



SCAPE

Singapore Centre for Applied and Policy Economics

**Time-Varying Currency Betas:
Evidence from Developed and Emerging
Markets**

by

Prabhath Jayasinghe and Albert K. Tsui

**Department of Economics
SCAPE Working Paper Series
Paper No. 2009/03 – Oct 2009**

<http://www.fas.nus.edu.sg/ecs/pub/wp-scape/0903.pdf>

**Time-Varying Currency Betas:
Evidence from Developed and Emerging Markets**

Prabhath Jayasinghe* and Albert K. Tsui**

*Department of Business Economics
University of Colombo, Sri Lanka

**Department of Economics
National University of Singapore
10 Kent Ridge, Singapore 119260

October 2009

Abstract

This paper examines the conditional time-varying currency betas from five developed markets and four emerging markets. A tri-variate BEKK-GARCH-in-mean model is used to estimate the time-varying conditional variance and covariance of returns of stock index, the world market portfolio and changes in bilateral exchange rate between the US dollar and the local currency of each country. It is found that currency betas are more volatile than those of the world market betas. Currency betas in emerging markets are more volatile than those in developed markets. Moreover, we find evidence of long-memory in currency betas. The usefulness of time-varying currency betas are illustrated by two applications.

Key Words: time-varying currency betas; multivariate GARCH-M models; international CAPM; fractionally integrated processes; stochastic dominance

JEL Classification: C22; F31; F37; G12; G15

Acknowledgements: The authors would like to thank SCAPE at the Department of Economics, NUS for providing research support.

1. Introduction

In the past decade, studies of exchange rate exposure have mainly focused on three approaches. The first approach uses conventional methods such as subsampling, dummy variables, and overlapping moving window regression to capture exchange rate exposure. See Williamson (2001), Entoff and Jamin (2003), Bodner and Wong (2003), and Dominguez and Tesar (2006), among others. The second approach uses pre-specified determinants of exposure coefficients to analyze the time-variation of exchange rate exposure. For example, Allayannis (1997) suggests that currency beta is determined by export and import shares, and finds support for time-variation of exposure in some 4-digit level SIC industries. Similar approaches are used by Chiao and Hung (2000), Allayannis and Ihrig (2001), and Bodner et al. (2002). But Bodner et al. did not find evidence of time-varying exposure. The third approach employs time-varying second moments to derive time-varying exchange rate exposure. For example, Hunter (2005) analyzes the time-varying exchange rate exposure of small and large firms using size-based portfolios of the Fama-French-type. Lim (2005) derives both market and currency betas at country level, with allowance for non-orthogonality between risk factors.

Apparently the third approach is more appealing as the well-documented bivariate GARCH-type models are often employed to estimate the time-varying exchange rates conditional on available information. Among others, they include: (a) VECH models (For example, Choudhry (2001, 2002), Giannopoulos (1995) and McClain et al. (1996)); (b) BEKK models (For example, Choudhry (2005), Gonzalez-Rivera (1996)); and (c) Constant Conditional Correlation GARCH (CCC-GARCH) models (For example, Brooks et al. (2000 and 2002)), respectively. However, the VECH model is less popular because of the difficulty in maintaining positive definiteness of the variance and covariance matrix and other computational hindrance on convergence during estimation. The CCC-GARCH model is too restrictive as the computed covariance between returns and exchange rate changes could be either negative or positive in all periods, depending on the sign of the constant conditional correlation coefficient. In reality, exchange rate changes may affect returns on stock index either positively and/or negatively in different time periods. Hence, it is inappropriate to assume time-constancy in the conditional correlation coefficient.

In this paper, we adopt the general framework of conditional ICAPM proposed by Adler and Dumas (1983) and De Santis and Gerard (1998) to estimate the time varying currency betas and the time-varying market betas for nine developed and emerging countries. A trivariate BEKK-GARCH-type model is used to estimate the conditional variance and covariance of return variables using the daily data. The main advantage of BEKK parameterization is that it guarantees the variance and covariance matrix to be positive definiteness during estimation. The often alleged difficulty of interpreting parameters in BEKK models is not an issue.

We compute the time-varying currency betas and market betas using estimates of the conditional variance and covariance of returns from country stock index, world market portfolio and changes in exchange rate of the trading country. To the best of our knowledge, this is the first study that estimates such betas from a BEKK-GARCH-type specification based on daily returns. It is found that currency betas are generally more volatile than that of the world market betas. In addition, currency betas in emerging markets, such as Korea, Taiwan and Thailand are more volatile than those in developed markets. We also find some evidence of long-memory in the estimated currency betas. Our findings have important implications for investment and hedging strategies.

The rest of this paper is organized as follows. The conditional version of international CAPM is outlined in Section 2. Section 3 highlights BEKK-GARCH-in mean-type models to estimate currency betas and market betas from the conditional variance and covariance of return variables. Section 4 presents the sample data and preliminary results. Section 5 reports the main empirical findings, including evidence of mean-reverting currency betas. A comparison of currency betas among countries by stochastic dominance and patterns of the time-varying currency premiums is explored. Some concluding remarks are given in Section 6.

2. The ICAPM Framework

The standard capital asset pricing models (CAPM) analyses how investors are compensated for investing in risky assets in their country of residence. Adler and

Dumas (1983) and others¹ extend the CAPM to international settings with deviation from the purchasing power parity. In the extended model (ICAPM), a representative investor is concerned about variance of the return on the world market portfolio, and about the covariance of the invested asset return with each of the exchange rates of various countries. Some salient features of ICAPM conditional on the available information are highlighted as follows.

In a world of $(L + I)$ countries, the expected excess returns on equity/asset i can be expressed as:

$$E_{t-1}(r_{i,t}) = \lambda_{m,t-1} Cov_{t-1}(r_{i,t}, r_{m,t}) + \sum_{l=1}^L \lambda_{\pi,l,t-1} Cov_{t-1}(r_{i,t}, \pi_{l,t}) \quad (1)$$

where $E_{t-1}(\cdot)$ and $Cov_{t-1}(\cdot)$ denote the expectation and covariance, conditional on the available information set I_{t-1} at time $(t - 1)$. $r_{i,t}$ denotes the excess return on asset i denominated in any numeraire currency; $r_{m,t}$ denotes the excess return on the world market portfolio denominated in the base currency; $\pi_{l,t}$ denotes the inflation rate in country l which includes the domestic inflation and changes in exchange rate; $\lambda_{m,t-1}$ is the price of world market risk. The covariance between $r_{i,t}$ and $r_{m,t}$ measures the world market risk. In addition, $\lambda_{\pi,l,t-1}$ denotes the price of asset risk in country l and the covariance between $r_{i,t}$ and $\pi_{l,t}$ is used to gauge the inflation risk and the risk of exchange rate changes.

For practical applications, we consider two simplifications to the Adler and Dumas-type model. First, following Dumas and Solnik (1995) and De Santis and Gerard (1997), we assume non-stochastic inflation² so that the PPP deviations are mostly reflected in the exchange rate changes. This could be a plausible simplification since we use daily data so that changes in price levels are negligible as compared to volatilities of exchange rate changes (Cappiello et al., 2003). Hence, $\pi_{l,t}$

¹ Their model was initially known as international asset pricing model. Dumas and Solnik (1995) and De Santis and Gerard (1998) test the validity of conditional ICAPM

² When inflation in a country is treated as stochastic, the expected returns are dependent on three premiums, namely, market, currency and inflation. See Moerman and van Dijk (2006) for details. However, we do not consider the inflation factor here.

is effectively reduced to currency risk ($\pi_{x,t}$). Accordingly, $\lambda_{\pi,l,t-1}$ is reduced to $\lambda_{x,l,t-1}$, which is the price of currency risk associated with country l .

Second, for parsimonious purposes, we assume that returns on a country stock index is a reasonable proxy for returns on assets or portfolios in that country, and that investors in each country will invest in assets in the United States. With this assumption, the second term on the right hand side equation (1) is reduced to only one bilateral exchange rate between the US dollar and currency of the trading country. This may lead to incomplete specification of the Adler and Dumas model since other currency premiums are still in the expected return equation. However, we can ignore this as the main objective of this paper is to investigate properties of time-varying currency betas, but not to test the validity of ICAPM³. As returns on assets in each country is gauged by changes in the exchange rate with the US dollar, the proposed parsimonious structure is able to serve as a common yard stick to compare exposure to currency risk in each country. The conditional ICAPM relationship in (1) can thus be rewritten as sum of the product of time varying betas and the respective expected returns of risk factors.

$$E_{t-1}(r_{i,t}) = \beta_{m,t-1} E_{t-1}(r_{m,t}) + \beta_{x,t-1} E_{t-1}(r_{x,t}). \quad (2)$$

$$\text{where } \beta_{m,t-1} = \frac{Cov_{t-1}(r_{i,t}, r_{m,t})}{Var_{t-1}(r_{m,t})} \text{ and } \beta_{x,t-1} = \frac{Cov_{t-1}(r_{i,t}, r_{x,t})}{Var_{t-1}(r_{x,t})}.$$

The world market beta ($\beta_{m,t-1}$) measures the asset's exposure to world market risk while the currency beta ($\beta_{x,t-1}$) measures its exposure to currency risk.

Following Lim (2005), we allow for possible non-orthogonality relationship between the world market returns and exchange rate changes. The expected returns for stock index, world market portfolio and changes in exchange rates can be further expressed as below.

³ See De Santis and Gerard (1998) and Capiello et al (2003) for testing the validity of ICAPM by a set of exchange rates.

$$E_{t-1}(r_{i,t}) = \lambda_{X,t-1} Cov_{t-1}(r_{i,t}, r_{x,t}) + \lambda_{M,t-1} Cov_{t-1}(r_{i,t}, r_{m,t}) \quad (3)$$

$$E_{t-1}(r_{x,t}) = \lambda_{X,t-1} Var_{t-1}(r_{x,t}) + \lambda_{M,t-1} Cov_{t-1}(r_{x,t}, r_{m,t}) \quad (4)$$

$$E_{t-1}(r_{m,t}) = \lambda_{X,t-1} Cov_{t-1}(r_{m,t}, r_{x,t}) + \lambda_{M,t-1} Var_{t-1}(r_{m,t}) \quad (5)$$

where $r_{i,t}$ is return on country i 's stock index at time t ; $r_{m,t}$ is return on the world market portfolio at time t ; $r_{x,t}$ is the change in bilateral nominal exchange rate between the US dollar and currency of country i at time t ; $\lambda_{M,t-1}$ is market price of risk; and $\lambda_{X,t-1}$ is the currency price of risk. Owing to non-orthogonality between the world market returns and exchange rate changes, a non-zero $Cov_{t-1}(r_{m,t}, r_{x,t})$ term is included in the mean equations in (4) and (5).

Moreover, as specified in equation (2), the expected return on asset/portfolio at time t is proportional to the world market returns and changes in exchange rates, conditional on the information available at time $(t - 1)$. Intuitively, the proportionality factors (i.e. the world market and exchange rate exposure) should be time-varying because investors are sensitive to the new information periodically available and are able to adjust their investment strategies accordingly⁴.

3. Empirical Methodology

The currency betas and market betas are to be obtained from estimates of the conditional second moments of various returns. We adopt a trivariate BEKK (k)-GARCH (p, q)-M (in mean) model to achieve such purposes. The mode is specified as follows:

$$r_{j,t} = \lambda_{0,j} + \lambda_X h_{X,t} + \lambda_M h_{M,t} + \theta_j \varepsilon_{j,t-1} + \varepsilon_{j,t} \quad j = i, m, x \quad (6)$$

$$\varepsilon_t = z_t H_t^{1/2} \quad (7)$$

$$\varepsilon_t | I_{t-1} = (\varepsilon_{i,t}, \varepsilon_{m,t}, \varepsilon_{x,t})' | I_{t-1} \sim N(0, H_t)$$

⁴ See Harvey (1991) for details.

$$H_t = C'C + \sum_{k=1}^K \sum_{l=1}^q A'_{kl} \varepsilon_{t-1} \varepsilon'_{t-1} A_{kl} + \sum_{k=1}^K \sum_{n=1}^p B'_{kn} H_{t-1} B_{kn} \quad (8)$$

$$H_t = \begin{bmatrix} H_t^{qq} & H_t^{qk} \\ H_t^{kq} & H_t^{kk} \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} \beta_{x,t-1} \\ \beta_{m,t-1} \end{bmatrix} = [H^{kk}]^{-1} H^{kq} \quad (10)$$

Here $r_{j,t}$ is the 3×1 vector consisting of returns from time $(t-1)$ to time t on country index ($r_{i,t}$), return on the world market portfolio ($r_{m,t}$) and changes in bilateral nominal exchange rate between the US dollar and currency of the trading country i ($r_{x,t}$)⁵. Parameters λ_M and λ_X denote constant market price of risk and currency price of risk⁶. Also, $h_{x,t}$ and $h_{M,t}$ are both 3×1 column vectors containing elements from the second and third columns of H_t ⁷. Note that $h_{x,t}$ represents the conditional covariance of changes in exchange rate with returns of the world market portfolio, with itself and with returns on country index, respectively. Similarly, $h_{M,t}$ represents the conditional covariance of returns of world market portfolio with returns on country index, with changes in exchange rate, and with itself, respectively.

Consistent with Hamao et al. (1990), an intercept and a MA (1) term are added to each of the mean equations to capture possible market inefficiencies associated with the non-synchronous closure of various markets. However the beta version of the ICAPM in the mean equations (3) to (5) is not followed strictly for two reasons. First, including contemporaneous dependent and independent variables would complicate estimation and create identification problems. Second, the current

⁵ Exchange rate is expressed as the US dollar price of foreign currency. An increase implies a depreciation of US dollar relative to the relevant currency.

⁶ These constant prices can be regarded as data generating processes formulated under the broad ICAPM framework, with GARCH-type structure in the variance equations. The idea is to capture the time-varying conditional second moments of returns of country index, the world market portfolio and changes in exchange rate of the bilateral trade between the US and a trading country.

⁷ Since we allow for non-orthogonality between market returns and exchange rate changes, a non-zero conditional covariance term between returns of x and m ($h_{xm,t}$) is included in the mean equations.

approach is more parsimonious than the beta version as fewer parameters are involved.

As regards the disturbances, z_t denote the standardized residuals assumed to be identically and independently distributed with mean 0 and variance 1. And $\varepsilon_t | I_{t-1}$ denote the 3×1 vector of random errors at time t given all available information at time $(t-1)$, which is assumed to follow a normal distribution with mean 0 and variance H_t , whereas H_t is the corresponding 3×3 conditional variance and covariance matrix.

Turning to the right hand side of equation (8), C denotes an upper triangular 3×3 matrix that contains constant parameters in the conditional variance and covariance matrix. Note that both A_{kl} and B_{kn} are 3×3 parameter matrices. We have restricted A_{kl} and B_{kn} to be diagonal for two reasons. First, the full BEKK model contains too many parameters and is less parsimonious but more computationally demanding in estimation⁸. Second, as will be discussed in Section 5, the diagnostic tests indicate that the diagonal version of BEKK model is sufficiently adequate to capture the non-linearity in stock returns and exchange rate changes. For parsimony, we have set $K = 1$ in the trivariate BEKK-GARCH-M model. Moreover, as indicated by the Ljung-Box statistics on standardized residuals, the optimal lag orders for the GARCH and ARCH terms are $p = 1$ and $q = 2$, respectively. Hence, for $K = 1$, the variance and covariance matrix of the proposed trivariate BEKK (I)-GARCH ($I, 2$)-M model can be simplified as follows:

$$H_t = \begin{bmatrix} h_{i,t} & h_{ix,t} & h_{im,t} \\ h_{xi,t} & h_{x,t} & h_{xm,t} \\ h_{mi,t} & h_{mx,t} & h_{m,t} \end{bmatrix} = \begin{bmatrix} c_i & 0 & 0 \\ c_{xi} & c_x & 0 \\ c_{mi} & c_{mx} & c_m \end{bmatrix} \begin{bmatrix} c_i & c_{ix} & c_{im} \\ 0 & c_x & c_{xm} \\ 0 & 0 & c_m \end{bmatrix} \\ + \begin{bmatrix} b_i & 0 & 0 \\ 0 & b_x & 0 \\ 0 & 0 & b_m \end{bmatrix} \begin{bmatrix} h_{i,t-1} & h_{ix,t-1} & h_{im,t-1} \\ h_{xi,t-1} & h_{x,t-1} & h_{xm,t-1} \\ h_{mi,t-1} & h_{mx,t-1} & h_{m,t-1} \end{bmatrix} \begin{bmatrix} b_i & 0 & 0 \\ 0 & b_x & 0 \\ 0 & 0 & b_m \end{bmatrix}$$

⁸ In our initial round of regressions, we found that the full BEKK model did not converge in some cases.

$$\begin{aligned}
& + \begin{bmatrix} a_i & 0 & 0 \\ 0 & a_x & 0 \\ 0 & 0 & a_m \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t-1} \\ \varepsilon_{x,t-1} \\ \varepsilon_{m,t-1} \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t-1} & \varepsilon_{x,t-1} & \varepsilon_{m,t-1} \end{bmatrix} \begin{bmatrix} a_i & 0 & 0 \\ 0 & a_x & 0 \\ 0 & 0 & a_m \end{bmatrix} \\
& + \begin{bmatrix} d_i & 0 & 0 \\ 0 & d_x & 0 \\ 0 & 0 & d_m \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t-2} \\ \varepsilon_{x,t-2} \\ \varepsilon_{m,t-2} \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t-2} & \varepsilon_{x,t-2} & \varepsilon_{m,t-2} \end{bmatrix} \begin{bmatrix} d_i & 0 & 0 \\ 0 & d_x & 0 \\ 0 & 0 & d_m \end{bmatrix} \quad (11)
\end{aligned}$$

And the corresponding elements of H_t are:

$$\begin{aligned}
h_{i,t} &= c_i^2 + b_i^2 h_{i,t-1} + a_i^2 \varepsilon_{i,t-1}^2 + d_i^2 \varepsilon_{i,t-2}^2 \\
h_{x,t} &= (c_{ix}^2 + c_x^2) + b_x^2 h_{x,t-1} + a_x^2 \varepsilon_{x,t-1}^2 + d_x^2 \varepsilon_{x,t-2}^2 \\
h_{m,t} &= (c_{im}^2 + c_{xm}^2 + c_m^2) + b_m^2 h_{m,t-1} + a_m^2 \varepsilon_{m,t-1}^2 + d_m^2 \varepsilon_{m,t-2}^2 \\
h_{ix,t} &= c_i c_{ix} + b_i b_x h_{ix,t-1} + a_i a_x \varepsilon_{i,t-1} \varepsilon_{x,t-1} + d_i d_x \varepsilon_{i,t-2} \varepsilon_{x,t-2} \\
h_{im,t} &= c_i c_{im} + b_i b_m h_{im,t-1} + a_i a_m \varepsilon_{i,t-1} \varepsilon_{m,t-1} + d_i d_m \varepsilon_{i,t-2} \varepsilon_{m,t-2} \\
h_{xm,t} &= (c_{ix} c_{im} + c_x c_{xm}) + b_x b_m h_{xm,t-1} + a_x a_m \varepsilon_{x,t-1} \varepsilon_{m,t-1} + d_x d_m \varepsilon_{x,t-2} \varepsilon_{m,t-2} \quad (12)
\end{aligned}$$

Note that an ARCH (2) term is included in the conditional variance equation whenever appropriate. But in most of the cases, it suffices to have $q = 1$ and $d_j = 0$ for $j = i, m, x$.

As specified in (10), the time-varying market betas and exchange rate exposure can be estimated using sub-matrices $H_t^{kk} = \begin{bmatrix} h_{x,t} & h_{xm,t} \\ h_{xm,t} & h_{m,t} \end{bmatrix}$ and

$H_t^{kq} = \begin{bmatrix} h_{ix,t} \\ h_{im,t} \end{bmatrix}$. When market returns and exchange rate changes are not orthogonal,

the market betas and currency betas are expressed as below:

$$\beta_{m,t-1} = \frac{h_{x,t} h_{im,t} - h_{xm,t} h_{ix,t}}{h_{x,t} h_{m,t} - [h_{xm,t}]^2} \quad (13)$$

$$\beta_{x,t-1} = \frac{h_{m,t} h_{ix,t} - h_{xm,t} h_{im,t}}{h_{x,t} h_{m,t} - [h_{xm,t}]^2}. \quad (14)$$

If they are orthogonal, H_t^{kk} become a diagonal matrix, and the market beta and currency beta are reduced to:

$$\begin{aligned}\beta_{m,t-1} &= \frac{h_{im,t}}{h_{m,t}} \\ \beta_{x,t-1} &= \frac{h_{ix,t}}{h_{x,t}}\end{aligned}\tag{15}$$

We note that (13) and (14) provide more precise estimates of betas than those models using pre-specified determinants (e.g. see Allayanis (1997); Allayannis and Ihrig, (2001)). In addition, our model is more adequate than those employing less appropriate mean structures to obtain the time-varying betas. For instances, Brooks et al. (2000) and (2002) take zero as the expected value of returns ($r_{i,t} = \varepsilon_{i,t}$). McClain et al. (1996) assume constant expected returns ($r_{i,t} = c + \varepsilon_{i,t}$). And Chaudhry (2002 and 2005) uses the MA(1) process⁹.

Assuming that the standardized residuals of the proposed trivariate BEKK (1)-GARCH (1, 2)-M model are conditionally normally distributed, the conditional log-likelihood of residual vector ε_t at time t can be written as follows:

$$\ell(\theta)_t = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln|H_t| - \frac{1}{2}\varepsilon_t'H_t^{-1}\varepsilon_t\tag{16}$$

The log-likelihood function of the sample becomes $L(\theta) = \sum_{t=1}^T \ell(\theta)_t$, with T denoting the number of observations. The parameter vector θ can be estimated by maximizing L with respect to θ . To accommodate non-normal country stock returns and the exchange rate changes, we estimate the parameters using the quasi-maximum likelihood (QML) estimation method as proposed by Bollerslev and Wooldridge (1992). Under certain regularity conditions, the QML estimates are consistent and asymptotically normal. Hence, statistical inference can be made using the robust

⁹ This comment does not apply to studies like Giannopoulos (1995), Gonzales-Rivera (1996) and Choudry (2005).

standard errors. The required computer programs are coded in GAUSS and the BHHH optimization algorithm is employed to compute QML estimates.

4. Data and Preliminary Results

Our sample dataset is drawn from five developed markets (the United States, United Kingdom, Canada, Japan, and Australia) and four emerging markets (Korea, Singapore, Taiwan and Thailand). For each country, we use a set of 1824 daily closing prices from 5 January 1999 to 30 December 2005.¹⁰ The series are culled from Morgan Stanley Capital International (MSCI) and DataStream. The country level portfolios are proxied by the MSCI country indexes measured in local currency. The world market portfolio is represented by the MSCI world market index, which is a value-weighted index free from exchange rate fluctuations¹¹ (see Giannopoulos (1995) and MSCI (1998)). Bilateral exchange rates for the non-US countries are represented by MSCI rates. These rates are then converted to the dollar price. A trade-weighted exchange rate compiled by the Bank of England is used to measure exposure of the US assets.

The daily returns (in percentage) of country stock index (i), world market index (m), and the bilateral exchange rate (x) are computed, on a continuously compounding basis, as follows:

$$r_{j,t} = \ln\left(\frac{R_{j,t}}{R_{j,t-1}}\right) * 100 \quad j = i, m, x \quad (17)$$

where $R_{j,t}$ and $R_{j,t-1}$ are the closing prices for trading days and $(t - 1)$ and t , respectively.

Table 1 displays the summary statistics for daily returns of country indexes, the world market index and the exchange rate changes. As can be observed in Panel A, all stock returns indicate excess kurtosis, ranging from the lowest 1.773 (Japan) to

¹⁰ In order to avoid the impact of unusual currency fluctuation, we have excluded the Asian financial crisis period from our sample data.

¹¹ Our approach is consistent with Giannopoulos (1995) to the effect that the market risk and currency risk should not be aggregated, and conversion of country index returns into a common currency will have an adverse impact on their volatility.

the highest 7.06 (Thailand). The Jarque-Bera test statistics for normality is extremely high in all cases, thereby exceeding the 1% level of significance. In addition, the exchange rate changes are less skewed than stock returns and have smaller kurtosis. Except for Taiwan and Thailand, the excess kurtosis of exchange rate changes is lower than that of the stock returns of other countries. The Jarque-Bera test statistics are all significant at the 1% level, attesting to non-normal distribution of the exchange rate changes. Such empirical evidence of non-normality in stock returns and changes in exchange rates provides some justification for estimating parameters by the quasi-maximum likelihood method.

We now present the preliminary tests performed on the return series. As indicated by the augmented Dicky-Fuller test statistics in Tables 2 and 3, returns of stock indexes, the world market index and exchange rate changes are stationary at the 5% level. The Ljung-Box statistics for returns at 20 lags ($Q(20)$) are statistically significant, indicating that stock returns in Canada, Korea, Taiwan, Thailand, UK and the world market are not free from linear dependencies. Exchange rate changes in the remaining seven countries do not indicate significant linear dependencies, except for Taiwan and Thailand. Moreover, the Ljung-Box test for squared returns at 20 lags ($Q^2(20)$) are significant at the 5% level for all returns and exchange rate series, thereby indicating some degree of non-linear dependency. Our findings provide some empirical support for employing GARCH-type models to capture the time-varying conditional variance and covariance.

A battery of tests is conducted for constancy in exchange rate exposure based on the OLS estimates of the conventional augmented market model¹². The first test is the cumulative sum of squared recursive residuals (CSSRR) as suggested by Brown et al. (1975). The CSSRR cross the critical value boundaries in all cases at the 5% level of significance, thereby providing evidence of parameter instabilities¹³. To conserve

¹² This refers to the constant parameter version of the regression equation in (2)

¹³ As two slope coefficients are involved in the regression, one may argue that this instability may stem from the market beta, but not from the exchange rate exposure beta. To address this issue, we have regressed country returns on exchange rate changes only and obtained the cumulative sum of squares of recursive residuals. The diagrams are very similar to those displayed in Figure 1. As such, it is more likely that the CSSRR crosses the critical value boundaries in all cases.

space, we display those CSSRR for Australia and Canada only. See Figure 1. The other two tests (White's and ARCH-LM) are for heteroskedasticity.

Table 1
Panel A: Summary statistics of returns of stock indexes by country

Coefficient	Aus	Can	Jap	Kor	Sing	Taiw	Thai	UK	US	World
Mean	0.028	0.034	0.021	0.059	0.028	0.003	0.043	-0.0018	-0007	0.0043
Maximum	3.670	5.081	6.273	8.484	5.524	9.172	15.861	5.589	5.610	4.752
Minimum	-5.372	-9.261	-6.512	-13.097	-9.095	-10.309	-8.073	-6.011	-6.161	-4.121
S D	0.767	1.1389	1.221	2.165	1.202	1.773	1.8156	1.143	1.170	0.918
Skewness	-0.454	-0.399	-0.209	-0.191	-0.349	0.075	0.720	-0.215	0.093	0.028
Kurtosis	6.458	8.522	4.773	5.668	7.622	5.271	10.067	5.936	5.244	5.379
J-B stat	971.4	2365.7	252.3	552.1	1660.4	393.5	3952.5	669.1	385.2	430.5
$Q(20)$	22.37	32.30	17.00	36.44	27.68	35.23	69.01	73.33	29.23	67.32
$Q^2(20)$	231.6	343.6	180.8	186.4	218.4	432.9	266.6	1885.6	782.9	959.7
ADF (ind) ^a	0.65	-1.07	-0.27	-0.47	-1.51	-1.73	-0.69	-1.54	-1.59	-1.26
ADF (ret) ^b	-43.52	-41.83	-41.11	-41.71	-39.73	-41.75	-36.95	-27.98	-43.63	-37.27

Panel B: Preliminary statistics of exchange rate changes by country

Coefficient	Aus	Can	Jap	Kor	Sing	Taiw	Thai	UK	US
Mean	0.0093	0.0146	-0.0026	0.0088	-0.00009	-0.0012	-0.0068	0.0021	0.0032
Maximum	2.7314	1.6294	2.7474	1.9041	2.1123	2.6210	3.6473	2.0549	1.7880
Minimum	-3.0104	-1.6559	-2.2680	-1.9811	-1.3086	-2.0572	-2.2332	-1.8415	-2.0409
S D	0.6706	0.4457	0.6233	0.4240	0.2716	0.2379	0.3595	0.5148	0.4231
Skewness	-0.2707	-0.0230	0.2773	-0.337	0.2976	-0.0962	0.0361	-0.0214	0.0594
Kurtosis	4.257	3.826	4.597	5.513	6.622	19.180	12.451	3.634	4.169
J-B stat	142.29	52.06	217.12	514.71	1024.10	1989.0	6788.13	30.67	104.86
$Q(20)$	28.80	20.96	27.68	17.32	25.69	38.17	70.79	10.51	22.94
$Q^2(20)$	113.88	432.63	58.91	261.62	32.14	27.07	440.25	79.86	61.83
ADF (rate) ^a	-0.72	0.02	-2.00	-0.31	-1.93	-1.33	-2.26	-1.12	-0.82
ADF (chan) ^b	-40.57	-42.72	-43.14	-41.40	-42.60	-43.81	-32.41	-43.68	-44.22

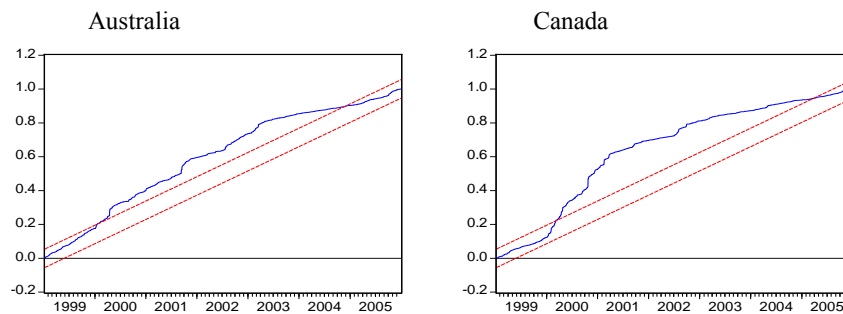
Notes: $Q(20)$ and $Q^2(20)$ are Ljung-Box statistics of returns and squared returns for 20 lags. They follow a χ^2 distribution and the critical value at the 5% level of significance with 20 degrees of freedom is 31.41. ^a and ^b - Augmented Dikey-Fuller statistic for exchange rate (level) and changes in exchange rate, respectively.

Table 2
Results of Heteroskedasticity test using OLS estimates by country

Test statistic	Aus	Can	Jap	Kor	Sing	Taiw	Thai	UK	US
White's test ^a	18.59*	43.21*	31.50*	15.15*	18.14*	109.84*	10.65	44.97*	41.88*
ARCH- LM (5) ^a	65.71*	70.74	48.95*	37.58*	45.20*	82.76*	28.28*	239.99*	141.97*

Notes: Regression equation used: $r_{i,t} = \beta_0 + \beta_m r_{m,t} + \beta_x r_{x,t} + \xi_i$; Both White's Heteroskedasticity (with cross terms) and ARCH LM test statistics are assumed to follow χ^2 distribution; ^a Critical value at the 5% level with 5 degrees of freedom is 11.07; * indicates significance at the 5% level.

Figure 1
Cumulative sum of squared recursive residuals (CSSRR) test results



A battery of tests is conducted for constancy in exchange rate exposure based on the OLS estimates of the conventional augmented market model¹⁴. The first test is the cumulative sum of squared recursive residuals (CSSRR) as suggested by Brown et al. (1975). The CSSRR cross the critical value boundaries in all cases at the 5% level of significance, thereby providing evidence of parameter instabilities¹⁵. To conserve space, we display those CSSRR for Australia and Canada only. See Figure 1. The other two tests (White's and ARCH-LM) are for heteroskedasticity. As can be observed in Table 2, the White's test is statistically significant in all cases at the 5% level with 5 degrees of freedom, except for Thailand. And the ARCH-LM test with 5 lags is significant for all cases at the 5% level of significance. Hence, these findings

¹⁴ This refers to the constant parameter version of the regression equation in (2)

¹⁵ As two slope coefficients are involved in the regression, one may argue that this instability may stem from the market beta, but not from the exchange rate exposure beta. To address this issue, we have regressed country returns on exchange rate changes only and obtained the cumulative sum of squares of recursive residuals. The diagrams are very similar to those displayed in Figure 1. As such, it is more likely that the CSSRR crosses the critical value boundaries in all cases.

consistently indicate that parameters specified in equation (2) are likely to be unstable.

5. Empirical Findings

In this section, we report estimates of the time-varying parameters specified in the trivariate BEKK-GARCH-M model. We then compute the time-varying currency betas and market betas, and check the adequacy of the proposed model. This is followed by a brief investigation of the stochastic structure of time-varying currency betas. Finally, we present two applications to illustrate the usefulness of time-varying exposure series.

Table 3 tabulates estimates of parameters specified in the proposed trivariate BEKK (1)-GARCH (2, 1)-in-mean model for the nine financial markets using the quasi-maximum likelihood method of estimation. Under the general framework of ICAPM, the market price of risk (λ_M) are positive for all countries with no restriction on signs of the currency price of risk (λ_X).

As can be gleaned from Table 3, all signs of λ_M are uniformly positive across all countries, with magnitudes ranging from the smallest 0.0237 (Canada) to the largest 0.0573 (US). However, all these estimates are statistically insignificant at the 5% level. Unlike the estimated market price of risk, estimates of the currency price of risk (λ_X) vary remarkably in sign and magnitude across countries ranging from -0.4539 (UK) to 0.0405 (Canada). The estimates of currency price of risk are not statistically significant at the 5% level. Our results are consistent with previous findings¹⁶.

¹⁶ For example, De Santis and Gerard (1998) and Cappiello et al. (2003) also find that both market and currency premiums are insignificant as long as the prices are time-variant.

Table 3
Estimation results for the trivariate BEKK(1)-GARCH(1,2)-M model

Coefficient	Australia	Canada	Japan	Korea	Singapore	Taiwan	Thailand	UK	US
λ_{iu}	0.0336 (0.76)	0.0237 (0.50)	0.0346 (0.79)	0.0531 (1.16)	0.0477 (0.98)	0.0486 (1.06)	0.0400 (0.80)	0.0345 (0.70)	0.0573 (1.05)
λ_x	-0.0648 (-0.45)	0.0405 (0.34)	-0.0453 (-0.22)	-0.0106 (-0.14)	-0.3089 (-0.75)	-0.1658* (-1.85)	-0.0848 (-1.05)	-0.4539* (-1.94)	-0.1209 (-0.56)
b_i	0.9800*** (75.76)	0.9765*** (153.41)	0.9606*** (123.82)	0.9848*** (235.71)	0.9423*** (32.68)	0.9737*** (169.72)	0.9888*** (168.24)	0.9516*** (101.41)	0.9637*** (150.79)
a_i	0.1833*** (3.40)	0.2083*** (7.28)	0.2427*** (8.93)	0.1633*** (6.98)	0.2603*** (2.31)	0.1976*** (6.54)	0.1323*** (4.53)	0.2792*** (10.82)	0.2363*** (12.26)
d_i	-	-	-	-	0.1700 (0.66)	-0.0912 (-1.31)	0.0481 (0.69)	-	-
b_x	0.9877*** (319.16)	0.9922*** (456.86)	0.9728*** (101.23)	0.9531*** (93.62)	0.9748*** (108.60)	0.8617*** (20.88)	0.9212*** (50.71)	0.9837*** (174.85)	0.9855*** (184.02)
a_x	0.1197*** (7.15)	0.1175*** (7.68)	0.1351*** (5.63)	0.2664*** (9.57)	0.1465*** (4.19)	0.5043*** (4.60)	0.3518*** (8.80)	0.1267*** (6.44)	0.1319*** (6.10)
b_m	0.9712*** (72.89)	0.9757*** (122.30)	0.9753*** (107.35)	0.9617*** (77.74)	0.9652*** (65.57)	0.9620*** (90.58)	0.9644*** (83.39)	0.9642*** (111.07)	0.9610*** (139.43)
a_m	0.2308*** (4.55)	0.2111*** (6.32)	0.2119*** (5.58)	0.1014*** (3.23)	0.1863* (1.84)	-0.0039 (0.06)	0.1114*** (3.11)	0.2080*** (11.34)	0.2410*** (12.46)
d_m	-	-	-	-	0.2349*** (6.09)	-0.1531 (-0.85)	-0.2603*** (-7.58)	-0.2250*** (-4.79)	-0.1149*** (-4.21)

Notes: *t*-values are in parenthesis; ***, **, * and * indicate significance at the 1%, 5% and 10% levels, respectively.

The estimates of GARCH parameters (denoted by b_j for $j = i, m, x$) are statistically significant at the 5% level, thereby suggesting that conditional variances are highly correlated with the previous ones. For Singapore, Taiwan and Thailand, an ARCH (2) term (denoted by d_j) was included in the conditional variance equations of country returns. Except for Australia, Canada and Japan, another ARCH (2) term is included in the conditional variance equation for the world market returns. However, no additional ARCH terms are necessary in the conditional variance equation for

exchange rate changes. Except for the world market return for Singapore, it can be observed from Table 3 that at least one of the estimated ARCH terms is significant in each of the remaining eight cases. As such, our sample data provides some support for volatility clustering in country stock markets and in exchange rate markets.

Turning to diagnostic checks, Table 4 reports the summary statistics of standardized residuals: Panel A for stock returns and Panel B for exchange rate changes. The Ljung-Box statistics for standardized and squared standardized residuals at 20 lags ($Q(20)$ and $Q^2(20)$) are significantly lower as compared to those of stock returns and changes in exchange rate series reported in Table 1. Except for Korea and Thailand, the Ljung-Box $Q(20)$ and $Q^2(20)$ statistics of residuals from the remaining countries are smaller than the critical value (31.481) at the 5 % level¹⁷. As such, our findings indicate that the proposed trivariate BEKK(1)-GARCH (1,2)-in-mean model is reasonably adequate for capturing the conditional volatility of stock returns and changes in exchange rates. In the next subsection, we discuss the characteristics of the time-varying market betas and currency betas computed from estimates of the conditional variance and covariance matrix H_t .

5.1 Time-varying currency betas and market betas

The time-varying market betas and currency betas by country are computed using equations (13) and (14), respectively. Table 5 compares mean values of estimated time-varying market betas and the time-varying currency betas with their corresponding OLS point estimates. The average of each currency betas is quite close to the corresponding OLS point estimate across countries (e.g. Australia (0.1049, 0.1057) and the US (0.1566, 0.1427)). The estimated currency betas associated with the bilateral exchange rate between the US dollar and the currency of each country are positive in seven cases, except for UK. Interestingly, the exposure beta of the US, which is associated with a trade-weighted exchange rate, is also positive.

¹⁷ It is worth making a special comment on two cases which do not satisfy this requirement: country returns for Thailand and exchange rate changes for Korea. As for Thailand, the $Q^2(\cdot)$ statistics is below the critical value up to 11 lags ($Q^2(11) = 13.71$). For Korea, the $Q^2(\cdot)$ statistics is below the critical value up to 15 lags ($Q^2(15) = 14.51$).

Table 4**Panel A: Diagnostics for return on stock indexes by country**

Coefficient	Aus	Can	Jap	Kor	Sing	Taiw	Thai	UK	US
Mean	-0.0265	-0.0314	-0.0161	-0.0279	-0.0339	-0.0231	-0.0169	-0.0549	-0.0536
Maximum	3.9285	4.1928	4.8650	3.9242	4.2083	5.1874	7.3946	3.0403	3.5236
Minimum	-6.1078	-5.5514	-4.7516	-7.5811	-7.9686	-5.0443	-4.7672	-5.1153	-5.3602
S D	0.9901	0.9964	0.9852	0.9999	0.9992	0.9953	1.0041	0.9979	1.0045
Skewness	-0.3933	-0.3076	-0.1846	-0.3357	-0.3358	0.0028	0.3999	-0.3553	-0.2431
Kurtosis	4.8198	4.5228	4.3655	5.5817	6.6696	4.4066	6.3975	3.6614	4.0960
J-B Stat	295.6	204.8	151.9	540.2	1056.5	150.2	924.8	71.6	109.1
$Q(20)$	13.9	27.8	11.3	19.5	18.7	17.2	21.2	26.3	57.3
$Q^2(20)$	26.8	17.6	24.5	10.7	8.33	24.1	70.1	23.2	15.7

Panel B: Diagnostics for bilateral exchange rate changes by country

Coefficient	Aus	Can	Jap	Kor	Sing	Taiw	Thai	UK	US
Mean	-0.0071	0.0027	0.0093	-0.0048	-0.0057	-0.0214	-0.0267	0.0167	0.0160
Maximum	4.1895	4.1091	4.7277	5.3546	7.9610	5.8243	4.6672	3.8210	3.9319
Minimum	-3.8022	-4.1363	-3.3134	-5.4964	-4.2677	-10.5147	-4.5227	-3.3602	-4.5527
S D	0.9925	0.9901	1.0030	1.0035	0.9984	0.9564	0.9994	0.9969	0.9945
Skewness	-0.2702	-0.0674	0.3201	-0.3489	0.3551	-1.2779	-0.1642	-0.0070	0.1184
Kurtosis	3.9225	3.3905	4.5389	5.7320	6.6529	21.3539	5.2742	3.5356	3.8741
J-B Stat	86.8	12.9	210.9	603.6	1051.3	26070.0	400.8	20.9	62.2
$Q(20)$	22.3	13.4	22.7	21.2	25.1	39.9	34.7	11.0	16.3
$Q^2(20)$	18.4	29.2	17.7	33.6	13.2	4.1	9.6	20.1	20.0

Notes: $Q(20)$ and $Q^2(20)$ are Ljung-Box statistics of residuals and squared residuals for 20 lags.

They follow a χ^2 distribution and the critical value at the 5% level of significance with 20 degrees of freedom is 31.41. ^a An outlier is removed to get these summary statistics.

Table 5
Comparison between OLS point estimates of betas and mean time-varying
currency betas by country

Country	Market beta		Currency beta	
	OLS β_m	Mean of $\beta_{m,t}$	OLS β_x	Mean of $\beta_{x,t}$
Australia	0.1970	0.2158	0.1057	0.1049
Canada	0.8398	0.8661	0.0142	0.0274
Japan	0.4457	0.5443	0.1117	0.0792
Korea	0.5448	0.5243	1.0318	0.8660
Singapore	0.4179	0.3620	0.2058	0.1249
Taiwan	0.3579	0.3022	1.3406	1.5765
Thailand	0.3149	0.2197	0.9412	0.9114
UK	0.8620	0.7993	-0.1187	-0.1292
US	1.1660	1.2044	0.1427	0.1566

Notes: Time-varying market and exchange rate exposure betas are obtained using $\beta_{m,t-1} = \frac{h_{x,t}h_{im,t} - h_{xm,t}h_{ix,t}}{h_{x,t}h_{m,t} - [h_{xm,t}]^2}$ and $\beta_{x,t-1} = \frac{h_{m,t}h_{ix,t} - h_{xm,t}h_{im,t}}{h_{x,t}h_{m,t} - [h_{xm,t}]^2}$, respectively.

At the risk of over-simplification, one possible interpretation is that a US exporter can hedge against currency risk by investing only in UK assets, whose returns are negatively correlated with depreciation of local currency. Importers or investors whose consumption basket consisting of imported goods from relevant countries can hedge against currency risk by investing in assets in any country except for UK. Moreover, we note that the US dollar exchange rate is highly related with returns on assets in Taiwan (1.5765), Thailand (0.9114) and Korea (0.8600), respectively.

Summary statistics of the estimated betas by country are reported in Panel A of Table 6. For currency betas, the mean and standard deviation ranges from (0.1049, 0.1156) for Australia to (1.5766, 1.4060) for Taiwan. The emerging markets like Korea (0.8661, 1.2186), Taiwan (1.5766, 1.4060), Singapore (0.1249, 0.4623) and Thailand (0.9098, 0.6993) have larger mean and wider volatility in conditional currency betas than those in the developed markets like the US (0.1566, 0.1157), UK (-0.1292, 0.1675), Japan (0.0792, 0.1952) and Canada (0.0272, 0.2487). In addition, currency betas of seven countries are positively skewed, except for Singapore and the UK. Moreover, all currency betas are leptokurtic.

Table 6
Panel A: Summary statistics of time-varying currency betas by country

Coefficient	Aus	Can	Jap	Kor	Sing	Taiw	Thai	UK	US
Mean	0.1049	0.0272	0.0792	0.8661	0.1249	1.5766	0.9098	-0.1292	0.1566
Maximum	0.7151	1.2227	0.8646	4.8935	1.9394	7.2367	4.1299	0.5521	0.7431
Minimum	-0.1946	-0.8770	-0.7595	-3.5040	-2.2617	-3.0963	-1.1436	-1.1903	-0.2164
S D	0.1156	0.2487	0.1952	1.2186	0.4623	1.4060	0.6993	0.1675	0.1157
Skewness	1.1676	1.2890	0.0432	0.4595	-0.1745	0.2801	0.6413	-0.6004	0.7654
Kurtosis	6.9273	11.0618	4.6935	3.9570	6.2937	3.9092	4.3961	7.9853	4.9590
J-B stat	1584.93	5438.64	218.28	133.64	832.84	86.58	272.87	1996.26	469.25

Panel B: Summary statistics of time-varying market betas by country

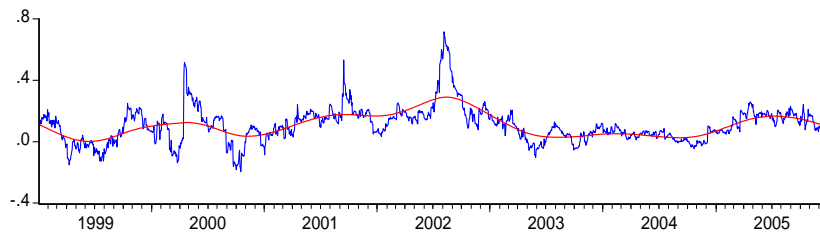
Coefficient	Aus	Can	Jap	Kor	Sing	Taiw	Thai	UK	US
Mean	0.2142	0.8664	0.5445	0.5039	0.3573	0.3161	0.2112	0.7851	1.2160
Maximum	0.8762	1.8926	1.3883	1.2852	0.9876	1.0048	0.6305	1.4009	1.5342
Minimum	-0.0366	0.3811	-0.2390	-0.3974	-0.0157	-0.3414	-0.0996	0.3141	0.8252
S D	0.1264	0.2578	0.2573	0.2590	0.1697	0.2075	0.1365	0.1273	0.1003
Skewness	0.9869	0.9417	0.3113	0.4304	0.3451	0.4033	0.6894	-0.1131	0.3863
Kurtosis	4.8983	4.3717	2.9397	2.4022	2.3236	3.1833	2.5337	4.3847	3.6682
J-B stat	563.38	412.15	29.70	83.39	70.90	51.95	160.82	149.46	79.20

As can be observed in Panel B of Table 6, there are no clear-cut patterns for the mean and standard deviation of estimates of conditional market betas by country. Upon comparison of currency betas and market betas tabulated in Panels A and B, except for Australia, Canada and Japan, the standard deviations of currency betas for the remaining 6 countries are higher than those of the market betas. Regardless of whether the economy is developed or emerging, the market beta in each case is relatively less volatile than the currency beta. This feature is remarkably prominent for countries like Korea (0.2590, 1.2186), Singapore (0.1697, 0.4623), Taiwan (0.2075, 1.4060) and Thailand (0.1365, 0.6993). In addition, the sample kurtosis of currency beta by country is always greater than the corresponding market beta. This suggests that the distribution of currency betas tends to have thicker tails than that of the market beta.

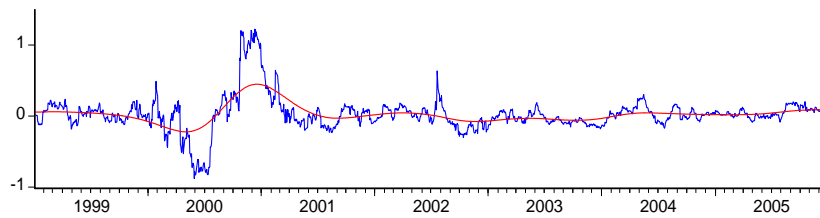
Figure 2 displays the time-varying currency betas for all nine countries. As can be observed, the fitted currency betas of Taiwan, Korea and Thailand fluctuate within wider ranges than those of Australia, Canada, Japan, and the UK; and the US displaying somewhat meager fluctuations. Understandably estimates of the time-varying currency betas are still subject to estimation errors. For easy reference, the Hodrick-Prescott filtered trends are computed for each currency betas series.

Figure 2
Time-varying currency betas by country

(a) Australia



(b) Canada



(c) Japan

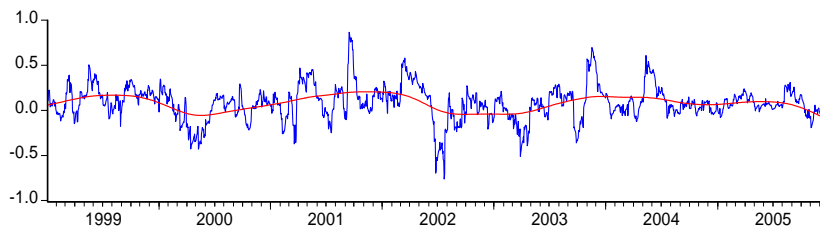
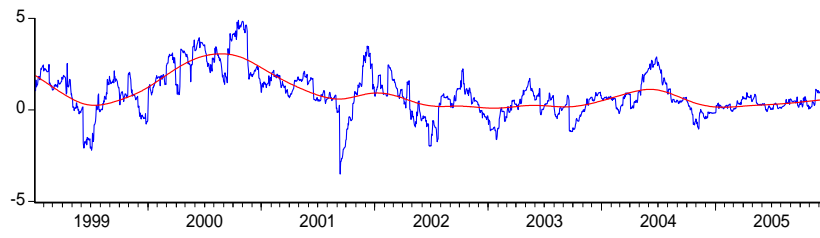
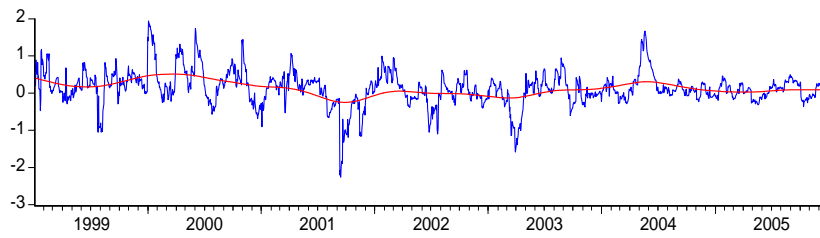


Figure 2 (continued)
Time-varying currency betas by country

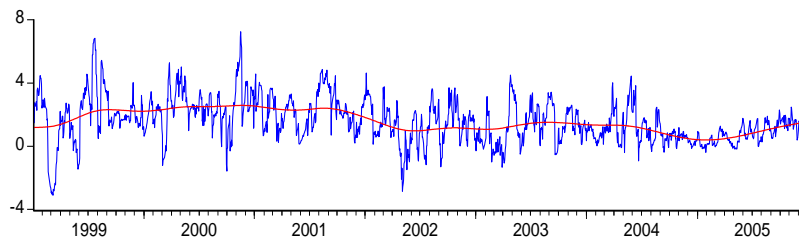
(d) Korea



(e) Singapore



(f) Taiwan



(g) Thailand

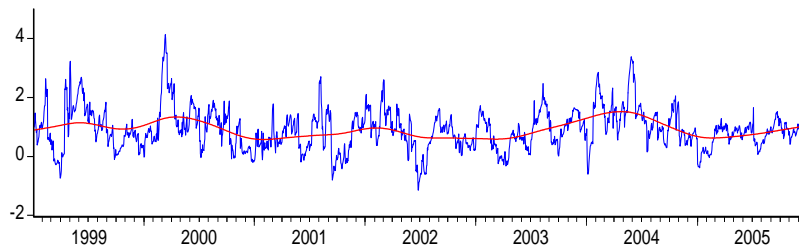
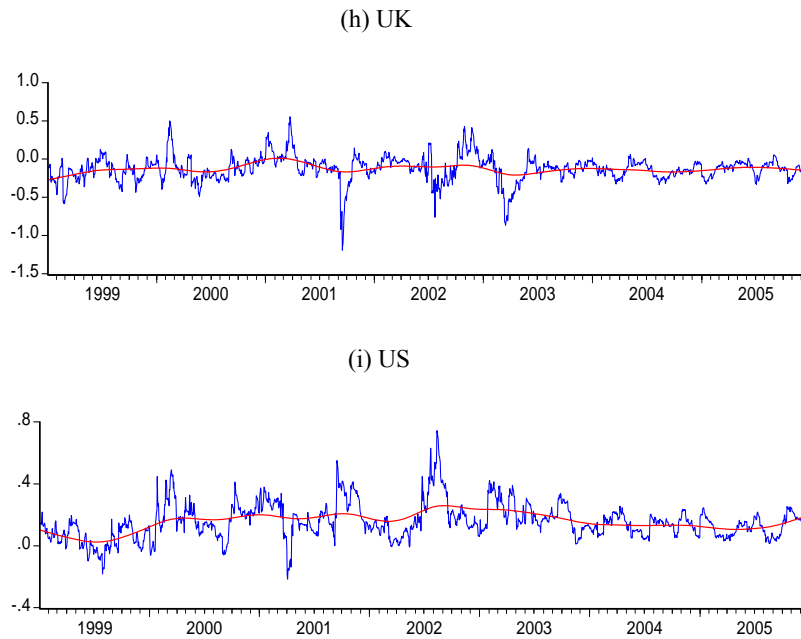


Figure 2 (continued)
Time-varying currency betas by country



Note: The Hodrick-Prescott filtered trend is indicated by the roughly flat line across the time-varying currency betas.

Next, we examine whether the time-varying currency betas are mean-reverting and stationary. We employ a widely used semi-nonparametric test proposed by Gewek and Porter-Hudak (1983) for such a purpose¹⁸. First, we perform a one-sided test to check the validity of the null hypothesis that the fractional differencing parameter (d) equals to 0 versus the alternative hypothesis that d is greater than 0. It is found that the null hypothesis is rejected at the 5% level of significance for all cases except for Japan and Taiwan when $\alpha = 0.50$ (to save space, the results are not reported). In addition, a second one-sided test is performed under the null hypothesis that d is equal to 0 versus the alternative hypothesis that d is less than 1.

¹⁸ The test is based on the following spectral regression equation:

$$\ln I(\omega_s) = c + \phi \ln(4 \sin^2(\omega_s/2)) + \zeta \quad \text{for } s = 1, 2, \dots, n(T)$$

where T is the number of observations in the series; $I(\omega_s)$ is the periodogram of a series at harmonic frequency $\omega_s = (2\pi s/T)$ with $s = 1, 2, \dots, T-1$; ζ is random error; n represents the number of low frequency ordinates and is usually determined as $n = T^\alpha$. OLS estimation of ϕ provides a consistent estimate of $-d$ in the ARFIMA process $\Phi(L)(1-L)^d y_t = \Theta(L)v_t$ with $v_t \sim (0, \sigma^2)$.

Table 7
GPH test results for estimates of time-varying currency betas by country

Country	Value of differencing parameter d		
	$\alpha = 0.50$	$\alpha = 0.55$	$\alpha = 0.60$
Australia	0.6267* (-4.54)	0.7715* (-3.31)	0.8601* (-2.24)
Canada	0.7234* (-2.33)	0.7583* (-2.43)	0.7669* (-3.02)
Japan	0.1607* (-5.43)	0.2775* (-6.25)	0.4072* (-6.41)
Korea	0.8270 (-1.34)	0.8028* (-1.91)	0.8462 (-1.92)
Singapore	0.3463* (-6.34)	0.4405* (-6.86)	0.5257* (-6.84)
Taiwan	0.1445* (-8.50)	0.3564* (-6.00)	0.4058* (-7.46)
Thailand	0.3621* (-5.81)	0.4813* (-5.84)	0.5411* (-6.74)
UK	0.2825* (-4.45)	0.3994* (-4.90)	0.4495* (-5.86)
US	0.3145* (-6.92)	0.4947* (-4.98)	0.6015* (-5.01)

Notes: d refers to the differencing parameter in the fractional integration process $\Phi(L)(1-L)^d Y_t = c + \Theta(L)v_t$ and is represented by ϕ in the regression: $\ln I(\omega_j) = c - \phi \ln(4 \sin^2(\omega_j/2)) + \zeta$
Values of t -statistics are in parentheses; * indicates significance at least at the 5% level

As can be gleaned from Table 7, the null hypothesis is rejected at the 5% level for all cases at different values of α , except for Korea. Even in the case of Korea, the null hypothesis is accepted only when α is 0. Hence, we find some evidence of long memory in currency betas. Moreover, for Japan, Taiwan and UK, the respective currency beta series may follow an ARFIMA process with d less than 0.5 for different α values. As for Singapore, Thailand and the US, d is less than 0.5 when $\alpha = 0.5$ and 0.55, but greater than 0.5 when $\alpha = 0.6$. For the remaining countries (Australia, Canada and Korea), d is greater than 0.5 for all values of α .

Based on results of the GPH tests, some discussions are in order. First, all time-varying currency betas series consistently reject both $I(0)$ and $I(1)$ processes. This implies that the betas series may follow a long memory process or an AFIMA

process $I(d)$, with $0 < d < 1$. Second, the currency betas for Japan, Taiwan and UK are covariance stationary as well as mean-reverting. The currency betas for Singapore, Thailand and the US are more likely to follow similar patterns, whereas currency betas for Australia, Canada and Korea indicate covariance non-stationary, but mean-reverting dynamics. Third, investors may exploit the mean-reverting feature of currency betas for forecasting purposes. This could be very useful in formulating hedging strategies against currency risk.

5.2 Usefulness of time-varying currency betas

In this subsection, we illustrate the usefulness of the conditional time-varying betas series as source of information for making decision. First, currency betas among countries are compared by using the stochastic dominance criterion. Second, we discuss the usefulness of time-varying currency premiums computed by using currency betas.

5.2.1 Dominance of currency betas among countries

The rules of stochastic dominance have been widely used to compare risk of stock returns. For example, Gonzales-Rivera (1996) applies the stochastic dominance criterion to compare risks associated with the time-varying market betas of firms. And Brooks et al. (2000) employ the same approach to analyzing impacts of regulatory changes on the risk and returns of the US banking industry.

However, in order to have a meaningful comparison of the distribution of currency betas, we have to modify¹⁹ the conventional first order stochastic dominance inequalities. For instance, when an investor wants to identify the exchange rate exposure in the nine countries, he/she needs to consider both negative and positive values of time-varying currency betas for each country. This is because equal

¹⁹ Let $F_x(\beta_{x,t})$ and $G_y(\beta_{x,t})$ be the cumulative distribution functions (CDFs) of the time-varying exchange rate exposure (currency betas) of two countries x and y , respectively. Country x 's currency beta first order stochastically dominates country y 's exposure beta, if two CDFs do not cross and $F_x(\beta_{x,t}) \geq G_y(\beta_{x,t})$ for all $\beta_{x,t}$ with at least one strict inequality. Graphically, $F_x(\beta_{x,t})$ lies above and to the left of $G_y(\beta_{x,t})$. Country x 's exposure beta is said to second order stochastically dominate country y 's exposure beta, if $\int_{-\infty}^{\beta_{x,t}} (F_x(\beta_{x,t}) - G_y(\beta_{x,t})) d\beta_{x,t} \geq 0$ for all $\beta_{x,t}$ with at least one strict inequality.

magnitudes of currency betas irrespective of their signs indicate similar risks. As such, it is more appropriate to compare distributions of currency betas in absolute values.

Figure 3 plots the empirical cumulative distribution (ECD) of currency betas in absolute value. Apparently, the ECDs of currency betas in three emerging markets including Taiwan, Korea and Thailand consistently lie below the right side of those ECDs of other six countries, with Taiwan on the further right. This indicates that Taiwan has the highest currency exposure during the sample period. Though Singapore is less exposed to currency risk than Taiwan, Korea and Thailand, it is more exposed to currency risk than those of Australia, Canada, Japan, UK and the US. Admittedly it is not easy to rank the cases without using the second order stochastic dominance as some of the EDFs cross over each other. However, Australia and Canada seem to be less exposed than Japan, UK and the US²⁰ as their EDFs lie to the further left of other EDFs.

For practical considerations, consider an importer from the US looking for means of hedging against currency risk through investment in foreign assets. The selection rule based on absolute values of currency betas may not be helpful in choosing the proper country for allocating funds. In this case, the empirical distribution of nominal values of currency betas may be more appropriate. As depicted in Figure 4, investors will be more likely to hedge against currency risk by investing in emerging markets including Korea, Thailand and Taiwan, which are highly positively exposed to the depreciation of the US dollar. By the same token, assets in country like UK would be the appropriate choice for exporters seeking means of hedging against currency risk.

¹⁷ This result based on CDFs of time-varying exposure betas is not fully reflected in the mean values of time-varying exposure betas. For instance, Thailand is more exposed to exchange rate changes than Korea (mean values for the two countries are -0.9114 and -0.8660, respectively). However, as shown in Figure 3, Korea seems to be second order stochastically dominated by Thailand, suggesting that Korea is more exposed to exchange rate changes than Thailand.

Figure 3
 Cumulative distribution of time-varying currency betas in absolute values by country

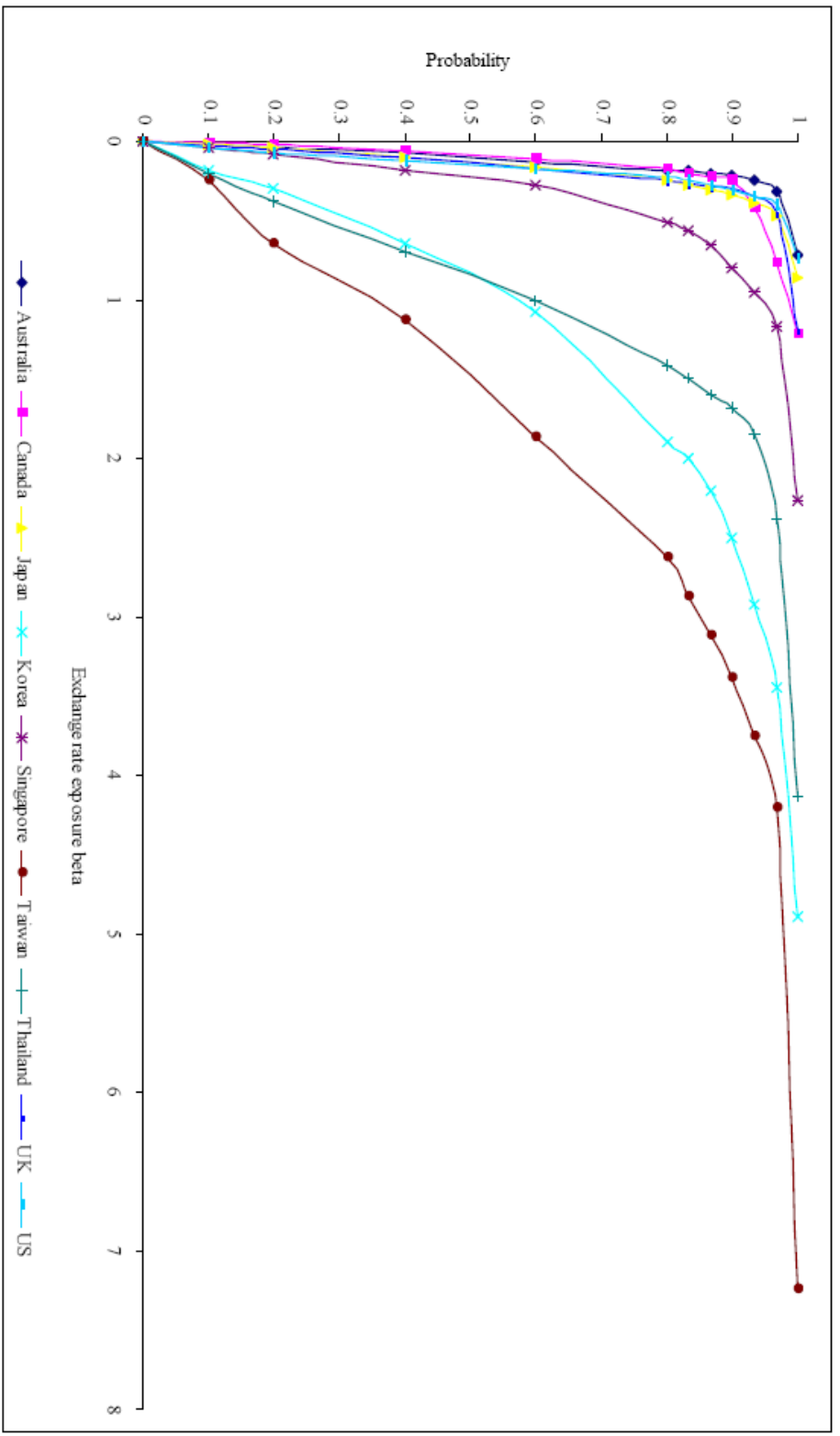
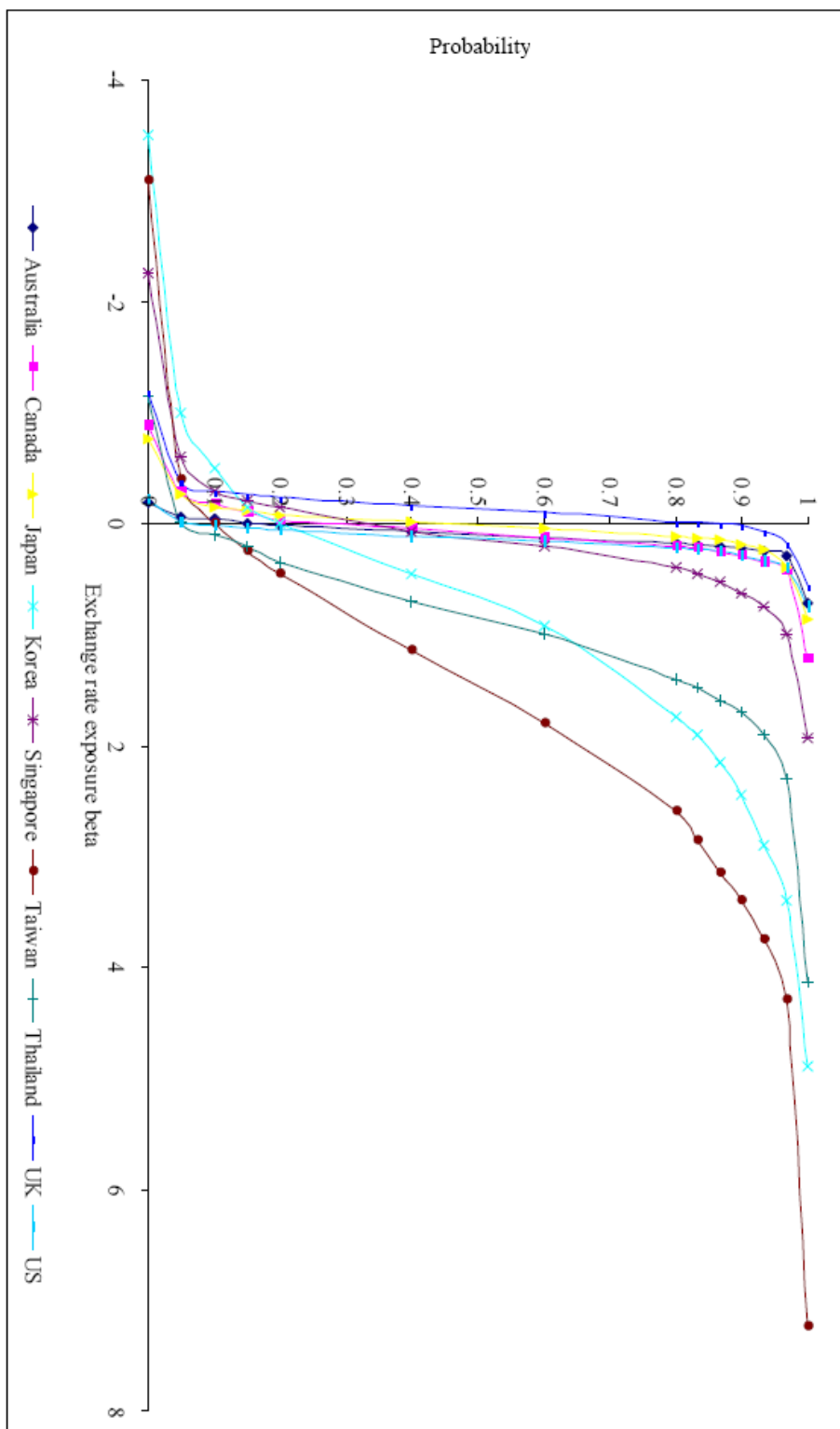


Figure 4
Cumulative distribution of time-varying currency betas by country



5.2.2 Time-varying currency premiums

As discussed in Section 3, the time-varying market and currency betas can be estimated under the broad ICAPM framework using equations (6) to (10). It is natural to explore the relationship among currency, market and total risk premiums by country. For each country, the market premium (*MP*) and currency premium (*CP*) can be expressed as follows:

$$MP = \beta_{m,t-1} E(r_{m,t}) \quad (18)$$

$$CP = \beta_{x,t-1} E(r_{x,t}) \quad (19)$$

Here the market premium is proportional to the expected return of world market portfolio and the market beta; whereas the currency premium is proportional to the expected return of changes in exchange rates and the currency beta. According to our model, the conditional proportionality factors (market beta and currency beta) vary over time. Hence, the total risk premium can be computed as the sum of conditional market premium and currency premium²¹.

Table 8 displays the computed mean values of conditional market, currency and total risk premiums, and their standard deviations by country for three sub-periods: 5 January 1999 to 30 April 2001; 1 May 2001 to 31 August 2003; 1 September 2003 to 31 December 2005 (see columns 1 - 3) and the entire sample period: 1 May 1999 to 31 December 2005 (see column 4), respectively. For easy comparison, all risk premiums are expressed in percentage. Apart from a few cases, the mean and standard deviation for three sub-periods are reasonably similar to those of the entire period. In addition, the average currency premiums of assets during the entire sample period are positive in seven cases and negative for the US and Japan.

At the risk of over-simplification, we attempt to provide an interpretation as follows. During the sample period, a representative US investor may demand a negative risk premium for holding Japanese and local assets as a means of hedging.

²¹ Strictly speaking, the first mean equation in (6) states that the total expected return consists of market premium, currency premium, an intercept and the moving average term. However, the intercept and the moving average term are ignored in this analysis.

Table 8
Mean and volatility of risk premiums by country in various sub-sample periods

Country	(1) 5/1/1999 – 30/4/2001			(2) 1/5/2001 – 31/8/2003			(3) 1/9/2003 – 31/12/2005			(4) 1/5/1999 – 31/12/2005		
	CP	MP	TP	CP	MP	TP	CP	MP	TP	CP	MP	TP
Australia	0.09 (0.53)	0.97 (0.72)	1.06 (0.96)	0.21 (0.90)	0.88 (0.86)	1.10 (1.30)	0.10 (0.39)	0.81 (0.53)	0.92 (0.70)	0.14 (0.65)	0.89 (0.72)	1.02 (1.02)
Canada	0.04 (0.49)	4.37 (8.45)	4.41 (8.44)	-0.02 (0.18)	3.21 (6.52)	3.18 (6.52)	0.03 (0.20)	2.48 (3.57)	2.51 (3.59)	0.02 (0.32)	3.35 (6.53)	3.37 (6.54)
Japan	-0.08 (0.39)	1.96 (0.92)	1.88 (0.96)	-0.02 (0.50)	2.32 (1.25)	2.30 (1.29)	-0.06 (0.31)	1.73 (0.63)	1.67 (0.65)	-0.05 (0.41)	2.00 (1.00)	1.95 (1.03)
Korea	1.80 (3.08)	1.50 (2.92)	3.30 (4.55)	0.46 (1.88)	1.65 (2.73)	2.11 (3.41)	0.56 (1.31)	1.63 (2.92)	2.19 (3.26)	0.94 (2.30)	1.59 (2.86)	2.53 (3.82)
Singapore	0.06 (0.19)	1.23 (2.07)	1.29 (2.08)	0.10 (0.35)	1.29 (1.96)	1.39 (2.06)	-0.06 (0.25)	1.33 (1.69)	1.27 (1.73)	0.03 (0.28)	1.29 (1.91)	1.32 (1.96)
Taiwan	1.61 (3.90)	0.88 (3.23)	2.49 (5.26)	1.05 (2.33)	0.99 (3.54)	2.04 (3.72)	0.16 (2.26)	1.03 (3.10)	1.19 (3.59)	0.94 (2.99)	0.97 (3.05)	1.91 (4.29)
Thailand	-0.18 (1.90)	0.43 (1.96)	0.25 (2.76)	0.46 (1.20)	0.44 (2.82)	0.91 (2.19)	0.48 (1.24)	0.74 (2.43)	1.22 (2.69)	0.25 (1.51)	0.54 (2.10)	0.79 (2.59)
UK	0.01 (0.54)	4.08 (3.00)	4.09 (3.11)	0.06 (0.54)	7.46 (6.16)	7.52 (6.30)	0.29 (0.63)	2.07 (1.72)	2.36 (1.93)	0.12 (0.59)	4.54 (4.64)	4.66 (4.72)
US	0.02 (0.20)	7.21 (3.21)	7.23 (3.21)	-0.16 (0.37)	9.49 (6.00)	9.33 (5.88)	-0.07 (0.20)	3.36 (1.18)	3.29 (1.19)	-0.07 (0.28)	6.69 (4.72)	6.62 (4.66)

Notes: Figures within parenthesis are volatility of each beta series for the relevant periods.

However, for assets in the other seven countries, investors could demand a higher compensation for a positive currency risk premium as investing in such assets is not useful in hedging against currency risk.

Several interesting patterns can be observed from the computed currency premium and the total premium. First, for countries like Canada, Japan, UK and the US, the currency premium occupies a very small fraction of the total premium, with an average of less than 3%. However, for three emerging markets including Taiwan, Korea and Thailand, the currency premium occupies a much larger fraction of the total premium, with an average of more than 50%. This implies that investors should focus more on currency premium in these emerging markets. Second, on average the absolute value of currency premium in Korea (0.94), Taiwan (0.94) and Thailand (0.25) is considerably much higher than those currency premiums in the developed markets like Japan (0.05), Canada (0.02), UK (0.12) and the US (0.07). Third, as evidenced by the standard deviations, currency premium is more volatile in the emerging markets including Korea (2.30), Taiwan (2.99) and Thailand (1.51) than those of the remaining markets such as Australia (0.65), Canada (0.32), Japan (0.41), Singapore (0.28), UK (0.59) and the US (0.28), respectively. Our findings may be useful for investors when formulating currency hedging strategies.

6. Concluding Remarks

We have studied the time-varying currency betas and market betas for developed and emerging markets including the US, UK, Canada, Japan, Australia, Korea, Singapore, Taiwan and Thailand, respectively. A trivariate BEKK-GARCH-in mean model is used to estimate the time-varying conditional variance and covariance of returns of stock index by country, the world market portfolio and changes in bilateral exchange rate between the US dollar and currency of each country, respectively. Our approach is within the broad conditional ICAPM framework and does not require prior knowledge of the determinants of time-variation of currency betas.

The time-varying currency betas are computed from the conditional variance and covariance of the return variables, thereby accommodating the conditional

correlation between the bilateral exchange rate changes and market returns. As such, the estimated time-varying currency betas are more adequate than those estimates without taking the possible correlations into account. We find that currency betas are generally more volatile than the world market betas. Such currency betas are also more volatile in emerging markets like Korea, Taiwan and Thailand than those in developed markets. Based on results of GPH tests, we find evidence of long-memory of the estimated currency betas and mean-reverting, thereby displaying slow decay.

We have illustrated two applications of the estimated time-varying currency betas: a comparison of exposures among the developed and emerging markets by stochastic dominance criterion; and an analysis of time-varying currency premium. Both applications demonstrate the usefulness of the time-varying exposures in strategic investment.

Our study is not without caveats. Within the general framework of ICAPM, we have made two simplifications: non-stochastic inflation in each country and constant prices of the world portfolio risk and currency risk. Future research should look into the robustness of currency betas without holding such constancy.

References

- Adler, M. and B. Dumas (1983) "International Portfolio Choice and Corporation Finance: A Synthesis", *Journal of Finance*, 88(3), 925-984.
- Allayannis, G. (1997) "The Time-Variation of the Exchange Rate Exposure: An Industry Analysis", *Working Paper 97-29*, University of Virginia.
- Allayannis, G. and J. Ihrig (2001) "Exposure and Mark-Ups", *Review of Financial Studies*, 14(3), 905-935.
- Bodnar, G. M. and M. F. H. Wong (2003) "Estimating Exchange Rate Exposures: Issues in Model Structure", *Financial Management (Spring)*, 35-67.
- Bodnar, G. M., B. Dumas and R. C. Marston (2002) "Pass-Through and Exposure", *Journal of Finance*, 107, 199-231.
- Bodnar, G. M. and W. M. Gentry (1993) "Exchange Rate Exposure and Industry Characteristics: Evidence from Canada, Japan and the USA", *Journal of International Money and Finance*, 12, 29-45.
- Bollerslev, T. and J. M. Wooldridge (1992) "Quasi-maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances", *Econometric Reviews*, 11, 143-179.
- Brooks, R. D., R. W. Faff, M. D. McKenzie and Y. K. Ho (2000) "U.S. Banking Sector Risk in an Era of Regulatory Change: A Bivariate GARCH Approach", *Review of Quantitative Finance and Accounting*, 14, 17-43.
- Brooks, R. D., R. W. Faff and M. McKenzie (2002), "Time-Varying Country Risk: an Assessment of Alternative Modeling Techniques", *The European Journal of Finance*, 8, 249-279.
- Brown, R., J. Durbin and J. M. Evans (1975) "Techniques for Testing the Constancy of Regression Relationships Over Time", *Journal of the Royal Statistical Society*, 37, 145-164.
- Cappiello, L., O. Castren and J. Jaasakela (2003) "Measuring the Euro Exchange Risk Premium: the Conditional International CAPM Approach", *Working Paper*, European Central Bank.
- Chiao, C. and K. Hung (2000) "Exchange Rate Exposure of Taiwanese Exporting Firms", *Review of Pacific Basin Financial Markets and Policies*, 3(2), 201-233.
- Choudhry, T. (2001) "The Long Memory of Time-Varying Beta: Examination of Three Emerging Asian Stock Markets", *Managerial Finance*, 27(1/2), 5-14.

- Choudhry, T. (2002) “The Stochastic Structure of the Time-Varying Beta: Evidence from UK Companies”, *The Manchester School*, 70(6), 768-791.
- Choudhry, T. (2005) “Time-Varying Beta and the Asian Financial Crisis: Evidence from Malaysian and Taiwanese Firms”, *Pacific-Basin Finance Journal*, 13, 93-118.
- Chow, E. H. and H. L. Chen (1998) “The Determinants of Foreign Exchange Rate Exposure: Evidence from Japanese Firms”, *Pacific Basin finance Journal*, 6, 153-174.
- De Santis, G. and B. Gerard (1998) “How Big is the Premium for Currency Risk?” *Journal of Financial Economics*, 49, 375-412.
- Dominguez, K. M. E. and L. L. Tesar (2006) “Exchange Rate Exposure”, *Journal of International Economics*, 68, 188 – 218.
- Dumas, B. and B. Solnik (1995) “The World Price of Foreign Exchange Risk”, *Journal of Finance*, 100, 445-479.
- Entorf, H. and G. Jamin (2003) “German Exchange Rate Exposure at DAX and Aggregate Level, International Trade, and the Role of Exchange Rate Adjustment Costs”, *Darmstadt Discussion Papers in Economics # 126*.
- Geweke, J. and S. Porter-Hudak (1983) “The Estimation and Application of Long Memory Time Series Models”, *Journal of Financial and Quantitative Analysis*, 13, 101-116.
- Giannopoulos, K (1995) “Estimating the Time Varying Components of International Stock Markets’ Risk”, *European Journal of Finance*, 1, 129-164.
- Gonzales-Rivera, G. (1996) “Time-Varying Risk: The Case of the American Computer Industry”, *Journal of Empirical Finance*, 2, 333-342.
- Granger, C. and R. Joeyux (1980) “An Introduction to Long Memory Time Series Models and Fractional Differencing”, *Journal of Time Series Analysis*, 1, 15-39.
- Hamao, Y, R. Masull and V. Ng (1990) “Corrections in Price Changes and Volatility across International Stock Markets”, *The Review of Financial Studies*, 3, 281-307.
- Harvey, C. R., (1991) “The World Price of Covariance Risk”, *Journal of Finance* 46, 111-157.
- Hosking, J. (1981) “Fractional Differencing”, *Biometrika*, 68, 165-176.

- Hunter, D. M. (2005) “Time-Varying Exchange Rate Exposure of Small and Large Firms”, *Working Paper*, University of South Florida.
- Lai, K. S. (1997) “Long-Term Persistence in the Real Interest Rate: Some Evidence on a Fractional Unit Root”, *International Journal of Financial Economics*, 2, 225-235.
- Lim, G. C. (2005) “Currency Risk in Excess Equity Returns: a Multi Time-Varying Beta Approach”, *International Financial Markets, Institutions and Money*, 15, 189-207.
- Moerman, G. A. and M. A. van Dijk (2006) “The World Price of Inflation Risk”, *Working Paper*, SSRN.
- MSCI, (1998) *Methodology and Index Policy*, Morgan Stanley Capital International.
- White, H. (1980) “A Heteroskedasticity-Consistent Covariance Estimator and a Direct Test for Heteroskedasticity”, *Economica*, 48, 817-838.
- Williamson, R. (2001) “Exchange Rate Exposure and Competition: Evidence from the Automotive Industry”, *Journal of Financial Economics*, 59, 441-475.