t^4) = \mu_4 < \infty$, under the null hypothesis of $\theta = 1$ and for *m>p,* $\sqrt{T}\,(\hat{\sigma}_{m,T}^2 - 2\hat{\sigma}_T^2) \xrightarrow{d} N(0, 4\sigma^4)$. In small samples $Var[\sqrt{T}\,(\hat{\sigma}_{m,T}^2 - 2\hat{\sigma}_T^2)] = 4\sigma^4 + 4\mu_4 (T / T_a - 1) - 2(\mu_4 - \sigma^4)mT / T_a^2$, where T_a is the effective sample size.

Proof: see Appendix.

The test procedure in practice is the following. Assuming p+1 pre-sample values $y_{-p},...,y_0$ are available, estimate the ARMA(p,1)[4] for Δy_t in (2) by ML and obtain $\hat{\theta}$ and $\hat{\sigma}^2 = \sum_{t=1}^{T} \hat{u}_t^2 /(T - p - 2)$ (these are provided by standard computer software procedures). Then obtain $\hat{V}_t = \hat{u}_t - \hat{\theta}^m \hat{u}_{t-m}$ and $\hat{\sigma}_m^2 = \sum_{t=m+1}^{T} (\hat{V}_t - \bar{\hat{V}})^2 /(T_a - 1)$, where $\bar{\hat{V}}$ is the sample mean of $\hat{V}$ series, $T_a = T - m$, and compute the z score. If u_t is assumed to be Normal then $z = \sqrt{T}(\hat{\sigma}_m^2 - 2\hat{\sigma}^2)/[2\hat{\sigma}^2(1 + 3(T/T_a - 1) - mT/T_a^2)^{1/2}]$ and reject the null hypothesis $\theta = 1$ if $z \le c$ where c is a left-hand critical value from the standard normal distribution. We term this z(MA) test to differentiate it from a z(AR) test that can be obtained by extending our test procedure to the AR unit root case.[5]

In estimating θ there are two problems that we have to guard against. One is the well known pile-up problem of the ML estimator at the invertibility boundary (see Breidt et al., 2006, for references). The pile-up problem is an issue that is being addressed by a number of researchers. In particular Davis and Dunmuir (1996) have explored the possibility of using a Laplace likelihood with a local maximizer to estimate an MA(1) model with a unit root or a near unit root. It is very likely that an estimator of θ that will overcome the pile-up problem will emerge in due course. From a practical point of view, the pile-up problem of the Gaussian likelihood may not be a serious problem. Although

[4] Model selection criteria and the usual diagnostics may be used for determining the structure of the model (see the empirical exercise in Section 5).

[5] We extended the test procedure to the AR unit root case, which provides a generalization to the variance-ratio test developed by Lo and MacKinlay (1988, 1989). Although the extension works very well (better power) in the AR(1) case, we still need further work on the AR(p) case.

over-differenced stationary series produce $\theta = 1$, AR unit-root series are likely to produce a θ well away from unity. Many empirical estimates of θ from non-stationary series hardly exceed 0.9 and do not exhibit the presence of the pile-up problem. As we shall see, our test offers sufficient power against the alternative of θ=0.9 in moderate-sized samples.

The other difficulty is the near common factor problem. Although the ML estimator of θ under the null is T-consistent (see Davis and Dunmuir, 1996, and reference therein), an AR factor with a root close to unity may render a highly unreliable estimate of θ in certain samples. The near common factor problem can easily be spotted by fitting an AR(p) model to y_t and ARMA(p,1) to Δy_t (see the application in Section 4). If y_t is stationary with an AR root near unity and if it is not well estimated in the ARMA model then it is important to re-estimate the model using different starting values for θ, including θ=1.[6]

3.2 Monte Carlo Results

In this section we present the results of a Monte Carlo experiment to highlight the size and power properties of the test under near unit root alternatives. Since our primary interest is in cointegrating relationships we use the following model for the simulation exercise.

$$
\begin{aligned}
y_t &= \delta_0 + \delta_1 x_t + z_t \\
x_t &= x_{t-1} + \varepsilon_t .
\end{aligned}
\qquad (5)
$$

[6] It is instructive to use a dedicated ARIMA software procedure for estimation. We used SAS PROC ARIMA in our exercise by removing the default boundary constraint.

$$(1-\phi_1 L)\Delta z_t = \beta + (1-\theta L)u_t \qquad (6)$$

where u_t and ε_t are generated from independent $N(0,1)$ distributions. If δ_1 is known then (6) represents the case of testing for the stationarity of a known long run relationship. If δ_0 and δ_1 are estimated then (6) represents the case of testing for the stationarity of regression residuals. The size of the test is obtained when $\theta = 1$. For this we set $\phi_1 = 0.5,\ 0.9,\ 0.95$. For power, we use $\theta = 0.8,\ 0.9$ with $\phi_1 = 0.5$. In the case of known z_t $\beta = 1$ and when z_t is estimated residuals $\beta = 0$ and $\delta_0 = \delta_1 = 1$. We obtained the simulation results for *m*=2,4,6,8,10,12 and the size and power of the test are reported in Tables 1 and 2 respectively. We exclude the results for m=10,12 from the tables because they do not add much new information. Moreover, when $\theta = 1$ and ϕ_1 large, convergence problems occur in some replications in small samples, so we obtained only the large sample results for these cases in Table 1. For the sample sizes considered here we noticed that the results do not change much whether we use the small sample variance or the asymptotic variance given in the theorem; so we use the asymptotic variance to obtain the results in these tables.

We observe that as *m* increases the size tends to increase especially when regression residuals are involved. Since the test relies on the consistency of $\hat{\theta}$, the small-sample bias of the estimator tends to distort the distribution of the test statistic as *m* increases. Table 2 shows that the power of the test is quite impressive. However, the gain in power when *m* increases beyond 4 is rather small. Therefore, based on both size and power properties an

m=4 seems to be an optimal choice. We also examined the results by over-fitting the AR order upto p=3; the results remain very much unaffected by this over-fitting.

These results may be compared with those in Table A.1. In particular, it is worth making a comparison with the variance-difference (VD) test that Breitung (1994) developed. This asymptotically normal VD test, derived based on the assumption of an MA(q) process, produces desirable small-sample size and power properties for finite order MA processes. However, when the process involved was an ARMA(1,1) that needed to be approximated by a finite order MA process, Breitung observed substantial size distortions. For example, when ϕ_1=0.9 (θ=1), T=100, α =5%, Breitung reported empirical size of 0.907 for MA(4) approximation and 0.215 for MA(12) approximation. This problem does not arise in our test as we can see in Table 1. The table also shows that near AR unit root cases which manifest with low power in AR unit root tests come under the control of type I error in this MA unit root test.

=================

Insert Table 1, Table 2

=================

4. Some Empirical Results

As empirical illustrations, we present two sets of results. The first is a representative group of variables from Abeysinghe and Choy (2007) who present a 62-equation macroeconometric model (ESU01 model) of the Singapore economy and the second is a test of stationarity of the average propensity to consume (APC) in OECD countries.

Abeysinghe and Choy (2007) estimated all the key behavioral equations in their model individually in the form of error correction models by crafting out the underlying long-run (cointegrating) relationships, paying careful attention to specific features of the Singapore economy, economic theory, and parameter stability. Table 3 presents test results for two groups of cointegrating relationships: (i) cointegrating regression residuals[7] and (ii) relations with known coefficients. In the latter group, the oil price equations were designed to check the extent of exchange rate pass-through.[8] Relative unit business cost (RUBC) and the real exchange rate (RER) are both measures of competitiveness. Although the RER presented in the table is not a variable in the ESU01 model, we use it here for further illustration of the performance of the test.

=============

Insert Table 3

=============

In Table 3, all series except for RER clearly qualify as AR(1) processes and it is worth noting that the estimates of ρ from AR model and ARMA(1,1) model for the first differences are very close. Therefore, first estimating an AR(p) model provides a good check against the ARMA(p,1) estimation for the MA unit root test. It is also useful to note that when over-differencing is not involved as in the RER case (also those in Table 4

---

[7] Readers interested in the regression equations and data are referred to Abeysinghe and Choy (2007).

[8] As the third largest oil refining center and trading hub in the world Singapore may have some price setting power on its oil market in which case the stationarity of the long-run relationship with unity restriction has to be rejected. Note that short-run pass through is well below unity.

below) the MA root is likely to be a distance away from unity in many practical cases and as a result our test carries a lot of power against such alternatives.

The test results in Table 3 show that if we were to use the ADF test to test for cointegration only three equations (consumption, exports and oil export price) qualify as cointegrating relationships (the null of AR unit root is rejected). Our z(MA) test, on the other hand, does not reject the null of stationarity (and cointegration) in all the cases except the last one. The RER series with $\hat{\rho}$ =0.98 clearly comes out as a non-stationary process. Since Abeysinghe and Choy (2007) have already studied these cointegrating relationships in detail, and the fact that the z(MA) test results concur with their findings represents a strong case in favor of the new test.

As a further illustration of the test, Table 4 presents the test results from three popular tests and the z(MA) test on APC for 21 OECD countries.[9] Because of the non-availability of sufficiently long data series on non-durable consumption and disposable income we measure APC by the ratio of total consumption expenditure to GDP. Although the APC is expected to be stationary for developed economies on the grounds that long-run departures of consumption expenditure from income is less likely, some countries show local trends in their APCs over the sample period. This is reflected in large values of $\hat{\rho}$ (the sum of AR coefficients) in Table 4. This is where many tests may misconstrue the APC to be an I(1) process.

[9] Data for this exercise are from the IFS database except for France. IFS data for France show some irregularities; therefore, France data were taken from the OECD database which covers a shorter time span than the IFS database.

As in Table 3, we notice in Table 4 the close correspondence between AR(p) coefficients and ARMA(p,1) coefficients in identifiable stationary cases. It is also worth noting that in stationary cases $\hat{\theta}$ turns out to be almost unity. This means that the size distortion we noticed in the Monte Carlo experiment resulting from under estimation of θ may not be a serious problem in practice.

Again the ADF test turns out to be the least powerful against near unit root alternatives, as it renders an I(1) verdict for 18 of the 21 APC series. The KPSS test and the Johansen test fair better, recognizing eight cases as cointegrating relationships. Unfortunately the eight cases do not necessarily overlap. Our z(MA) test, on the other hand, takes 15 of the APC series to be stationary. It rejected stationarity only when $\hat{\rho} \geq 0.97$ and when the local trend dominated the series; see the cases of Canada and Korea for a comparison, both with $\hat{\rho} = 0.97$, while one is assessed to be I(0), the other I(1). Like many fast growing developing economies Korea experienced a falling APC till the mid 1980s before stabilizing to fluctuate around a constant mean. Rejecting the null of stationarity of APC is, therefore, an indication of the interplay of other variables that need to be considered instead of taking APC to be an I(1) process.

=================

Insert Table 4

===============

5. Conclusion

This exercise addresses three important issues. First, it highlights the importance of formulating tests based on the null of stationarity. Unfortunately the profession has not paid enough attention to this. What is of general importance is whether a regression relationship produces stationary residuals regardless of the nature of non-stationarities of the individual series. Moreover, an AR unit root in an individual series is hard to pin down because an apparent unit root could be a manifestation of some other forms of non-stationarity. We present an MA unit root test based on the null of stationarity. Unlike the AR unit root which is a behavioral outcome, the MA unit root is created by over-differencing and therefore easier to pin down.

Although testing for an MA unit root is not new to the literature the existing tests require non-standard distributions. The second important aspect of the exercise is that the proposed test brings us back to the normal distribution and makes specifications searches easier. The third aspect of the exercise is that the test procedure entails a mechanism to increase power without necessarily having to increase the sample size. This addresses the problem of low power at near null alternatives of many unit root tests that are currently available.

An important objection one could raise against our test is the difficulty of estimating an MA root on or near the unit circle. Some researchers are actively working on this problem and a better estimation method is likely to emerge in due course. Nevertheless, as our empirical exercise highlights, the estimation problem may not be that serious in cases encountered in practice. An alternative would be to devise a method that avoids estimating θ. We tried to obtain the transformed residuals in (4) through a couple of methods that utilized only the AR parameter estimates, but they did not improve power much despite producing impressive size properties.

Appendix

Proof of the Theorem

Here we derive the distribution of $\sqrt{T}(\hat{\sigma}^2_{m,T} - 2\hat{\sigma}^2_T)$ under the null hypothesis $\theta = 1$. The ML estimates of the parameters are obtained by running the model in (2). Using the results below it can easily be verified that $\sqrt{T}(\hat{\sigma}^2_{m,T} - 2\hat{\sigma}^2_T) \xrightarrow{p} 0$. To derive the variance, this can be expressed as

$$E[\sqrt{T}\,(\hat{\sigma}^2_{m,T} - 2\hat{\sigma}^2_T)]^2 = T\,E[(\hat{\sigma}^2_{m,T} - 2\sigma^2) - 2(\hat{\sigma}^2_T - \sigma^2)]^2. \qquad \text{(A1)}$$

It is well established that $\hat{\sigma}^2_T = (1/T)\hat{\boldsymbol{u}}'_T\hat{\boldsymbol{u}}_T \xrightarrow{p} (1/T)\boldsymbol{u}'_T\boldsymbol{u}_T \xrightarrow{p} \sigma^2$ and $\sqrt{T}(\hat{\sigma}^2_T - \sigma^2) \xrightarrow{d} N(0,(\mu_4 - \sigma^4))$. (See, for example, Hamilton, 1994, p. 212.)

For $\hat{\sigma}^2_{m,T}$, with reference to model (2) define $\boldsymbol{\beta} = (\beta, \phi_1, ..., \phi_p)'$, the $T \times (1+p)$ matrix $\boldsymbol{X}$ with the t-th row given by $(1, \Delta y_{t-1}, ..., \Delta y_{t-p})$, $\boldsymbol{u} = (u_1, u_2, ..., u_T)'$, $\boldsymbol{u}_{-1} = (u_0, u_1, ..., u_{T-1})'$, $\boldsymbol{u}_{-m} = (u_m, u_{m+1}, ..., u_T)'$, and the $(T-m+1)\times T$ aggregation matrix $\boldsymbol{A}$:

$$\boldsymbol{A} = \begin{pmatrix} \theta^{m-1} & . & \theta^2 & \theta & 1 & 0 & 0 & 0 & . & . & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta^{m-1} & . & \theta^2 & \theta & 1 & 0 & 0 & . & . & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta^{m-1} & . & \theta^2 & \theta & 1 & 0 & . & . & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & \theta^{m-1} & . & \theta^2 & \theta & 1 \end{pmatrix} \qquad \text{(A2)}$$

Model (2) now can be written in vector-matrix notation as $\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u} - \theta\boldsymbol{u}_{-1}$. Pre-multiplying this by $\boldsymbol{A}$ and using the subscript a to indicate aggregation, we obtain

$\boldsymbol{y}_a = \boldsymbol{X}_a\boldsymbol{\beta} + \boldsymbol{u} - \theta^m \boldsymbol{u}_{-m}$ which can be re-arranged to give $\boldsymbol{y}_a = \boldsymbol{X}_a\boldsymbol{\beta} + \boldsymbol{V} - (\theta^m - 1)\boldsymbol{u}_{-m}$ where $\boldsymbol{V} = \boldsymbol{u} - \boldsymbol{u}_{-m}$ under the null. Now we can obtain

$$\begin{aligned}\hat{\boldsymbol{V}} &= \boldsymbol{V} - \boldsymbol{X}_a(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) + (\hat{\theta}^m - 1)\boldsymbol{u}_{-m} \\ &= \boldsymbol{V} - \boldsymbol{X}_a^*(\hat{\boldsymbol{\beta}}^* - \boldsymbol{\beta}^*)\end{aligned} \tag{A3}$$

where $\boldsymbol{X}_a^*$ is augmented $\boldsymbol{X}_a$ with the first element of the tth row given by u_{t-m} and $(\hat{\boldsymbol{\beta}}^* - \boldsymbol{\beta}^*)$ is augmented $(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$ with the first element given by $(\hat{\theta}^m - 1)$. Now defining the diagonal scaling matrix Υ of dimension $(2+p)\times(2+p)$ with the first diagonal element given by T and the rest by $T^{1/2}$ (Sims et al., 1990; Note, $\hat{\theta}$ is T-consistent) we obtain under the null:

$$\begin{aligned}\hat{\sigma}^2_{m,T} &= (1/T)\hat{\boldsymbol{V}}_T'\hat{\boldsymbol{V}}_T = (1/T)\boldsymbol{V}_T'\boldsymbol{V}_T - (2/T)(\hat{\boldsymbol{\beta}}_T^* - \boldsymbol{\beta}^*)'\boldsymbol{X}_{aT}^{*\prime}\boldsymbol{V}_T \\ &\qquad + (\hat{\boldsymbol{\beta}}_T^* - \boldsymbol{\beta}^*)'(\boldsymbol{X}_{aT}^{*\prime}\boldsymbol{X}_{aT}^*/T)(\hat{\boldsymbol{\beta}}_T^* - \boldsymbol{\beta}^*) \\ &= (1/T)\boldsymbol{V}_T'\boldsymbol{V}_T - 2(\hat{\boldsymbol{\beta}}_T^* - \boldsymbol{\beta}^*)'(\Upsilon_T/\sqrt{T})(\Upsilon_T^{-1}/\sqrt{T})\boldsymbol{X}_{aT}^{*\prime}\boldsymbol{V}_T \\ &\quad + (\hat{\boldsymbol{\beta}}_T^* - \boldsymbol{\beta}^*)'(\Upsilon_T/\sqrt{T})(\Upsilon_T^{-1}\boldsymbol{X}_{aT}^{*\prime}\boldsymbol{X}_{aT}^*\Upsilon_T^{-1})(\Upsilon_T/\sqrt{T})(\hat{\boldsymbol{\beta}}_T^* - \boldsymbol{\beta}^*) \\ &\xrightarrow{p} 2\sigma^2.\end{aligned} \tag{A4}$$

This result holds because $(\Upsilon_T/\sqrt{T})(\hat{\boldsymbol{\beta}}_T^* - \boldsymbol{\beta}^*) \xrightarrow{p} \boldsymbol{0}$ while the rest converge to bounded quantities.

Now we have to consider the distribution of $\sqrt{T}(\hat{\sigma}^2_{m,T} - 2\sigma^2)$. Multiplying (A4) through by $\sqrt{T}$ shows that the last term of (A4) converges in probability zero and in the second term, $T(\hat{\theta}^m - 1)(T^{-3/2}\sum X_{a1t}V_t) \xrightarrow{p} 0$ and $\sqrt{T}(\hat{\beta} - \beta)(1/T)\sum X_{a2t}V_t \xrightarrow{p} 0$. Thus we have to consider the distribution of

$$\sqrt{T}(\hat{\sigma}^2_{m,T} - 2\sigma^2) \\ = \sqrt{T}(\boldsymbol{V_T}'\boldsymbol{V_T}/T_a - 2\sigma^2) - 2\sqrt{T}(\hat{\boldsymbol{\beta}}_T - \boldsymbol{\beta})'(\boldsymbol{X}_{aT}{}'\boldsymbol{V}_T/T_a) \tag{A5}$$

where the presence of the constant term in the $\boldsymbol{X}_a$ matrix is inconsequential.

Now consider the variance of the first term on the RHS of (A5):

$$(T/T_a^2)E[(\sum_{t=1}^{T}(V_t^2 - 2\sigma^2)]^2 \\ = (T/T_a^2)[\sum E(V_t^2 - 2\sigma^2)^2 + 2\sum E((V_t^2 - 2\sigma^2)(V_{t-k}^2 - 2\sigma^2)] \tag{A6}$$

where k =1,2,...

From the first term of (A6):

$$E(V_t^2 - 2\sigma^2)^2 = E(V_t^4 - 4\sigma^2 V_t^2 + 4\sigma^4) = E(V_t^4) - 4\sigma^4$$

and

$$E(V_t^4) = E[(u_t - u_{t-m})^2]^2 \\ = E[(u_t^2 - 2u_t u_{t-m} + u_{t-m}^2)(u_t^2 - 2u_t u_{t-m} + u_{t-m}^2)] \\ = 2\mu_4 + 6\sigma^4.$$

Thus

$$E(V_t^2 - 2\sigma^2)^2 = 2(\mu_4 + \sigma^4).$$

From the second term of (A6):

$$E[(V_t^2 - 2\sigma^2)(V_{t-k}^2 - 2\sigma^2)] = E(V_t^2 V_{t-k}^2) - 4\sigma^4.$$

Now for k=m

$$E(V_t^2 V_{t-m}^2) = E[(u_t^2 - 2u_t u_{t-m} + u_{t-m}^2)(u_{t-m}^2 - 2u_{t-m}u_{t-2m} + u_{t-2m}^2)] \\ = \mu_4 + 3\sigma^4.$$

Thus

$$E[(V_t^2 - 2\sigma^2)(V_{t-k}^2 - 2\sigma^2)] = \mu_4 - \sigma^4, \;\; \text{if } k = m$$
$$= 0, \;\text{otherwise.}$$

Combining the two terms of (A6) we obtain:

$$\begin{aligned}(T / T_a^2)E[(\sum_{t=1}^{T}(V_t^2 - 2\sigma^2)]^2 \\ &= (T / T_a^2)[T_a(2\mu_4 + 2\sigma^4) + (T_a - m)(2\mu_4 - 2\sigma^4)] \qquad \text{(A7)} \\ &= 4\mu_4 T / T_a - 2(\mu_4 - \sigma^4)mT / T_a^2] \\ &\to 4\mu_4.\end{aligned}$$

Note that $(V_t^2 - 2\sigma^2)$ is a stationary process and therefore by the central limit theorem $\sqrt{T}(\boldsymbol{V}_T'\boldsymbol{V}_T / T_a - 2\sigma^2) \xrightarrow{d} N(0, 4\mu_4)$.

Now consider the second term on the RHS of (A5). To obtain its variance first note that $\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \xrightarrow{d} N(\boldsymbol{0}, \sigma^2(\boldsymbol{X}'\boldsymbol{X} / T)^{-1})$ and $\boldsymbol{X}_{aT}'\boldsymbol{V}_T / T_a \xrightarrow{p} \boldsymbol{c}(\sigma^2, \phi_1, ..., \phi_p, m)$ where $\boldsymbol{c}$ is a $(1+p)\times 1$ vector. This vector can be derived easily by noting that the aggregated form of model (2) under the null provides, $\Delta_m y_t = \phi(L)^{-1}V_t = \psi(L)V_t$, where $\phi(L)^{-1} = \psi(L) = 1 + \psi_1 L + \psi_2 L^2 + ...$ Note that the first term of $\boldsymbol{X}_{aT}'\boldsymbol{V}_T / T_a$ that corresponds to the constant term of the model is zero. Now consider the second term in the $\boldsymbol{X}_{aT}'\boldsymbol{V}_T / T_a$ vector:

$$\begin{aligned}(1/T_a)\sum X_{a2t}V_t &= (1/T_a)\sum \Delta_m y_{t-1}V_t = (1/T_a)\sum \psi(L)V_{t-1}V_t \\ &= (1/T_a)\sum[(1 + \psi_1 L + \psi_2 L^2 + ...)(1 - L^m)u_{t-1}](u_t - u_{t-m}) \\ &= -\sigma^2\psi_{m-1}.\end{aligned}$$

Proceeding in this way, we obtain for p=m: $\boldsymbol{c}' = -\sigma^2(0, \psi_{m-1}, \psi_{m-2}, ..., 1)$. If p>m the $\boldsymbol{c}$ vector will have zero entries for the excess terms. Using these results the variance of the second term of (A5) can be written as

$$Var[2\sqrt{T}(\hat{\boldsymbol{\beta}}_T - \boldsymbol{\beta})'(\boldsymbol{X}'_{aT}\boldsymbol{V}_T / T_a)] = 4\sigma^2 \boldsymbol{c}'(\boldsymbol{X}'\boldsymbol{X}/T)^{-1}\boldsymbol{c} . \tag{A8}$$

Monte Carlo simulation results show that when m>p, $\sigma^2 \boldsymbol{c}'(\boldsymbol{X}'\boldsymbol{X}/T)^{-1}\boldsymbol{c}$ contributes very little to the overall variance of (A5) and therefore can be dropped safely from the derivation.

Now using Hausman's approach (Hausman, 1978) the variance of $\sqrt{T}(\hat{\sigma}^2_{m,T} - 2\hat{\sigma}^2)$ can be obtained as the difference of the variances of $\sqrt{T}(\hat{\sigma}^2_{m,T} - 2\sigma^2)$ and $\sqrt{T}(\hat{\sigma}^2_T - \sigma^2)$. Thus we obtain:

$$E[\sqrt{T}(\hat{\sigma}^2_{m,T} - 2\hat{\sigma}^2_T)]^2 = 4\mu_4 - 4(\mu_4 - \sigma^4) = 4\sigma^4 \tag{A9}$$

This is the variance of the difference of two asymptotically normally distributed variables, hence we establish that

$$\sqrt{T}(\hat{\sigma}^2_{m,T} - 2\hat{\sigma}^2_T) \xrightarrow{d} N(0, 4\sigma^4) . \tag{A11}$$

In small samples from (A7):

$$\begin{aligned} &Var[\sqrt{T}(\hat{\sigma}^2_{m,T} - 2\hat{\sigma}^2_T)] = \\ &\qquad 4\sigma^4 + 4\mu_4(T/T_a - 1) - 2(\mu_4 - \sigma^4)mT/T_a^2 . \end{aligned} \tag{A12}$$

QED

References

Abeysinghe, T. (1998), "Forecasting Singapore's quarterly GDP with monthly external trade," *International Journal of Forecasting,* 14, 505-513.

Abeysinghe, T. (2000), "Modeling variables of different frequencies," *International Journal of Forecasting,* 16, 117-119.

Abeysinghe, T., and Choy, K.M. (2007), *The Singapore Economy: An Econometric Perspective*, Routledge, London.

Bhargava, A. (1986), "On the theory of testing for unit roots in observed time series," *Review of Economic Studies,* 53, 369-384.

Blough, S.R. (1992), "The relationship between power and level for Generic Unit Root tests in finite samples," *Journal of Applied Econometrics,* 7, 295-308.

Breidt, F.J., Davis, R.A., Nan-Jung Hsu, and Rosenblatt, M. (2006), "Pile-up probabilities for the Laplace likelihood estimator of a non-invertible first order moving average," *IMS Lecture Notes (Monograph Series)*, Institute of Mathematical Statistics.

Breitung J. (1994), "Some simple tests of the moving-average unit root hypothesis," *Journal of time series analysis,* 15, 351-370.

Cochran, J.H. (1991), "What s*hould* macroeconomists know about unit roots? Comments on 'Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots" by John Campbell, *NBER Macroeconomics Annual,* 6, 201-210

Choi, I. (1992), "Durbin-Hausman tests for a unit root," *Oxford Bulletin of Economics and Statistics,* 54, 289-304.

Choi I. (1994), "Residual-based tests for the null of stationary with application to US macroeconomic time series," *Econometric Theory,* 10, 720-746.

Davis, R.A., and Dunsmuir, W.T.M. (1996), "Maximum likelihood estimation for MA(1) processes with a root on or near the unit circle," *Econometric Theory,* 12, 1-29.

Dicky, D.A., and Fuller, W.A. (1979), "Distribution of Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association,* 74, 427-431.

DeJong, D.N., Nankervis, J.C., Savin, N.E., and Whiteman, C.H. (1992), "Integration versus trend stationary in time series," *Econometrica,* 60, 423-433.

Elliott, G., Rothenberg, T.J., Stock, J.H., 1996. Efficient Tests for an Autoregressive Unit Root. *Econometrica* 64, 813-836.

Gonzalo, J. and Lee, T. H. (1996), "Relative power of t type tests for stationary and unit root processes," *Journal of Time Series Analysis*, 17, 37-47.

Hamilton, J.D. (1994), *Time Series Analysis*, Princeton University Press.

Harvey, A. C. (1989), *Forecasting Structural Time Series Models and the Kalman Filter,* Cambridge: Cambridge University Press.

Hausman, J. (1978), "Specification tests in econometrics," *Econometrica*, 46, 1251-1271.

Hwang, J. and Schmidt, P. (1996), "Alternative methods of detrending and the power of unit root tests," *Journal of Econometrics*, 71, 227-248.

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P., and Shin, Y. (1992), "Testing the null hypothesis of stationarity against the alternative of a unit root, How sure are we that economic time series have a unit root?" *Journal of Econometrics,* 54, 159-178.

Lanne, M. and Lutkepohl, H. (2002), “Unit root tests for time series with level shifts: a comparison of different proposals,” *Economics Letters*, 75,109-114.

Lanne, M., Lutkepohl, H., and Saikonen, P. (2003). “Test procedures for unit Roots in time series with level shifts at unknown time,” *Oxford Bulletin of Economics and Statistics*, 65, 91-115.

Lee, J., and Schmidt, P. (1994), “Unit root tests based on instrumental variables estimation,” *International Economic Review,* 35, 449-462.

Leybourne S.J., and McCabe, B.P.M. (1994), “A consistent test for a unit root,” *Journal of Business and Economic Statistics,* 12, 157-166.

Leybourne, S.J. (1995), “Testing for unit roots using forward and reverse Dickey-Fuller regressions,” *Oxford Bulletin of Economics and Statistics,* 57, 559-571.

Lo A.W., MacKinlay, A.C. (1988), “Stock prices do not follow random walks: Evidence from a simple specification test,” *The Review of Financial Studies,* 1, 41-66.

Lo A.W., MacKinlay, A.C. (1989), “The size and power of the variance ratio test in finite samples,” *Journal of Econometrics* 40, 203-238.

Maddala G.S., and Kim, I. (1998), *Unit roots, cointegration and structural change,* Cambridge: Cambridge University Press.

Pantula S.G., and Hall, A. (1991), “Testing for unit roots in autoregressive moving average models,” *Journal of Econometrics,* 48, 325-353.

Pantula, S.G., Gonzalez-Farias, G., and Fuller, W.A. (1994), “A comparison of unit-root test criteria,” *Journal of Business and Economic Statistics,* 12, 449-459.

Park J. (1990), “Testing for unit roots and cointegration by variable addition,” *Advances in Econometrics,* 8, 107-133.

Park, H.J., and Fuller, W.A. (1995), “Alternative estimators and unit root tests for the autoregressive process,” *Journal of Time Series Analysis,* 16, 415-429.

Perron, P., and Ng, S. (1996), “Useful modifications to some unit root tests with dependent errors and their local asymptotic properties,” *Review of Economic Studies,* 63, 435-465.

Phillips, P.C.B., and Perron, P. (1988), “Testing for a unit root in time series regression. *Biometrica,* 75, 335-346.

Phillips, P.C.B. (1998), “New tools for understanding spurious regressions,” *Econometrica,* 66, 1299-1325.

Phillips, P.C.B. (2002), “New unit root asymptotics in the presence of deterministic trends,” *Journal of Econometrics,* 111, 323-353.

Rajaguru, G. (2004), *Effects of temporal aggregation and systematic sampling on model dynamics and causal inference*, PhD Thesis, Department of Economics, National University of Singapore.

Rajaguru. G., and Abeysinghe, T. (2002), “The distortionary effects of temporal aggregation on Granger causality,” Working Paper wp0204, Department of Economics, National University of Singapore.

Saikkonen P., and Luukkonen, R. (1993), “Testing for a moving average unit root in ARIMA models,” *Journal of American Statistical Association,* 88, 596-601.

Schmidt, P., and Phillips, P.C.B. (1992), “LM tests for a unit root in the presence of deterministic trends,” *Oxford Bulletin of Economics and Statistics,* 54, 257-287.

Sims, C., Stock, J., and Watson, M. (1990), "Inference in Linear Time Series Models with Some Unit Roots," *Econometrica* 58, 113-144.

Yap S.F., and Reinsel, G.C. (1995), “Results on estimation and testing for a unit root in the non-stationary ARMA model,” *Journal of Time Series Analysis,* 16, 339-353.

Table 1
Size of the z(MA) test for an MA unit root (2000 replications)

Known long-run relation or single series

	m=2			m=4			m=6			m=8		
	ϕ_1=0.5, θ=1											
T	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
100	0.010	0.028	0.058	0.022	0.048	0.087	0.027	0.050	0.088	0.030	0.052	0.085
200	0.008	0.030	0.055	0.020	0.042	0.076	0.028	0.056	0.086	0.031	0.059	0.091
300	0.008	0.028	0.068	0.017	0.050	0.096	0.032	0.076	0.121	0.038	0.066	0.103
500	0.002	0.030	0.068	0.013	0.054	0.095	0.022	0.060	0.096	0.027	0.061	0.104
	ϕ_1=0.9, θ=1											
200	0.010	0.035	0.076	0.011	0.039	0.079	0.013	0.042	0.083	0.015	0.045	0.077
300	0.007	0.034	0.079	0.008	0.040	0.089	0.011	0.052	0.097	0.009	0.038	0.075
500	0.005	0.035	0.080	0.009	0.037	0.084	0.011	0.044	0.080	0.008	0.038	0.087
	ϕ_1=0.95, θ=1											
300	0.009	0.044	0.090	0.005	0.040	0.082	0.008	0.038	0.087	0.005	0.047	0.101
500	0.006	0.038	0.078	0.006	0.048	0.089	0.008	0.040	0.095	0.011	0.049	0.097

Regression residuals

	ϕ_1=0.5, θ=1											
100	0.014	0.050	0.083	0.041	0.081	0.113	0.056	0.094	0.137	0.075	0.111	0.150
200	0.007	0.032	0.060	0.026	0.064	0.103	0.041	0.088	0.132	0.062	0.105	0.149
300	0.009	0.030	0.056	0.024	0.057	0.095	0.040	0.080	0.130	0.053	0.091	0.132
500	0.006	0.028	0.055	0.015	0.057	0.093	0.023	0.069	0.122	0.047	0.093	0.140
	ϕ_1=0.9, θ=1											
200	0.019	0.059	0.099	0.035	0.085	0.130	0.055	0.103	0.149	0.074	0.118	0.163
300	0.012	0.048	0.096	0.025	0.073	0.117	0.051	0.100	0.145	0.062	0.118	0.165
500	0.009	0.037	0.084	0.016	0.054	0.106	0.033	0.082	0.137	0.037	0.085	0.142
	ϕ_1=0.95, θ=1											
300	0.020	0.064	0.114	0.038	0.085	0.135	0.050	0.104	0.153	0.065	0.114	0.169
500	0.012	0.056	0.099	0.027	0.070	0.117	0.038	0.092	0.142	0.048	0.095	0.142

Table 2
Power of the z(MA) test for an MA unit root (2000 replications)

Known long-run relation or single series												
	m=2			m=4			m=6			m=8		
	ϕ_1=0.5, θ=0.8											
T	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
100	0.432	0.518	0.553	0.556	0.595	0.618	0.579	0.604	0.630	0.582	0.600	0.624
200	0.756	0.826	0.851	0.867	0.885	0.893	0.881	0.888	0.897	0.887	0.892	0.899
300	0.911	0.946	0.957	0.969	0.973	0.977	0.972	0.974	0.976	0.972	0.974	0.977
500	0.988	0.994	0.994	0.997	0.997	0.997	0.996	0.997	0.997	0.998	0.998	0.998
	ϕ_1=0.5, θ=0.9											
100	0.162	0.242	0.288	0.286	0.346	0.384	0.334	0.359	0.395	0.339	0.367	0.402
200	0.350	0.508	0.584	0.618	0.678	0.702	0.665	0.696	0.718	0.688	0.706	0.727
300	0.566	0.714	0.785	0.813	0.855	0.872	0.854	0.876	0.884	0.866	0.879	0.890
500	0.828	0.924	0.952	0.970	0.980	0.983	0.982	0.987	0.987	0.984	0.988	0.990

Regression residuals												
	ϕ_1=0.5, θ=0.8											
100	0.421	0.494	0.530	0.529	0.558	0.581	0.546	0.559	0.581	0.550	0.568	0.589
200	0.732	0.790	0.809	0.817	0.837	0.852	0.829	0.843	0.856	0.834	0.845	0.856
300	0.888	0.927	0.937	0.942	0.951	0.955	0.948	0.953	0.959	0.950	0.955	0.960
500	0.988	0.993	0.994	0.995	0.996	0.996	0.995	0.996	0.996	0.995	0.995	0.996
	ϕ_1=0.5, θ=0.9											
100	0.169	0.241	0.272	0.276	0.310	0.347	0.305	0.332	0.357	0.310	0.341	0.370
200	0.358	0.494	0.553	0.561	0.622	0.657	0.613	0.643	0.674	0.630	0.653	0.675
300	0.575	0.717	0.775	0.792	0.827	0.848	0.831	0.855	0.866	0.842	0.856	0.864
500	0.850	0.912	0.929	0.947	0.957	0.959	0.955	0.958	0.963	0.959	0.961	0.963

Table 3

Cointegration test for selected equations from the ESU01 model of the Singapore economy (Abeysinghe and Choy, 2007)

Equation in the model	T	$\hat{\rho}$	ARMA(1,1)	ADF	z(MA)
(i) Regression Residuals					
Consumption	104	0.67	0.70, 0.99	-4.48*	-0.77
Exports (non oil domestic)	96	0.54	0.56, 0.99	-5.27*	0.63
Employment	96	0.86	0.88, 0.99	-2.41	0.51
Wages	96	0.89	0.87, 0.99	-2.94	0.49
CPI	96	0.93	0.95, 0.99	-2.01	0.05
(ii) Known coefficients (log form)					
Oil import price in S$	104	0.89	0.85, 0.99	-2.43	-1.49
Oil export price in S$	104	0.76	0.79, 0.99	-3.68*	0.42
RUBC	96	0.91	0.93, 0.99	-2.17	0.25
RER	336	0.98	0.00, -0.25	-2.39	-9.03*

RUBC=relative unit business cost. RER=real exchange rate (S$/US$, CPI based). Oil price relationships are: oil price in Singapore dollars equals oil price in US$ times the Sin/US exchange rate. The first eight series are quarterly from 1978Q1 or 1980Q1 to 2003Q4. RER is monthly over 1975-2003. The null for z(MA) is stationarity (MA unit root) and that for ADF is non-stationarity (AR unit root). * significant at the 5% level (left-tail test).

Table 4
Cointegration test on APC

Country	Sample period (quarterly)	T	AR Lags	AR Coefficients	$\hat{\rho}$	ARMA(p,1)	ADF	Johansen VAR(4)	KPSS	z(MA) m=4
Australia	1960-2007	192	1	0.92	0.92	0.94, 0.99	-2.71	yes	0.21	0.39
Austria	1965-2007	172	1,2,3	0.55,0.18,0.18	0.91	0.56, 0.19, 0.20, 0.99	-2.33	no	0.14	0.34
Belgium	1980-2007	111	1	0.98	0.98	0.00, 0.12	-0.77	no	1.09*	-5.37*
Canada	1957-2007	204	1	0.97	0.97	0.97, 0.99	-1.97	no	0.73*	-0.30
Denmark	1978-2007	124	1,4	0.75, 0.21	0.96	0.75, 0.17, 0.99	-1.71	yes	1.02*	-0.57
Finland	1970-2007	152	1,4	0.71, 0.21	0.92	0.72, 0.19, 0.99	-2.21	no	1.00*	-1.41
France	1978-2007	120	1	0.94	0.94	0.97, 0.99	-2.1	yes	0.49*	0.48
Germany	1961-2007	188	1,3	0.71, 0.23	0.94	0.72, 0.23, 0.99	-1.99	yes	0.87*	-1.13
Italy	1970-2007	151	1,4	0.70, 0.12	0.82	0.66, 0.99	-2.98*	yes	0.95*	-0.5
Japan	1965-2007	172	1	0.94	0.94	0.95, 0.99	-2.45	no	0.18	-1.29
Korea, South	1965-2007	172	1	0.97	0.97	0.00, 0.20	-2.45	no	1.38*	-6.51*
Mexico	1981-2007	108	1	0.88	0.88	0.88, 0.99	-2.62	no	0.40	-0.14
Netherlands	1977-2007	124	1,2	0.51, 0.46	0.97	0.35, 0.25	-0.78	no	1.03*	-5.75*
New Zealand	1987-2007	82	1	0.72	0.72	0.75, 0.99	-3.69*	yes	0.09	0.58
Norway	1961-2007	188	1,2	0.75, 0.23	0.98	0.00, 0.25	-0.83	no	1.31*	-6.52*
Spain	1970-2007	152	1,4	0.79, 0.20	0.99	0.00, 0.24	-0.06	no	1.41*	-6.66*
Sweden	1980-2007	112	1,2,4	0.66, 0.39, -0.17	0.88	0.61, 0.41, -0.17, 0.99	-2.21	no	0.43	-0.81
Switzerland	1970-2007	152	1,2,3	0.60, 0.51, -0.18	0.94	0.59, 0.53, -0.16, 0.99	-1.81	no	0.16	-1.11
Turkey	1987-2007	83	1	0.62	0.62	0.57, 0.99	-4.23*	yes	0.49*	0.36
UK	1957-2007	204	1,3	0.73, 0.24	0.97	0.73, 0.25, 0.99	-1.55	yes	0.44	1.21
US	1957-2007	204	1,2	0.83, 0.17	1.00	0.00, 0.17	-0.18	no	1.62*	-7.29*

Note that some data series end in Q2 or Q3 in 2007. Tests are based on log(APC) = log(C/Y), where C is total consumption expenditure and Y is GDP, both in nominal terms and seasonally adjusted. For the Johansen test "yes" means acceptance of cointegration between log(C) and log(Y) with the cointegrating vector (1, -1). For the KPSS test the default settings in Eviews were used. * Significant at the 5% level.

Table A.1

Power of unit root tests at the 5% level and T=100. Reference model: $y_t = \alpha + \beta t + \rho y_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1},\ \varepsilon_t \sim iid(0, \sigma^2)$

(When T=100 is not available 200 is used and marked with an asterisk against author's name)

(a) Non-stationary null (ρ = 1)

Name of Authors	**Year**	**Model Type**	**Test Type**	**ρ = 0.80**	**0.85**	**0.90**	**0.95**	**0.975**	**Remarks**
Dicky & Fuller	1979	θ=0, β=0	$\hat{\rho}$	0.86		0.30	0.10		
		θ=0, β=0	t	0.73		0.18	0.06		DF test, AR(1) process
Bhargava	1986	θ=0, β=0	DW	0.73	0.49	0.25	0.10		Also Sargan & Bhargava 1983
Phillips & Perron	1988	θ=0, β=0	t		0.47				ADF, Said & Dicky 1984
		θ=0.8, β=0	t		0.30				ADF
		θ=0, β=0	Z(t)		0.69				PP
		θ=0.8, β=0	Z(t)		0.35				PP
Pantula & Hall*	1991	θ=0, β=0	IV					0.09-0.33	Range of IV estimates. In general power > 0.05
		θ=0.8, β=0	IV					0.01-0.35	
DeJong et al.	1992	θ=0, $\beta\neq$0	$\tau(\rho)$	0.75	0.49	0.24	0.10		For starting value 0. Power drops slightly as starting value increases.
			$F(\beta,\rho)$	0.65	0.39	0.19	0.08		
Blough	1992	θ=0, β=0	ADF, IV						Graphical presentation. Power drops to 5% for ρ>0.5.
Schmidt & Phillips	1992	θ=0, $\beta\neq$0	LM			0.27	0.108		Reported is highest power under different specifications
Choi	1992	θ=0, $\beta\neq$0	DH	0.97	0.84	0.54	0.24		Durbin-Hausman
Lee & Schimidt	1994	θ=0.8, β=0	IV				0.22		Compares Hall-IV with SP-IV
Pantula et al.	1994	θ=0, β=0	WS			0.602	0.261		Compares OLS, MLE as well.
Yap & Reinsel *	1995	θ=0, β=0	LR	1.00		0.82	0.33		
		θ=0.8, β=0	LR	-		0.74	0.56		
Leybourne	1995	θ=0, β=0	DFmax	0.88		0.34			

Table A.1 continued

Name of Authors	**Year**	**Model Type**	**Test Type**	**ρ = 0.80**	**0.85**	**0.90**	**0.95**	**0.975**	**Remarks**
Park & Fuller	1995	$\theta=0, \beta=0$							Graphical. For intercept model: WS>SS>OLS. For interceptless model: OLS>SS>WS. (SS=simple symmetric, WS=weighted symmentric)
Perron & Ng *	1996	$\theta=0.8, \beta=0$	MZ(ρ)			0.75	0.42		
			MSB			0.79	0.46		
			MZ(t)			0.63	0.30		Modified PP
Elliot et al.	1996	$\theta=0.8, \beta=0$	t	0.51		0.30	0.15		Power at ρ=0.95 not very different across models
Hwang & Schmidt	1996	$\theta=0, \beta\neq0$	GLS	0.28	0.18				Power is roughly similar across different tests reported

Non-stationary null: Structural breaks

Lanne & Lutkepohl	2002	Perron				0.21			Known break, level shift. Power is very similar for slope change. See the article for model specification.
		Perron & Vogelsang				0.14			
		Amsler & Lee				0.12			
		Schmidt & Phillips				0.09			
		Lanne et al				0.23			
Lanne et al.	2003	Test 1, drift				0.28			Unknown break, level shift. Power is very similar for slope change. See the article for model specification.
		Test 2, drift				0.20			
		Test 3, trend				0.23			
		Test 3, trend				0.18			

Table A.1 continued

(b) Stationary null ($\rho = 1$, $\theta = 1$)

Name of Authors	Year	Model Type	Test Type	θ* = 0.80	0.85	0.90	0.95	0.975	Remarks
Park	1990		J1 test						No simulation results
Kwiatkowski et al.	1992	β=0	η(μ) l0			0.59		0.17	
		β=0	η(μ) l4			0.51		0.15	
		β=0	η(μ) l12			0.38		0.10	
		$\beta \neq 0$	η(τ) l0			0.35		0.05	KPSS test. The test basically
		$\beta \neq 0$	η(τ) l4			0.28		0.05	involves testing σ_η^2 = 0 in
		$\beta \neq 0$	η(τ) l12			0.17		0.04	model (1) in Section 3.
Saikkonen & Luukkonen	1993	β=0	R2	0.81	0.71	0.56	0.32		Authors also consider non-white errors.
Breitung	1994	β=0	Spectral	0.04		0.03	0.03		
			Var diff	0.87		0.43	0.16		
			Tanaka	0.86		0.62	0.32		
Leybourne and McCabe	1994	Extended KPSS	s(α) p=1			0.61		0.17	Show that KPSS is subject to
			s(α) p=2			0.59		0.17	severe size distortions in
			s(α) p=3			0.56		0.16	general ARIMA cases.
Choi	1994	β=0	w1 l=2	0.47					
			w1 l=3	0.38					
			w1 l=4	0.27					
			w1 l=5	0.06					Power remains low for other
		$\beta \neq 0$	w2 l=1	0.08					lags on w2 test

Note: * θ values given here are implicit of many of these models.