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Environmental and Resource Management under Myopia

Tomoki Fujii

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Tomoki Fujii*

Abstract

Myopia is important in environmental and resource management problems because they often involve intertemporal decisions over a long time horizon. We present a parsimonious extension of a standard dynamic programming equation in a continuous time and continuous state setting, which enables rich description of myopic behavior. In our model, the process of planning future controls and choosing current control are clearly distinguished. We illustrate the behavior of various types of myopic agents with a simple example of non-renewable resource extraction, and discuss the policy relevance of the expiry of extraction permits to resource management under myopia.

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Key words: non-constant discounting, resource management, myopia, time-consistency, observational equivalence

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*Singapore Management University. email:tfujii@smu.edu.sg. Phone: +65-6828-0279. Fax: +65-6828-0833. Address: School of Economics and Social Sciences, Singapore Management University, 90 Stamford Road, Singapore 178903. This research was supported by the SMU Research Grant (C208/MSS6E010).

1 Introduction

This paper considers a stylized environmental and resource management problem with a myopic agent. When the agent, or the body that makes decisions, is myopic, she discounts future payoffs at a rate higher than what would be deemed ideal in the long run. Myopic behavior may greatly affect trade-offs over a long time horizon. Thus, myopia is particularly an important issue in environmental and resource management problems because they often involve trade-offs between now and distant future.

It is not hard to find the cases where myopia resulted in poor environmental and/or resource management. The case of Minamata disease–a well known case of pollution in Japan–may be a good example. Chisso, a chemical company, released waste water from acetaldehyde production. The mercury contained in the waste water became concentrated in fish in the Minamata Bay through the process of food chain. People as well as cats consumed the contaminated fish suffered from organic mercury poisoning, a disease now known as the Minamata disease.

The first clinical case of Minamata disease was found in 1956. The cause of the disease was not known then. In 1959, a group of researchers at Kumamoto University concluded that the cause of the Minamata disease is organic mercury, most likely from the Chisso factory. However, it took another nine years until the Chisso company stopped operating the acetaldehyde production, and national government officially acknowledged that organic mercury is indeed the cause of Minamata disease.

Why did it take so long before the Chisso took any action? The company lost a number lawsuits started by the patients of the Minamata disease and the bereaved families. The loss was indeed so large that the company had to be rescued and controlled by the government to help it survive and pay the compensation to the plaintiffs. It is true that the cause of the Minamata disease was still unclear until the early 1960s. Yet, it wouldn't have taken so long to stop the spread of the pollution, had the company fully calculated the potential costs arising from the loss of lawsuits against the costs of curbing the emission of mercury-contaminated waste water. That is, had the company were not myopic, it would have taken action much earlier.

A case like this is not uncommon at all. We can find at least two reasons why the agent may be myopic. First reason is the psychological drive for instantaneous gratification. Suppose that you have a box of chocolates which you plan to finish over a week. You may end up finishing it in two days simply because you are not able to stop eating. A social planner may favor the present generation over future generations even in the absence of pure time preference for similar reasons.

Second reason is the incentive problem. That is, the incentive schedule that the agent faces may be different from the optimization problem that she is supposed to solve. For example, politicians are often reluctant to implement a policy that increases the burden of the present generation for the benefit of future generations, even if such a policy is supposed to benefit the country in the long run. This is because the present generation has a vote but future generations don't. Similarly, a manager of a mine would behave myopically if his compensation is determined solely by the current profit from the mine. The case of the Minamata may also fall in this category.

Indeed, the inefficiency arising from the incentive problem creates the room for policy intervention. Schmutzler (2001) studied the impact of environmental policy in the presence of managerial myopia and showed that environmental policy can only increase firm profits, provided that several conditions are met.

This paper is closely related to the literature on non-constant discounting problems. Non-constant discounting and its consequences have been given a renewed attention over the last decade with the advancement of behavioral economics (*e.g.* Laibson (1997),O'Donoghue and Rabin (1999)). It is relatively recent that economists have started studying the consequences of non-constant discounting in environmental and resource management. Weitzman (2001) shows the effective social discount rate decreases over time even when the distribution of correct discount rate is gamma-distributed. Karp (2005) studies the effect of non-constant discounting in the context of global warming. He shows that there may be multiple Markov perfect equilibria when the agent is a quasi-hyperbolic discounter. Fujii and Karp (2006) provide a numerical method to find a Markov perfect equilibrium under non-constant discounting and illustrate a numerical example of renewable resource management.

This study is distinguished from previous studies in two points. First, we offer a new way of handling myopia. Our model, therefore, can be deemed as an alternative to non-constant discounting models. It is a parsimonious extension of the standard dynamic optimization problem and can be numerically solved in a manner somewhat similar to the standard problems. While we focus only on environmental and resource management problem, our model is sufficiently general to be applicable to other problems involving intertemporal trade-offs. One such application is savings and investment decisions.

In our model, the process of planning future controls is strictly distinguished from the process of choosing the current control. We assume that every agent shares a constant *rational* discount rate $\gamma \in \mathbb{R}_{++}$, which reflects the long-run rate of time preference. A rational agent uses γ for both planning future controls and choosing the current control. In contrast, a myopic agent uses *myopic* discount rate $\rho (\geq \gamma)$ for choosing the current control.

Second, most of the studies on non-constant discounting are done in discrete time setting, whereas we use a continuous time setting. One notable exception for this is Karp (2006). We chose continuous time setting because this formulation allows us to distinguish more clearly the processes of planning controls and taking the current control. We also believe that environmental and resource management problem in many instances can be more realistically modelled in a continuous time setting.

In the next section, we describe our model. We start with a rational (standard) agent and then introduce two extreme types of myopic agents. One extreme is the *completely naïve* agent who is not aware of her myopia at all, and the other extreme is the *completely sophisticated* agent who is fully aware of her myopia. We then consider a generalized agent that includes rational, completely naïve and completely sophisticated agents as a special case. We derive some useful differential equations to find the control for the generalized agent. We also explain the algorithm for finding the solution.

In Section 3, we apply our model to a standard non-renewable extraction problem and derive analytical results when the terminal time is free as well as when it is fixed. We show that the consequences of myopia may critically depend on whether the planning period is fixed or free. This distinction is particularly important in resource management because permits, whether is for resource extraction or emission of pollutants, may or may not have an expiry. In Section 4, we provide a numerical example of the non-renewable extraction problem to illustrate interesting behavioral features that myopic agents manifest. We also analyze the consequences of the rationality assumption, an assumption that is often simply made in applied analysis without sufficient scrutiny. Section 5 provides discussion and conclusion.

2 The model

We start with a standard dynamic optimization problem. Let us denote the state space by $\mathcal{S}(\subset \mathbb{R})$ and the control space by $\mathcal{X}(\subset \mathbb{R})$. The stock variable may be, for example, the amount of pollutants in the air or the amount of oil in the field. Examples for the control variables include the emission of pollutants and the amount of resource extraction. The calender time starts at $t = 0$ and the initial stock is $S_0(\in \mathcal{S})$. The terminal time T may be fixed or free, but let us for now fix T. We denote the planning time horizon by $\mathcal{T} \equiv [0, T]$.

Let the stock and control at time be $t \in \mathcal{T}$ be $S(t) \in \mathcal{S}$ and $x(t) \in \mathcal{X}$. The transition function $g : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$ gives the derivative of stock with respect to time. We denote the instantaneous payoff function by $f : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$. Further, suppose in addition that there may be $N(< \infty)$ inequality constraints on the stock and control variables so that we must have $h^i(x(t), S(t)) \geq 0$ for $\forall i \in \mathcal{N}(\equiv \{1, \ldots, N\})$ and $\forall t \in \mathcal{T}$ where $h^i : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$. We shall use the subscript to denote partial derivatives (*e.g.* $f_S(x, S) \equiv \frac{\partial f(x,S)}{\partial S}$). For notational convenience, we use $\mathcal{H}_{t_1}^{t_2}$ to mean the constraints $\{h^i(x(t), S(t)) \geq 0, \forall t \in [t_1, t_2], \forall i \in \mathcal{N}\}$. We also use $\mathcal{G}_{t_1}^{t_2}$ for the transition equation $\{\dot{S}(t) = g(x(t), S(t)), \forall t \in [t_1, t_2)\}$ and $\mathcal{I}_\tau$ for the initial condition $\{S(\tau) = S_\tau\}$.

At time $\tau \in \mathcal{T}$, the agent takes the value of stock variable S_τ at time τ as given. The agent does not have a commitment technology and thus the agent's decision binds only the current control. Suppose that she is concerned about her "lifetime payoff" at each point in time, which is expressed in terms of the present-discounted value at the time of decision-making. Her lifetime payoff has two components, one coming from the stream of payoffs until the terminal time T and the other from leaving the stock $S(T)$ at time T. Then, we can write her current lifetime payoff

$U : \mathcal{X}^{\mathcal{T}} \times \mathcal{T} \times \mathcal{S} \times \mathbb{R}_{++} \to \mathbb{R}$ as a functional of x in the following way:

$$U(x, \tau, S_\tau, \gamma) \equiv \left(\int_\tau^T f(x(t), S(t)) e^{-\gamma(t-\tau)} \mathrm{d}t + \phi(S(T)) e^{-\gamma(T-\tau)} \right)$$

, where $\phi : \mathcal{S} \to \mathbb{R}$ is the salvage function and γ is the *rational* discount rate that reflects the agent's rate of time preference in the absence of myopia. Once can think of γ as the long-run rate of time preference. This is the discount rate used by rational agents for planning future controls and choosing the current control. We shall call the current lifetime payoff evaluated at $t = 0$ the total lifetime payoff. We make the following assumptions:

A1 f, g, h and ϕ are twice continuously differentiable,

A2 $g_x(x, S) \neq 0$ for any (x, S),

A3 For any $S \in \mathcal{S}$, $\mathcal{C}(S) \equiv \{\xi | h^i(\xi, S) \geq 0, \forall i \in \mathcal{N}\}$ is a compact set.

A1 is the smoothness assumptions we need for this analysis. Many environmental and resource management problems have this property. We allow for the possibility of discontinuity in the control, which we believe is more relevant in environmental and resource management problems. A2 means that the control always has some impact on the stock. A3 states that the set of permissible control for any given stock variable is convex and bounded. This assumption is clearly inappropriate when the control variable is discrete. Otherwise, A3 should also be a reasonable assumption because the control variable of most environmental and resource management is usually chosen from a continuous interval.

Now, let us consider the behavior of the rational agent at calendar time τ. She wants to maximize her current lifetime payoff subject to relevant constraints. Therefore, a rational agent plans her control $x^R(t; \tau, S_\tau, \gamma)$ for $t \in [\tau, T]$ by solving the following optimization problem:

$$\max_{\{x(t)\}_{t \in [\tau, T]}} U(x, \tau, S_\tau, \gamma) \quad \text{s.t. } \mathcal{I}_\tau, \ \mathcal{G}_\tau^T, \ \mathcal{H}_\tau^T \tag{1}$$

This is the standard dynamic optimization problem. So, letting the costate variable be $\lambda(t) \in \mathbb{R}$, we can define the present-value Hamiltonian $H : \mathcal{T} \times \mathcal{X} \times \mathcal{S} \times \mathbb{R} \to \mathbb{R}$ as follows:

$$H(t, x(t), S(t), \lambda(t)) \equiv f(x(t), S(t))e^{-\gamma t} + \lambda(t)g(x(t), S(t))$$

Letting $\mu_i : \mathcal{T} \to \mathbb{R}_+$ for $\forall i \in \mathcal{N}$ be the Lagrange multiplier and $\text{Ind}(\cdot)$ be the indicator function, we form the Lagrangian $L : \mathcal{T} \times \mathcal{X} \times \mathcal{S} \times \mathbb{R} \times \mathbb{R}_+^N \to \mathbb{R}$ as follows:

$$L(t, x(t), S(t), \lambda(t), \{\mu_i(t)\}_{i\in\mathcal{N}}) \equiv H(t, x(t), S(t), \lambda(t)) + \sum_{i=1}^{N} \mu_i(t)h^i(x(t), S(t)) + \text{Ind}(t = T)\phi(S(T)).$$

Suppose that $x^*(t)$ for $t \in [\tau, T]$ is the maximizing argument in Eq(1), and let its associated state trajectory be $S^*(t)$. Then, there must exist λ and $\{\mu_i(t)\}_{i\in\mathcal{N}}$ that satisfy the following conditions:

C1 $\dot{\lambda}(t) = -L_S(t, x^*(t), S^*(t), \lambda(t), \{\mu_i(t)\}_{i\in\mathcal{N}})$,

C2 $\lambda(T) = \sum_{i=1}^{N} \mu_i(T)h_S^i(x^*(T), S^*(T)) + \phi_S(S^*(T))$ for $\forall i \in \mathcal{N}$,

C3 $L_x(t, x^*(t), S^*(t), \lambda(t), \{\mu_i(t)\}_{i\in\mathcal{N}}) = 0$ for $\forall t \in [\tau, T]$,

C4 $H(t, x^*(t), S^*(t), \lambda(t)) \geq H(t, x(t), S^*(t), \lambda(t))$ at each $t \in [\tau, T]$ satisfying $h^i(x(t), S^*(t)) \geq 0$ for $\forall i \in \{1, \ldots N\}$,

C5 $S^*(\tau) = S_\tau$

C6 $\dot{S}^*(t) = g(x^*(t), S^*(t))$ for $\forall t \in [\tau, T)$

C7 $h(x^*(t), S^*(t)) \geq 0$ for $\forall i \in \mathcal{N}$ and $t \in [\tau, T]$

C1-C7 are just a variant of the standard results found in books on dynamic optimization (*e.g.* Chiang (1992); Kamien and Schwartz (1991); Seierstad and Sydsaeter (1987)). C1 is the costate equation. C2 is the transversality equation. C3 is the complementary-slackness condition for the inequality constraints. C4 is the optimality condition. C5, C6 and C7 are the initial condition $\mathcal{I}_\tau$, the transition equation $\mathcal{G}_\tau^T$ and the constraint $\mathcal{H}_\tau^T$ respectively. The triple $\{x^*(t), S^*(t), \lambda(t)\}$ that satisfy C1-C7 may not be unique in general. Also, there may be some points in the planning horizon when the set of binding constraints changes. At such points, λ and x^* may not be continuous and

$U(x^*, \tau, S_\tau, \gamma)$ may not be differentiable with respect to S_τ. In what follows, we assume the following two conditions.

A4 There is a set of $(0 \leq)K(< \infty)$ distinct points $\mathcal{J} \equiv \{j_1, \ldots, j_K\} \subset [\tau, \mathcal{T}]$ in time such that for $\forall l \in \{1, \ldots, K\}$ and $\forall \epsilon > 0$, there exists $\delta_{l,\epsilon} \in \mathbb{R}$ such that $\mathcal{B}(j_l) \neq \mathcal{B}(j_l + \delta_{l,\epsilon})$ and $|\delta_{l,\epsilon}| < \epsilon$, where $\mathcal{B}(t) \equiv \{i \in \mathcal{N} : h_i(x(t), S(t)) = 0\}$ is the set of binding constraints.

A5 There exists a unique triple $\{x^*(t), S^*(t), \lambda(t)\}$ that solve the maximization problem Eq(1) and satisfy C1-C7.

A4 states that there are at most a finite number of entry times, exit times and contact times at which the set of boundary conditions just changes. We allow $\mathcal{J}$ and $\mathcal{B}(\cdot)$ to be empty. Oniki (1973) showed that differentiability condition A1 and the uniqueness condition A5 are sufficient for $\mathcal{J}$ to be uniquely defined. One well-known condition that guarantees A5 is given by Arrow and Kurz (1970); A5 is satisfied when the maximized Hamiltonian H^* defined below is concave in $S(t)$:

$$H^*(t, S(t), \lambda(t)) \equiv \max_x H \quad \text{s.t.} \quad h^i(x(t), S(t)) \geq 0 \text{ for } \forall i \in \mathcal{N}$$

Under A5, the control schedule for a rational agent $x^R(t; \tau, S_\tau, \gamma) = x^*(t)$ uniquely exists and satisfy C1-C7. The control for a rational agent for time t planned at time τ is exactly the same as the control actually chosen at time t even in the absence of commitment technology. That is, $x^R(t; t, S_t, \gamma) = x^R(t; \tau, S_\tau, \gamma)$ for $\forall t \geq \tau$. Hence, we can simply write $x^R(t, S_t, \gamma)$ to mean the actual control at time t. We can also write the lifetime payoff for a rational agent as $V^R(\tau, S_\tau, \gamma) \equiv U(x^R(t; \tau, S_\tau, \gamma), \tau, S_\tau, \gamma)$. Oniki (1973) showed that $A1$ and $A5$ are sufficient for the following condition:

C8 $S(t)$, $\dot{S}(t)$, $\lambda(t)$ and $\dot{\lambda}(t)$ are continuous on $[\tau, T]$ and continuously differentiable on $[\tau, T] \setminus \mathcal{J}$.

We can now turn to the behavior of a myopic agent. A myopic agent uses the myopic discount rate ρ for choosing the current control. Let us first consider an extreme case, where the agent is completely naïve so that she is unaware that her decision is influenced by myopia. As a result, she makes plans for future control believing that she will behave like a rational agent in the future.

A completely naïve agent knows what she *should* do to behave like a rational agent. Because she evaluates the value of her control schedule using ρ instead of γ over the planning horizon, the control a myopic agent chooses may differ from the one chosen by a rational agent. This model is suitable for describing the behavior of the type of agents who have psychological drive for instantaneous gratification and say "I knew what I should have done (but I didn't)."

Suppose now that τ is not a junction time. Then, there exists $\delta > 0$ such that for $\forall \epsilon \in [0, \delta]$, we have $B(\tau) = B(\tau + \epsilon)$. Let us consider a very short period of time $(0 <)\Delta_\tau (\leq \delta)$, during which time she can commit her control variable. We shall later let $\Delta_\tau \downarrow 0$, so she can commit only her current control. She will choose her current control x_τ by solving the following problem:

$$\begin{aligned}
& \max_{x_\tau} \left(\int_\tau^T f(x(t), S(t)) e^{-\rho(t-\tau)} \mathrm{d}t + \phi(S(T)) e^{-\rho(T-\tau)} \right) \\
& \quad \text{s.t. } \mathcal{I}_\tau,\ \mathcal{G}_\tau^T,\ \mathcal{H}_\tau^T,\ x(t) = \begin{cases} x_\tau & \text{for} \quad t \in [\tau, \tau + \Delta_\tau) \\ x^R(t, S(t), \gamma) & \text{for} \quad t \in [\tau + \Delta_\tau, T] \end{cases} \\
= \ & \max_{x_\tau} \left(\int_\tau^{\tau+\Delta\tau} f(x_\tau, S(t)) e^{-\rho(t-\tau)} \mathrm{d}t + e^{-\rho\Delta_\tau} U(x^R(t, S(t), \gamma), \tau + \Delta_\tau, S(\tau + \Delta_\tau), \rho) \right) \\
& \quad \text{s.t. } \mathcal{I}_\tau,\ \mathcal{G}_\tau^T,\ \mathcal{H}_\tau^T
\end{aligned} \tag{2}$$

Note that the control $x(t)$ may be discontinuous at $t = \tau + \Delta_\tau$. This discontinuity arises because the process of planning future controls is different from the process of choosing the current control for a brief period of time Δ_τ. Of course, the control $x(t)$ for time $t(> \tau)$ planned at time τ is in general different from the control actually chosen when time t comes. This formulation may first seem peculiar if this problem is considered within the standard framework of dynamic "optimization". However, if we distinguish the underlying processes of planning the future controls from that of choosing the current control, this formulation should make sense.

As $\Delta_\tau \downarrow 0$, the first term in the maximand of Eq(2) drops out. Hence, the maximized value of Eq(2) approaches to $W(\tau, S_\tau, \gamma, \rho) \equiv U(x^R(t; \tau, S_\tau, \gamma), \tau, S_\tau, \rho)$. We shall call W the value function. One should note that $W(\tau, S_\tau, \gamma, \gamma) = V^R(\tau, S_\tau, \gamma)$ trivially holds. Further, A1 and C8 imply that W is differentiable with respect to S_τ on $[\tau, \tau + \Delta_\tau]$ for small enough $\Delta_\tau (< \delta)$ and that W_S is continuous. Given these, we can use an argument similar to the derivation of the standard dynamic programming equation as follows:

$$
\begin{aligned}
& W(\tau, S_\tau, \gamma, \rho) \\
= \; & \lim_{\Delta_\tau \downarrow 0} \max_{x_\tau} \left(\int_\tau^{\tau+\Delta\tau} f(x_\tau, S(t)) e^{-\rho(t-\tau)} \mathrm{d}t + e^{-\rho\Delta_\tau} W(\tau+\Delta_\tau, S_{\tau+\Delta_\tau}, \gamma, \rho) \right) \\
& \quad \text{s.t. } \mathcal{I}_\tau,\ \mathcal{G}_\tau^T,\ \mathcal{H}_\tau^T \\
= \; & \lim_{\Delta_\tau \downarrow 0} \max_{x_\tau} \left(f(x_\tau, S_\tau)\Delta_\tau + W(\tau, S_\tau, \gamma, \rho) - \rho W(\tau, S_\tau, \gamma, \rho)\Delta_\tau + W_\tau(\tau, S_\tau, \gamma, \rho)\Delta_\tau \right. \\
& \quad \left. + W_S(\tau, S_\tau, \gamma, \rho) g(x_\tau, S_\tau)\Delta_\tau + O(\Delta_\tau^2) \right) \quad \text{s.t. } \mathcal{I}_\tau,\ \mathcal{G}_\tau^{\tau+\Delta_\tau},\ \mathcal{H}_\tau^{\tau+\Delta_\tau} \qquad (3)
\end{aligned}
$$

The constraints $\mathcal{G}_{\tau+\Delta_\tau}^T$ and $\mathcal{H}_{\tau+\Delta_\tau}^T$ are automatically satisfied by the construction of x^R. Subtracting W from both sides of Eq(3), dividing them by Δ_τ and letting $\Delta_\tau \downarrow 0$, we have

$$
\begin{aligned}
\rho W(\tau, S_\tau, \gamma, \rho) - W_\tau(\tau, S_\tau, \gamma, \rho) \; &= \; \max_{x_\tau} (f(x_\tau, S_\tau) + W_S(\tau, S_\tau, \gamma, \rho) g(x_\tau, S_\tau)) \\
& \quad \text{s.t.} \quad h^i(x_\tau, S_\tau) \geq 0 \quad \text{for} \quad \forall i \in \mathcal{N} \qquad (4)
\end{aligned}
$$

Given γ and ρ, we define the Lagrangian $L^N : \mathcal{T} \times \mathcal{X} \times \mathcal{S} \times \mathbb{R}_+^N \to \mathbb{R}$ for the completely naïve agent as

$$
\mathfrak{L}^N(\tau, x_\tau, S_\tau, \{\mu_i^N\}_{i \in \mathcal{N}}) = f(x_\tau, S_\tau) + W_S(\tau, S_\tau, \gamma, \rho) g(x_\tau, S_\tau) + \sum_{i=1}^N \mu_i^N h^i(x_\tau, S_\tau),
$$

where $\mu_i^N \in \mathbb{R}_+$ is the Lagrange multiplier associated with the i-th constraint. The first order conditions for the control of the myopic agent at time τ is:

N1 $\frac{\partial L}{\partial x_\tau} = f_x(x_\tau, S_\tau) + W_S(\tau, S_\tau, \gamma, \rho) g_x(x_\tau, S_\tau) + \sum_{i=1}^N \mu_i^N h_x^i(x_\tau, S_\tau) = 0$

N2 $\mu_i^N h^i(x_\tau, S_\tau) = 0, \quad \mu_i^N \geq 0, \quad h^i(x_\tau, S_\tau) \geq 0$

The solution to Eq(4) exists because of A3. However, the solution may not be unique. One sufficient condition of the uniqueness is that the maximand of Eq(4) is strictly concave in x_τ. Hereafter we require the following:

A6 Eq(4) has a unique solution x_τ^{**}

Then, the control for the completely naïve agent at time τ is unique. Her control schedule $\tilde{x}^N$

planned at time τ is as follows:

$$\tilde{x}^N(t;\tau,S_\tau,\gamma,\rho) = \begin{cases} x_\tau^{**} & \text{for} \quad t=\tau \\ x^R(t,S(t),\gamma,T) & \text{for} \quad t\in(\tau,T] \end{cases}$$

$\tilde{x}^N(t;\tau,S_\tau,\gamma,\rho)$ contains the scheduled control for the future, which is in general different from the actual control. As with $x^R(\tau,S_\tau,\gamma)$, we write the actual control for the naïve agent at time τ by $x^N(\tau,S_\tau,\gamma,\rho)$. However, unlike the case of the rational agent, $\tilde{x}^N(\tau,S_\tau,\gamma,\rho)$ is in general different from $x^N(\tau;t,S_t,\gamma,\rho)$ for $t<\tau$.

When there is no binding constraint, the first order condition reduces to

$$f_x(x_\tau,S_\tau) + W_S(\tau,S_\tau,\gamma,\rho)g_x(x_\tau,S_\tau) = 0 \tag{5}$$

and the second order condition in this case can be written in terms of the partial derivatives with respect to x as follows: $SOC = f_{xx}(x_\tau,S_\tau) - \frac{f_x(x_\tau,S_\tau)}{g_x(x_\tau,S_\tau)}g_{xx}(x_\tau,S_\tau)$. When an interior solution is expected, we can simply use Eq(5) and check that the solution is indeed interior.

Eq(5) has a familiar form and permits the familiar interpretation; marginal gains from increasing one unit of control is equal to the opportunity cost of present-discounted future payoffs via changes in the stock. The difference is the presence of the myopic discount rate ρ. As with the standard setting, the myopic agent plans future control using the rational discount rate γ, because she believes she will behave like a rational agent. However, the computation of the present-discounted value of the flow of payoff generated from her control schedule, which is used for choosing the current control, is determined by the myopic discount rate ρ. As a result, the value function W has two discount rates in its arguments.

Several observations and cautions are in order. First, one should note that W is not the current lifetime payoff of the agent. W is the present-discounted value of the stream of payoffs used to determine the current control of myopic agents. The current lifetime payoff of the naïve agent is $U(x^N(t,S_t,\gamma,\rho),\tau,S_\tau,\gamma)$. Second, when $\Delta_\tau \downarrow 0$, the agent can make a binding decision only for the current control. Third, because the agent's plan is binding only for the current control, it has no first-order effect on S, W or U. This point is an important feature of the continuous-time model, because the current control would have a first-order impact on these variables in an analogous discrete-time model.

Fourth, the future plans for the rational and completely naïve agents are identical. However, the control for the rational agent x^R is different from that for the naïve agent x^N because they discount the same stream of future payoffs in different ways using different discount rates. Fifth, the naïve agent with $\rho = \gamma$ coincides with the rational agent, and thus $x^R(\tau, S_\tau, \gamma) = x^N(\tau, S_\tau, \gamma, \gamma)$. Sixth, the above discussion holds for autonomous problems without a major modification. One notable difference is that the arguments τ in x, W and U will be unnecessary and the salvage value function disappears from the equations.

Seventh, when $\rho = \gamma$, $N1$ is equivalent to $C4$ for time τ because $W_S(\tau, S_\tau, \gamma, \gamma)$ is the shadow value of the stock variable, which is exactly what $\lambda(\tau)$ means. Notice that $N1$ is a weaker condition than $C4$ because it is a static condition. This is because the future control schedule of the completely naïve agent is implied in the definition of W.

We have assumed so far that the completely naïve agent is not aware of her myopia at all. This assumption may be too strong to represent a myopic agent in reality, because many of myopic people seem aware that they tend to behave myopically. Let us consider another extreme in which the agent knows exactly how she will behave in the future. Such a completely sophisticated agent would in principle take its future behavior as given, realizing that the future action is based on the *myopic* discount factor ρ.

Hence, such a completely sophisticated agent would use ρ for planning her future control and for choosing the current control. In effect, her mode of behavior is the same as that of a rational agent except that the discount rate to be used is ρ instead of γ. This means that completely sophisticated agents and rational agents are observationally equivalent. The control x^S of completely sophisticated agent is $x^R(\tau, S_\tau, \rho)$. Notice that x^S does not have γ in its argument. The lifetime payoff of the sophisticated agent, of course, still depends on γ and is written as $U(x^R(\tau, S_\tau, \rho), \tau, S_\tau, \gamma)$.

Both completely naïve and sophisticated agents take future behavioral patterns as given. The difference is the discount rate that is used to plan their future control. The completely naïve agent incorrectly assume that she would be using γ in the future for choosing her future controls. On the other hand, the completely sophisticated agent correctly uses ρ for that purpose. At this point, it would be natural to consider a generalized myopic agent who uses the *myopic* discount rate ρ for choosing the current control, but the *planning* discount rate $\hat{\rho} \in [\gamma, \rho]$ for planning her future controls. The rational, completely naïve and completely sophisticated agents are subsumed into the generalized agent; a rational agent is the generalized agent with $\gamma = \rho = \hat{\rho}$, because she correctly predict her discount rate used for planning her future control, which corresponds to her rational discount rate. Similarly, the completely naïve agent corresponds to the one with $\gamma = \hat{\rho}$. For the completely sophisticated agent, we have $\rho = \hat{\rho}$.

The current control chosen by the generalized agent is based on the value function $W(\tau, S_\tau, \hat{\rho}, \rho)$. Thus, the maximization problem that the generalized agent uses to find her control is obtained by replacing γ by $\hat{\rho}$ in Eq(3). So far, we have assumed τ is not a junction time. Now let us see what happens if τ is a junction time. Because W is continuous on $[\tau, T]$ and continuously differentiable in S_τ on $[\tau, T] \setminus \mathcal{J}$, W has a right-hand derivative W_S^+ and a left-hand derivative W_S^-. Therefore, Eq(3) holds even when τ is a junction point once W_S is replaced by a one-sided derivative. We can use $W_S^+ (W_S^-)$ when $S_{\tau+\Delta\tau}$ approaches S_τ from above (below). Thus, when $g(x_\tau, S_\tau)$ is strictly positive (negative), we can replace W_S in Eq(3) by $W_S^+(W_S^-)$. When $g(x_\tau, S_\tau) = 0$, the term $W_s \cdot g \cdot \Delta_\tau$ drops out as $\Delta_\tau \downarrow 0$.

Therefore, now we can re-rewrite Eq(4) for a generalized agent as follows:

$$
\begin{aligned}
& \rho W(\tau, S_\tau, \hat{\rho}, \rho) - W_\tau(\tau, S_\tau, \hat{\rho}, \rho) \\
= \quad & \max_{x_\tau} \big[f(x_\tau, S_\tau) + (W_S^+(\tau, S_\tau, \hat{\rho}, \rho) \cdot \text{Ind}(g(x_\tau, S_\tau) > 0) + \\
& \quad W_S^-(\tau, S_\tau, \hat{\rho}, \rho) \cdot \text{Ind}(g(x_\tau, S_\tau) < 0)) \cdot g(x_\tau, S_\tau) \big] \quad \text{s.t.} \quad h^i(x_\tau, S_\tau) \geq 0 \quad \text{for} \quad \forall i \in \mathcal{N} \qquad (6)
\end{aligned}
$$

When there is no binding constraint and τ is not a junction time, we have the following first order condition:

$$0 = f_x(x_\tau, S_\tau) + W_S(\tau, S_\tau, \hat{\rho}, \rho) g_x(x_\tau, S_\tau) \qquad (7)$$

By solving Eq(6), we can again find the control $x^G(\tau, S_\tau, \hat{\rho}, \rho)$ for the generalized agent for the entire planning horizon. Plugging this in the definition of lifetime payoff, we get the current lifetime payoff of the generalized agent $U(x^G(t, S_t, \hat{\rho}, \rho), \tau, S_\tau, \gamma)$.

Finding an analytic solution for this problem is straightforward in principle especially when there is no binding constraint. We can first find the control for the rational agent whose rational discount rate is $\hat{\rho}$. Then we can calculate the value function W and solve the first order condition Eq(7). By plugging in the transition function the control expressed as a function of the stock and time, we obtain a differential equation for the stock. Solving this, we find the time-evolution of the stock, which in turn allows us to find the time-evolution of the control.

It is, however, often the case that analytic solution does not exist. In this case, we need to discretize the time and find the solution numerically. When we solve a non-autonomous problem, we can simply simulate the time evolution of stock and control in the following manner. First, we identify (an approximant of) the control for each state for the completely sophisticated agent whose planning discount rate is $\hat{\rho}$ using the standard numerical method. Second, we c numerically calculate the value function W for the myopic agent at $t = 0$. Third, we find the numerical derivative of W with respect to the (initial) stock. Fourth, we solve for the control at time $t = 0$ by plugging W_S in Eq(7). Fifth, substituting the initial control in the transition function, we calculate the new stock after a brief period of time. Sixth, taking the new stock as the initial stock, we advance the time by the time step. Repeating the same process until T, we can find the time-evolution of the stock and control.

So far, we have taken T as fixed. Now, suppose that T is free and there is a terminal condition for the management problem. For example, the agent's resource management problem disappears once the resource is exhausted (*i.e.* $S(T) = 0$). Even in this case, our discussion holds only with a minor modification. The Lagrangian now includes the term for the terminal condition and, T is now a choice variable. As we have seen, the agent finds the future plans. Hence, when T is free, she has to predict (forecast) the terminal time T. Rational and sophisticated agents correctly predict their future behavior, and thus T is also correctly predicted. In general, however, the agent's prediction $\tilde{T}$ may be different from the end of T. $\tilde{T}$ depends on $\hat{\rho}$, τ and S_τ, whereas T depends on $\hat{\rho}$, ρ and S_0. The decision-making is based on $\tilde{T}$ and not T at time τ. $\tilde{T}$ should be used instead of T where T is required for the fixed T. The argument we used to derive Eq(6), however, is still valid because the current control has no first-order effect on the stock variable and thus no first-order effect on $\tilde{T}$.

3 Application to Non-renewable Resource Extraction

As an illustration, let us consider a simple non-renewable resource extraction problem first considered by Hotelling (1931). We use this example because it is a well-known problem and analytically tractable. Further, non-renewable resource may be exploited in a fixed T or free T environment. So, the analysis we present here has some relevance to resource management problems in the real world.

Suppose that an agent in the public sector is responsible for managing non-renewable resource. She wants to supply the resource to the market over time to maximize the social surplus. To simplify the problem, we assume the marginal cost of extraction is constant at $c(\geq 0)$, and the agent observes the stock at each point in time. She faces a linear demand curve $d_0 - b \cdot (p(t) + c)$, where $p(t)(\geq 0)$ denotes the economic rent on one unit of the resource at time t, and d_0, b and c are positive constants. The agent chooses a non-negative amount of extraction $x(t)$ at each point in time. We also assume that there is no salvage value of the mine so that $\phi(\cdot) = 0$.

The instantaneous payoff of the agent, or the social surplus at time t, is $f(x(t)) = \frac{2ax(t) - x(t)^2}{2b}$, where $a \equiv d_0 - bc$ is assumed to be positive. The stock is depleted by the amount extracted so that $g(x(t)) = -x(t)$, where th control must satisfy $0 \leq x(t) \leq a$ by the non-negativity assumptions for the control and price. Further, we require the stock variable is always nonnegative so that $S(t) \geq 0$. In what follows, we only consider the cases where these constraints are not binding except for the end of the planning horizon. When these constraints are not binding, the stock must be scarce so that $S_0 < aT$. Further, the stock must be depleted at the end of the planning horizon so that the terminal condition is $S(T) = 0$. Note that f and g are independent of the stock level in this problem.

As with the previous section, we first let T be fixed and consider a rational agent. The problem that the rational agent is facing is a standard non-renewable resource extraction problem. That is, she wants to solve the following problem.

$$V^R(\tau, S_\tau, \gamma) \equiv \max_{x(t)} \int_\tau^T \frac{2ax(t) - x(t)^2}{2b} e^{-\gamma(t-\tau)} \mathrm{d}t \quad \text{s.t.} \;\; \dot{S} = -x(t), \; S(\tau) = S_\tau \tag{8}$$

So, we can use the standard technique of optimal control to solve the problem. The solution is as follows:

$$\begin{aligned} x^R(t; \tau, S_\tau \gamma) &= a - \frac{\gamma(a(T-\tau) - S_\tau)}{e^{\gamma(T-\tau)} - 1} e^{\gamma(t-\tau)} \\ V^R(\tau, S_\tau, \gamma) &= \frac{1}{2b\gamma} \left(a^2(1 - e^{-\gamma(T-\tau)}) - \frac{\gamma^2(a(T-\tau) - S_\tau)^2}{e^{\gamma(T-\tau)} - 1} \right) \end{aligned} \tag{9}$$

Now, let us consider the problem of the generalized agent with the rational discount rate γ, myopic discount rate ρ and planning discount rate $\hat{\rho}$. Then, her planned control at time τwill be $x^R(t; \tau, S_\tau, \gamma)$. Hereafter, we assume that $\rho \neq \hat{\rho}$ to keep our presentation simple. However, the results we present holds for $\rho \neq \hat{\rho}$ as a limiting case. The decision of the generalized agent is based on the following present-discounted value of the stream of future payoff evaluated under the myopic discount rate:

$$\begin{aligned} W(\tau, S_\tau, \hat{\rho}, \rho) &= \int_\tau^T f(x^R(t; \tau, S_\tau, \hat{\rho})) e^{-\rho(t-\tau)} \mathrm{d}t \\ &= \frac{1}{2b} \left(\frac{a^2(1 - e^{-\rho(T-\tau)})}{\rho} + \frac{\hat{\rho}^2(a(T-\tau) - S_\tau)^2(1 - e^{(2\hat{\rho}-\rho)(T-\tau)})}{(e^{\hat{\rho}(T-\tau)} - 1)^2(2\hat{\rho} - \rho)} \right) \end{aligned}$$

One can easily verify that $V^R = W$ when $\gamma = \rho = \hat{\rho}$. The first order condition Eq (7) gives us $\frac{a-x}{b} = W_S(\tau, S_\tau, \hat{\rho}, \rho)$. Hence, taking the partial derivative of W with respect to S and plugging the previous equation in, we obtain

$$x^G(\tau, S_\tau, \hat{\rho}, \rho) = a - \frac{\hat{\rho}^2(1 - e^{(2\hat{\rho}-\rho)(T-\tau)})(S_\tau - a(T-\tau))}{(2\hat{\rho} - \rho)(e^{\hat{\rho}(T-\tau)} - 1)^2} \tag{10}$$

By the transition equation, we know that $\frac{\mathrm{d}S(\tau)}{\mathrm{d}\tau} = -x^G(\tau, S(\tau), \hat{\rho}, \rho)$. This differential equation can be solved analytically, and we can express the stock variable as a function of t, S_0, $\hat{\rho}$ and ρ. The sketch of the derivation of the analytic solution is given in the Appendix. Plugging the stock variable back in Eq(10), we obtain the control as a function of time t, S_0, $\hat{\rho}$ and ρ, which in turn allows us to calculate the total lifetime utility $U(x^G(t, S_t, \hat{\rho}, \rho), 0, S_0, \gamma)$.

Now, let T be free and consider again a rational agent. In this case, T depends on her rational discount rate and the initial stock. Further, at each point in time, the remaining time until the end of the planning horizon predicted by a rational agent depends only on the rational discount rate and the remaining stock. Let the remaining time predicted by a rational agent be $r : \mathcal{S} \times \mathbb{R}_+ \to \mathbb{R}$. r does not have τ in its argument because the predicted remaining time does not depend on the calendar time at which the prediction is made. For a rational agent, the predicted remaining time is the same as the actual remaining time so that we have $r(S_\tau, \gamma) + \tau = T$ for $\forall \tau \in \mathcal{T}$. Noting that the Hamiltonian evaluated at time T along the optimal solution for Eq(8) is zero, we can use the standard dynamic optimization technique to obtain the following results:

$$
\begin{aligned}
x^R(t; S_\tau, \gamma) &= a(1 - e^{-\gamma(r(S_\tau,\gamma)-t)}) \\
V^R(S_\tau, \gamma) &= \frac{a^2}{2b\gamma}(1 - e^{-\gamma r(S_\tau,\gamma)})^2
\end{aligned}
\tag{11}
$$

,where $r(S_\tau, \gamma)$ satisfies

$$
S_\tau = ar(S_\tau, \gamma) - \frac{a}{\gamma}(1 - e^{-\gamma r(S_\tau,\gamma)}). \tag{12}
$$

Now, let us consider a generalized agent. Because a completely sophisticated agent is observationally equivalent to the rational agent with the rational discount rate of γ, we can just plug x^R in the definition of W to find its value:

$$
W(S_\tau, \hat{\rho}, \rho) = \frac{a^2}{2b} \cdot \left(\frac{1 - e^{-\rho r(S_\tau,\hat{\rho})}}{\rho} + \frac{e^{-2\hat{\rho} r(S_\tau,\hat{\rho})} - e^{-\rho r(S_\tau,\hat{\rho})}}{2\hat{\rho} - \rho} \right)
$$

Hence, using the first order condition Eq (7), we obtain

$$
x^G(S_\tau, \hat{\rho}, \rho) = a \left(1 - \frac{\hat{\rho}(e^{-\rho r(S_\tau,\hat{\rho})} - e^{-2\hat{\rho} r(S_\tau,\hat{\rho})})}{(2\hat{\rho} - \rho)(1 - e^{-\hat{\rho} r(S_\tau,\hat{\rho})})} \right) \tag{13}
$$

Differentiating Eq(12) by τ and equating it to Eq(13) yield a differential equation with respect to

r. This differential equation can be solved analytically up to a constant term. However, we cannot fully solve the equation analytically because we cannot find $r(S_0, \hat{\rho})$ analytically. It is obvious that the time evolution of the stock variable can be calculated as follows:

$$S(t, S_0, \rho, \hat{\rho}) = S_0 - \int_0^t x^G(S_\tau, \hat{\rho}, \rho) \mathrm{d}\tau$$
$$\text{s.t.} \quad S(0) = S_0,\ \dot{S}(\tau) = -x^G(S_\tau, \hat{\rho}, \rho) \tag{14}$$

It should be reminded that r is the predicted remaining time horizon when a completely sophisticated agent with the same planning discount rate as the generalized agent has the remaining resource of S_τ at time τ. The actual terminal time $T(S_0, \hat{\rho}, \rho)$ at which the agent finish the extracting the resource can be found by solving for T in $S(T, S_0, \rho, \hat{\rho}) = 0$.

4 Numerical Example

By changing the units in which the price and stock are expressed, we can set $a = b = 1$ without loss of generality. $\frac{S_0}{a}$ is the time that the resource lasts when the resource is open access. We set $S_0 = 9$ in this example. The rational discount rate is set at $\gamma = 0.01$ and we take 0.07, 0.20 and 0.50 for the values of the myopic discount rate ρ to see the effect of various degrees of myopia. In addition to the rational, completely naïve and completely sophisticated agents, we also consider partially naïve (and partially sophisticated) agents whose planning discount rate is $\hat{\rho} \in (\gamma, \rho)$. We call the unit of time a year. For the fixed T case, we let T be 10.00 years.

To compute the total lifetime payoff $U(x^G(0, S_0, \hat{\rho}, \rho); 0, S_0, \gamma)$, we first discretized the time, and then computed the stock and control at each point in time starting from $t = 0$. We computed the time-evolution of the stock using the fourth-order Runge-Kutta method for each type of individual. Then, we evaluated the flow of payoff in the middle of the time step and summed over the entire planning horizon. We set the time step to be small enough so that we have sufficiently accurate numbers.[1]

[1] The time step we used was at most 10^{-6} years for all the cases we considered. We compared the calculated figures with the analytic solution whenever possible and the comparison suggests that the reported figures are sufficiently accurate.

In Table 1, we report the total lifetime payoff for various agents. In the upper row of each cell, we give the label for each agent. For example, the agent with $\rho = 0.20$ and $\hat{\rho} = 0.07$ will be referred to as P1 agent. We use the suffix R, N, S and P for rational, completely naïve, completely sophisticated and partially naïve (and partially sophisticated) agents. In the lower row of each cell, the total lifetime payoffs for the fixed T (left) and free T (right) are reported. For example, P1's lifetime payoff is 2.2267 when T is fixed and 2.0788 when T is free.

[Table 1 about here.]

There are two points worth noting here. First, The agent's lifetime payoff depends on whether T is fixed or free. The difference is particularly large for the rational agent. R agent takes 45.651 years to deplete the resource when T is free, but she is forced to deplete the resource in 10 years when T is fixed. As a result, her lifetime payoff is much lower when T is fixed. It is, however, not always the case that the free T gives higher total lifetime payoff. For N3 agent, fixed T is better, because her myopia is so strong that she would deplete resources too quickly. As a result, she is better off by being forced to deplete the resource within a fixed amount of time.

Second, whether T is fixed or free also changes whether "knowing thyself" is a good thing. When T is fixed, naïve agents do better than sophisticated agents. For example, for agents with $\rho = 0.20$, N2 has the highest total lifetime payoff followed by P1 and S2 under the fixed T. When T is free, however, the converse is true. For agents with $\rho = 0.50$, the situation is not as simple. When T is fixed, N3 does better than P2, P3 and S3. On the other hand, when T is free, $P2$ has the highest payoff followed by N3, P3 and S3.

To understand this result, let us first consider a sophisticated agent under the fixed T environment. Her planned future controls are optimal under her myopic (and planning) discount rate $\rho(= \hat{\rho})$. Suppose that she had suddenly become naïve. Then, her new future planned controls x^N will be sub-optimal when it is evaluated with the myopic discount rate ρ. This makes the current marginal value of stock higher under ρ, which in turn makes the agent use less of the stock in the current period. This helps the naïve agent to extract the resources more smoothly over time, and as a result, increases the naïve agent's total lifetime payoff.

When T is free, the above argument may still hold. However, we also need to take into consideration the change in the planning horizon. When the agent is naïve, her predicted duration of extraction is much longer than that for the sophisticated agent with the same ρ. Because the rational agent extract resources very slowly, a marginal increase in stock in the current period results in a very small increase in extraction for the near future because extraction is averaged out over a long time horizon. This translates in a smaller current marginal value of stock under ρ. Hence, the naïve agent may extract more in the current period and decrease her total lifetime payoff.

Let us now look at the time evolution of control and stock variables for both cases. Figure 1 shows the time evolution of stock $S(t)$ for different types of agents when T is fixed. As the graph shows, the differences in the path $S(t)$ takes are small when T is free. This is not very surprising given that the planning period is relatively short, even though the discount rate used for each type of the agents substantially vary. Figure 2 shows the time evolution of $S(t)$ when T is free. We restricted the domain of the time for this graph to present the differences in control among the agents more clearly. Unlike the fixed T case, the graphs look quite different between the agents.

[Figure 1 about here.]

[Figure 2 about here.]

When we look at the control $x(t)$, the differences among agents are clearer. For the fixed T case, when ρ is close to γ, S1 and N1 agents behave in a similar manner. However, when ρ gets larger, their controls are strikingly different. N3 agents starts with less resource extraction as $S(t)$ does not decline as fast as S3 as shown in Figure 3.

When T is free, the controls for the naïve and sophisticated agents are quite different even when ρ is small as shown in Figure 4. The control for naïve agents declines much more rapidly than that for sophisticated agents. This is because of the time-inconsistency of naïve agents. That is, they think that they will extract slowly in the future and that they can extract more today. Indeed, N3's control starts below S3's control because of the optimism. N3 maintains $x(t)$ over 0.9 until just before the stock is depleted. The degree of myopia is so high that the graphs of $x(t)$ for S3 and N3 cross each other twice.

[Figure 3 about here.]

[Figure 4 about here.]

The time-inconsistency of myopic agents can be clearly seen in Figure 5. This shows the planned time of the termination of extraction at each given point in time against actual time. That is, the horizontal axis measures the actual time τ elapsed whereas the predicted terminal time of extraction $\tilde{T}(t, S(t), \hat{\rho}) \equiv t + r(S(t), \hat{\rho})$ depends on the time at which the prediction was made. When the graph hits the diagonal 45-degree line, the extraction ends. The rational and sophisticated agents are time-consistent, and thus have a horizontal graph of $\tilde{T}(t, S(t), \hat{\rho})$. On the other hand, $\tilde{T}$ for naïve agents declines over time because they extract resources faster than they planned. This is because they exhaust the resource faster than their plan. Table 2 gives the predicted extraction time $r(S_0, \rho)$ at time $t = 0$ (left) and actual duration of extraction (right).

[Figure 5 about here.]

[Table 2 about here.]

The results shown above indicate that the kind of myopia we considered cannot be simply assumed away. To further shed light on this point, let us look at the problem from a slightly different angle. Suppose that we had simply assumed that the agent is rational and let us ask ourselves what the consequences of such an assumption would be. To answer this question, let us suppose that we observed the control x_t^O and stock S_t^O at time t. Then, we can find out the observationally equivalent rational discount rate $\tilde{\gamma}(t)$ for time t that is consistent with these observations. That is, we can find $\tilde{\gamma}(t)$ such that $x_t^O = x^R(t, S_t^O, \tilde{\gamma}(t))$. One may expect that $\tilde{\gamma}(t)$ should be somewhere between γ and ρ for $\forall t \in \mathcal{T}$. This is indeed the case when T is fixed as shown in Figure 6. Surprisingly, however, this is not the case when T is free as shown in Figure 7. Indeed, the $\tilde{\gamma}(t)$ may be much higher than ρ towards the end of the extraction period.

[Figure 6 about here.]

[Figure 7 about here.]

5 Discussion and Conclusion

This study explored environmental and resource management under myopia. We explicitly included the process of planning future controls and choosing the current control while keeping the model being parsimonious. Despite the simplicity of the model, our model enables rich description of myopic agents. While our primary focus was on environmental and resource management, the model is general enough to be applicable to other economic issues.

The results we obtained have several features that did not appear in other models. First, because we can explicitly deal with time-inconsistency by distinguishing the planning future controls and choosing the current control, we can also see the time-evolution of variables related to planning in a continuous time setting. In particular, we offer a way to handle both the predicted and actual terminal time of extraction as shown in Figure 5. Second, our model does not in general suffer from the kind of multiplicity of the controls noted in Karp (2006), where there are multiple solutions associated with a range of steady states.

The simple problem of non-renewable resource extraction we considered offers some intriguing lessons. First, "knowing thyself" does not always pay. We saw that, when T is fixed, naïve agents do better than the sophisticated ones. Naïve agents rather optimistically think that they will behave like a rational agent, and their optimism realizes itself. That is, their optimism about the future unintentionally made them to commit to conserve the resource and smooth out the extraction over time. Indeed, "knowing thyself" is actually harmful when T is fixed as bad future prospect realizes itself.

On the other hand, when T is free, "knowing thyself" can be good. One may still benefit from optimism, but it is not always the case. When the agent is naïve and thinks that she behave like a rational agent, her evaluation of future payoffs under the myopic discount rate ρ is low because it involves far distant future. Marginal gains from increasing the current control relative to the opportunity cost become much higher when T is free. As a result, naïve agents tend to extract resources faster than sophisticated agents.

From the analysis of the consequences of rationality assumption we find that there is no observationally equivalent rational discount rate $\tilde{\gamma}$ for the generalized agent in general. Thus, we cannot simply assume away the myopia. One surprising result is that, while $\tilde{\gamma}$ is between γ and ρ when T is fixed, it is not the case when T is free. So, even when the difference between the myopic discount rate and the rational discount rate is very small, we cannot ignore the myopia.

The results we obtained also have some policy relevance. Suppose, for example, that the government is trying to sell resource extraction permits to agents who may be myopic. The government has an option sell the permit with or without expiration. Which option is better would depend on the degree of myopia of the agents. Table 1 is useful for this purpose. By comparing the total lifetime utility, we can see which option is more desirable for which agents. It should be obvious that free T is desirable for the rational agent. It turns out that this is the case for all the sophisticated agents but not others. This suggests that, when it is likely that the agents are more or less ignorant of their own myopia, the government should set the expiry. The logic is somewhat similar to the theory of second best; the government may be able to improve the social welfare by imposing a regulation when there is a pre-existing "distortion". Such a distortion includes myopia.

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Appendix

In this appendix, we shall consider the time-evolution of the stock and control for the general agent using Eq(10) when T is fixed. We denote the time remaining for extraction at time τ by $r \equiv T - \tau$. Further, let us define $M \equiv ar - S(\tau)$, which is a measure of the scarcity of the stock. It is the difference between the amount of extraction when the resource is not scarce and the remaining stock at time τ. Using the equation of motion, we have $\frac{\mathrm{d}S(\tau)}{\mathrm{d}r} = -\frac{\mathrm{d}S(\tau)}{\mathrm{d}\tau} = x^G(\tau, S_\tau, \hat{\rho}, T, \rho)$. Then, subtracting a from and dividing by M both sides, Eq(10) can be written as follows:

$$\frac{\mathrm{d}M}{\mathrm{d}r} \cdot \frac{1}{M} = \frac{\hat{\rho}^2 (e^{(2\hat{\rho}-\rho)r} - 1)}{(2\hat{\rho} - \rho)(e^{\hat{\rho}r} - 1)^2}$$

Integrating over r and noting that $e^{-\hat{\rho}r} < 1$ for $r > 0$, we have

$$\begin{aligned}
& \log M \\
= \; & \frac{\hat{\rho}^2}{(2\hat{\rho}-\rho)} \int \frac{(e^{(2\hat{\rho}-\rho)r} - 1)}{(e^{\hat{\rho}r} - 1)^2} \mathrm{d}r \\
= \; & -\frac{e^{(2\hat{\rho}-\rho)r} - 1}{\hat{\rho}(e^{\hat{\rho}r} - 1)} - r - \log(e^{\hat{\rho}r} - 1) + \frac{(\hat{\rho}-\rho)}{\hat{\rho}} \int \frac{e^{(\hat{\rho}-\rho)r}}{1 - e^{-\hat{\rho}r}} \mathrm{d}r \\
= \; & -\frac{e^{(2\hat{\rho}-\rho)r} - 1}{\hat{\rho}(e^{\hat{\rho}r} - 1)} - r - \log(e^{\hat{\rho}r} - 1) + \frac{(\hat{\rho}-\rho)}{\hat{\rho}} \sum_{k=0}^{\infty} \frac{e^{((\hat{\rho}-\rho)-k\hat{\rho})r}}{(\hat{\rho}-\rho) - k\hat{\rho}} + A \\
= \; & -\frac{e^{(\hat{\rho}-\rho)r} - 1}{\hat{\rho}(e^{\hat{\rho}r} - 1)} - r - \log(e^{\hat{\rho}r} - 1) + \frac{(\hat{\rho}-\rho)e^{(2\hat{\rho}-\rho)}}{\hat{\rho}} F\left(1, \frac{\rho}{\hat{\rho}} - 2, \frac{\rho}{\hat{\rho}} - 1; e^{-\hat{\rho}r}\right) + A \qquad (15)
\end{aligned}$$

, where $A \in \mathbb{R}$ is a constant and $F(a_1, a_2, a_3; z)$ is a hypergeometric function defined as follows:

$$F(a_1, a_2, a_3; z) \equiv \sum_{k=0}^{\infty} \frac{\Pi_{l=0}^{k-1}(a_1 + l) \cdot \Pi_{l=1}^{k-1}(a_2 + l)}{\Pi_{l=1}^{k-1}(a_3 + l)} \cdot \frac{z^k}{k!}$$

Hence, letting $r = T$ (*i.e.* t=0), we have

$$A = \log(aT - S_0) + \frac{e^{(\hat{\rho}-\rho)T} - 1}{\hat{\rho}(e^{\hat{\rho}r} - 1)} + T + \log(e^{\hat{\rho}T} - 1) - \frac{(\hat{\rho}-\rho)e^{(2\hat{\rho}-\rho)}}{\hat{\rho}} F\left(1, \frac{\rho}{\hat{\rho}} - 2, \frac{\rho}{\hat{\rho}} - 1; e^{-\hat{\rho}T}\right)$$

Plugging this back in Eq(15) and solving for $S(\tau)$, we can write the stock as a function of S_0, τ, ρ and $\hat{\rho}$, which, in turn, allows us to write the control as a function of S_0, τ, ρ and $\hat{\rho}$ using Eq(10).

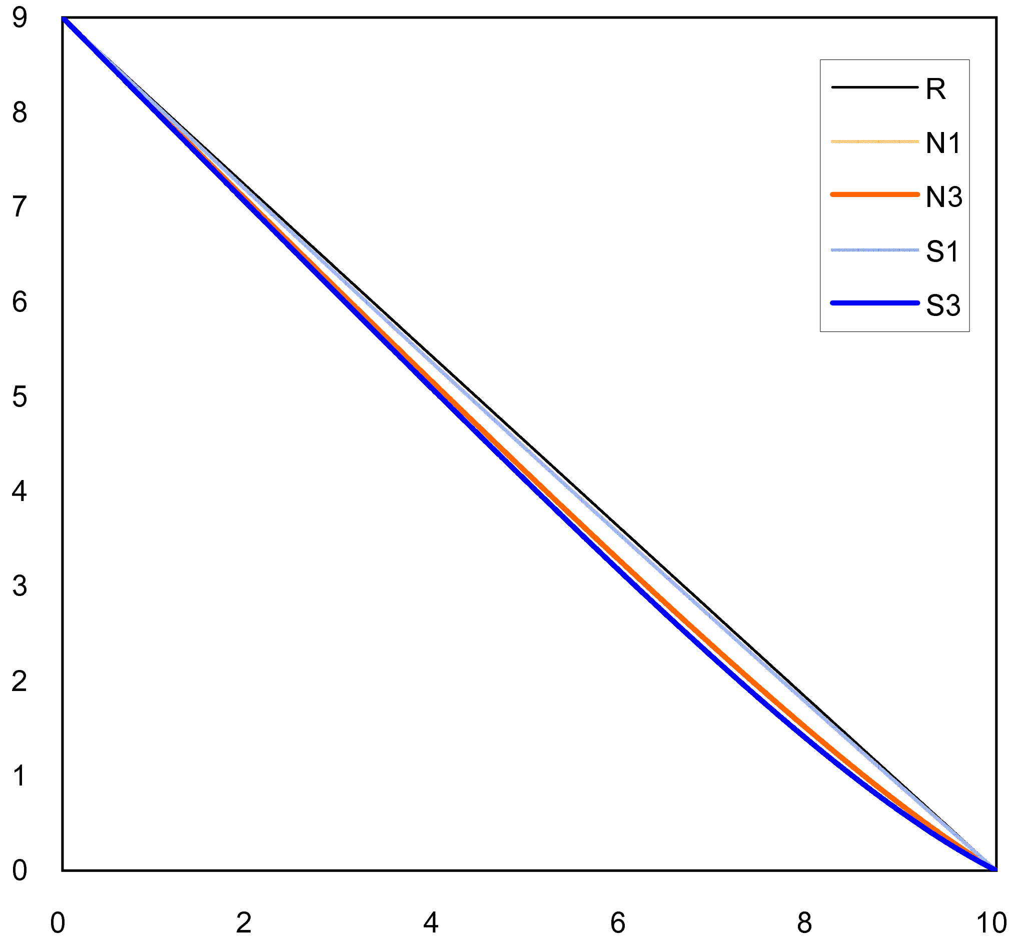

Figure 1: Graph of $S(t)$ for selected types of agents. T is fixed.

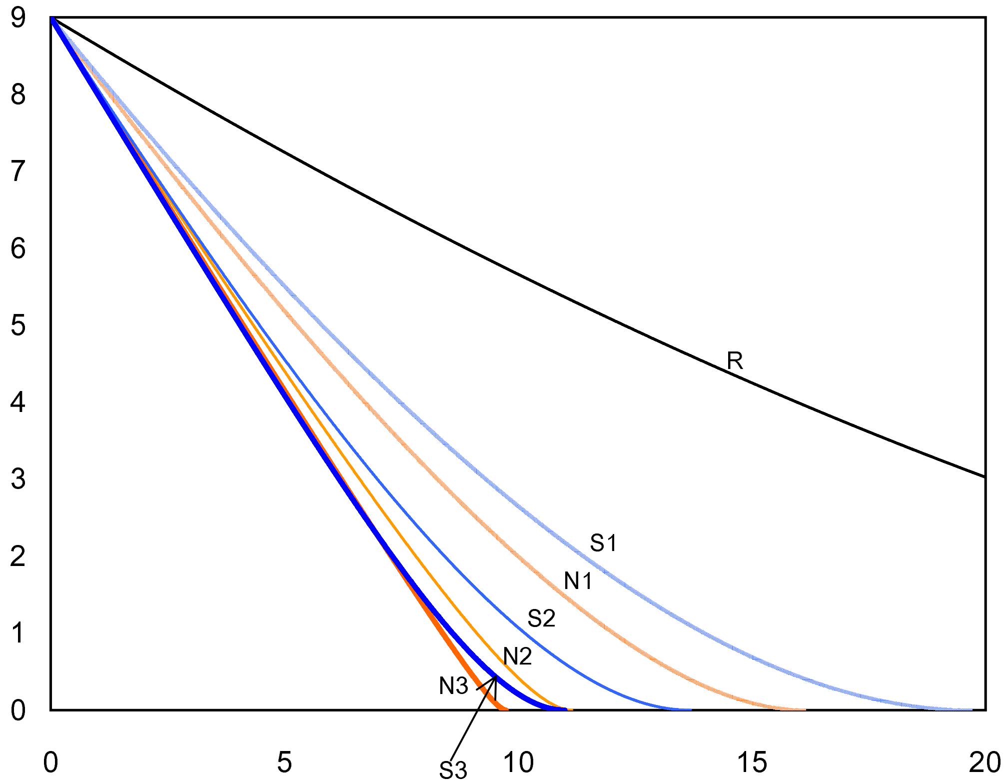

Figure 2: Graph of $S(t)$ for selected types of agents. T is free. The domain of t is restricted.

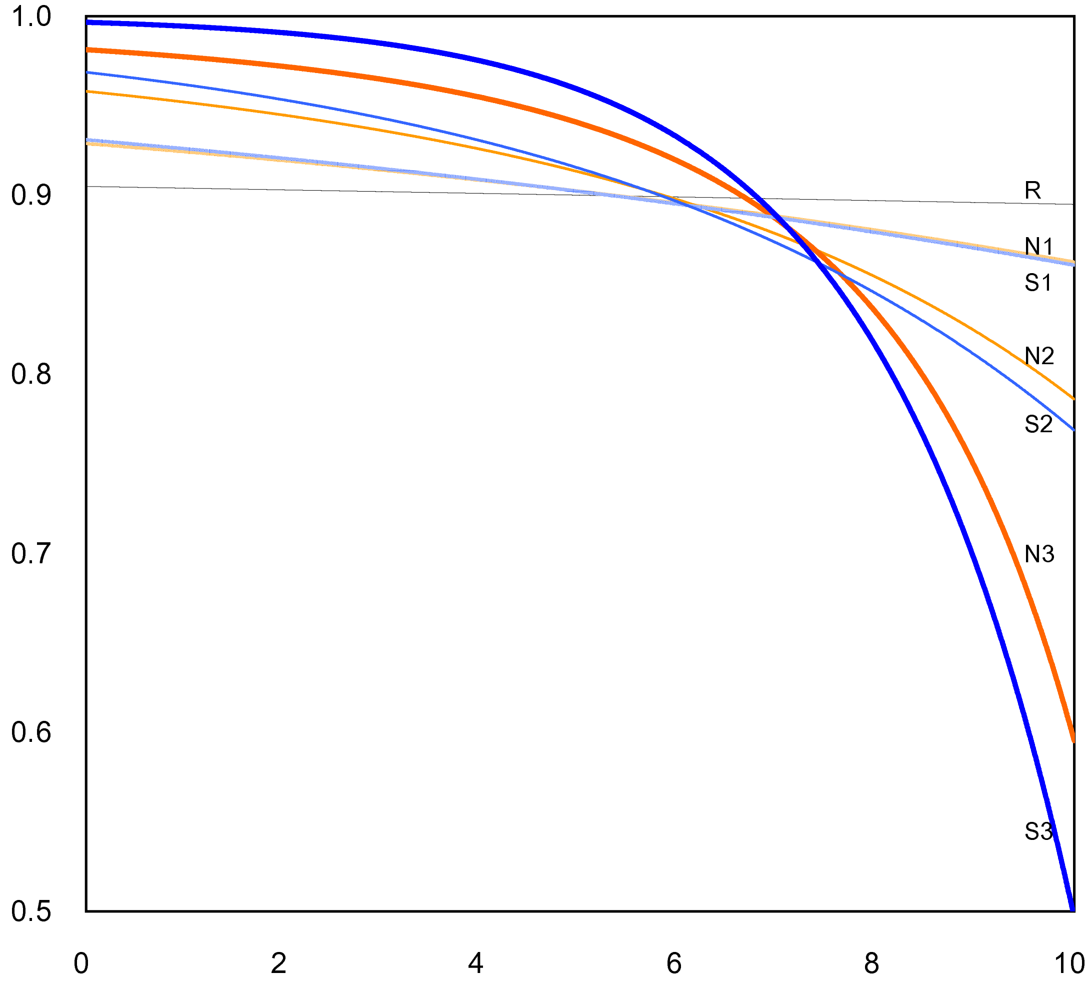

Figure 3: Graph of $x(t)$ for selected types of agents. T is fixed.

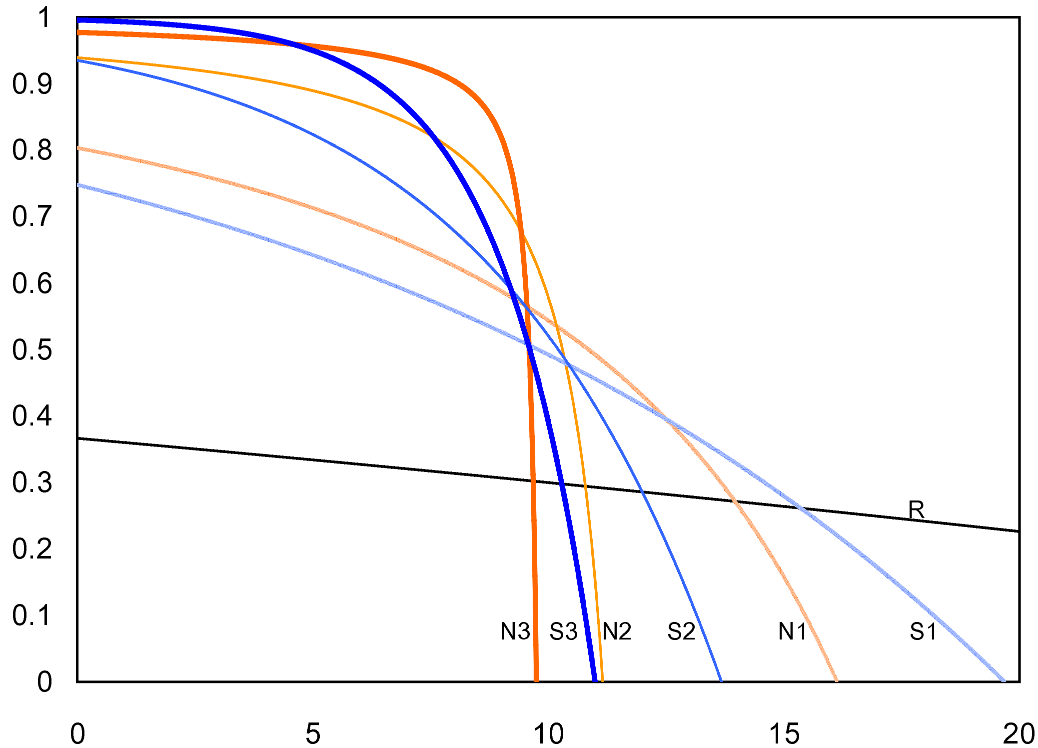

Figure 4: Graph of $x(t)$ for selected types of agents. T is free. The domain of t is restricted.

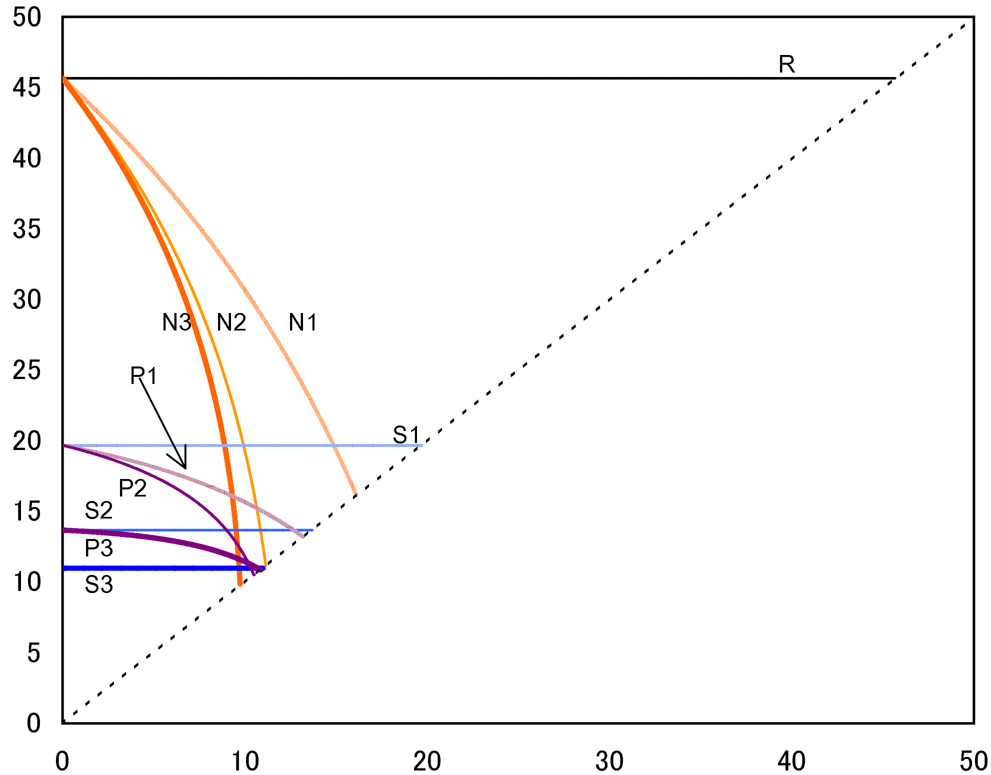

Figure 5: The graph of $\tilde{T}(t, S(t), \tilde{\rho})$ against the time t for all types of agents.

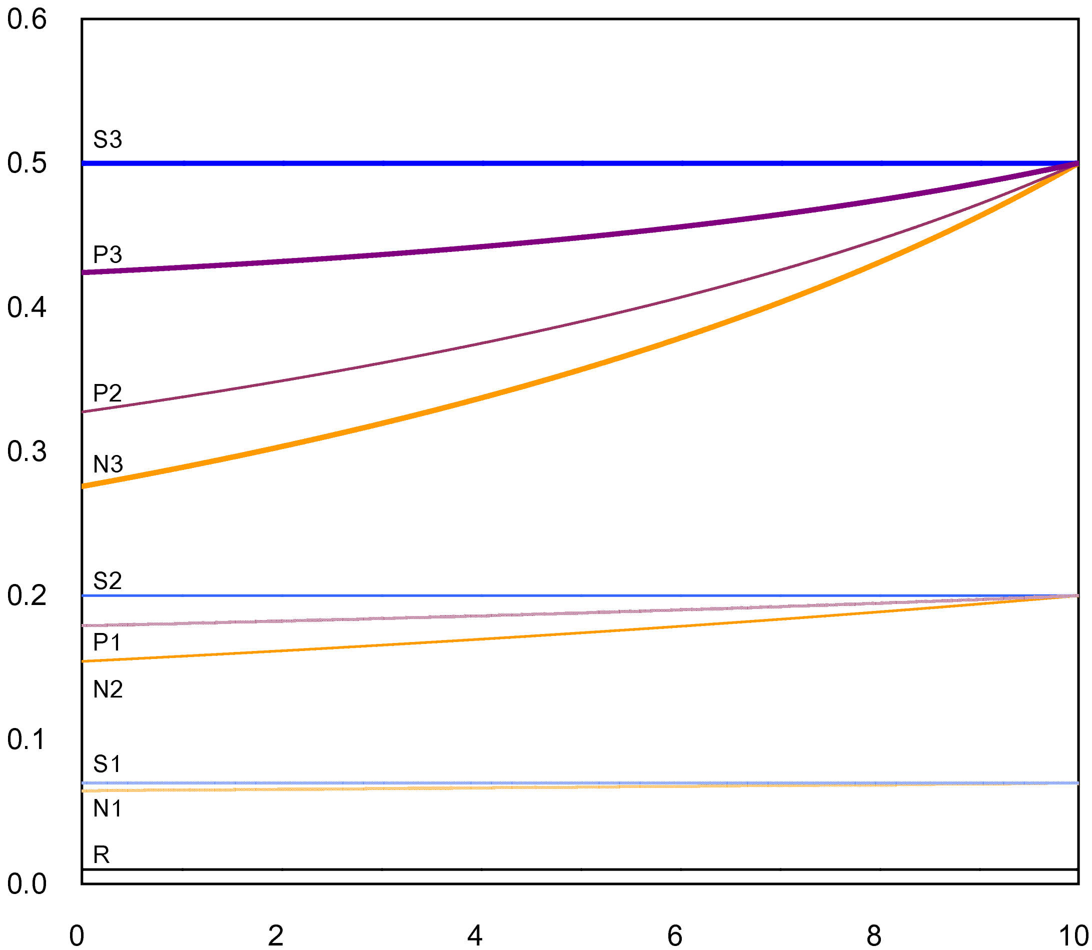

Figure 6: The graph of observationally equivalent rational discount rate $\tilde{\gamma}(t)$ against time t for all types of agents. T is fixed.

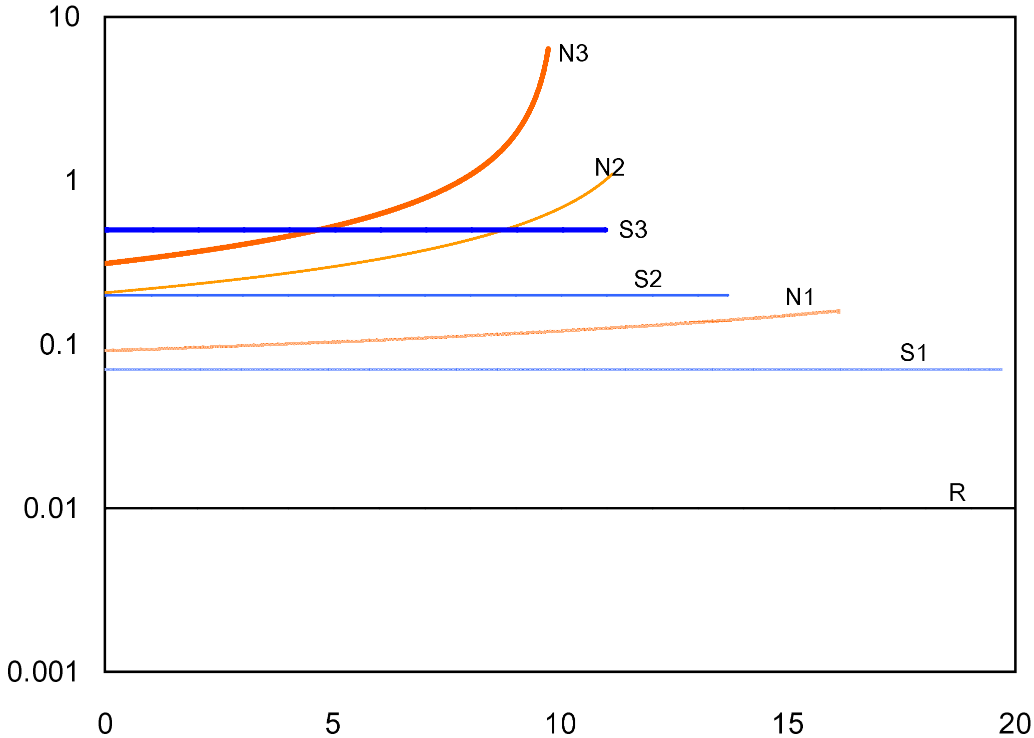

Figure 7: The graph of observationally equivalent rational discount rate $\tilde{\gamma}(t)$ against time t for selected types of agents. T is free. The vertical axis is on a log scale. The domain of t is restricted.

Table 1: Total lifetime payoff $U(x^G(0, S_0, \hat{\rho}, \rho); 0, S_0, \gamma)$ for various types of agents. In each cell, the label for each agent is given in the top row, the lifetime payoff for fixed $T(= 10.00)$ in the bottom left and the lifetime payoff for free T in the bottom right.

	$\hat{\rho} = 0.01$	$\hat{\rho} = 0.07$	$\hat{\rho} = 0.20$	$\hat{\rho} = 0.50$
$\rho = 0.01$	R 4.7106 6.7164	-	-	-
$\rho = 0.07$	N1 3.5948 3.1445	S1 3.5613 3.9953	-	-
$\rho = 0.20$	N2 2.3247 1.5860	P1 2.2267 2.0788	S2 2.1460 2.1861	-
$\rho = 0.50$	N3 1.3952 1.1120	P2 1.2833 1.1450	P3 1.1119 1.0470	S3 0.9916 0.9918

Table 2: The predicted terminal time $\tilde{T}(0, S_0)$ of extraction for time $t = 0$ and the actual terminal time of extraction for all types of agents.

	$\hat{\rho} = 0.01$	$\hat{\rho} = 0.07$	$\hat{\rho} = 0.20$	$\hat{\rho} = 0.50$
$\rho = 0.01$	R 45.651 45.651	-	-	-
$\rho = 0.07$	N1 45.651 16.128	S1 19.684 19.684	-	-
$\rho = 0.20$	N2 45.651 11.152	P1 19.684 13.204	S2 13.676 13.676	-
$\rho = 0.50$	N3 45.651 9.744	P2 19.684 10.497	P3 13.676 10.870	S3 10.992 10.992