

# **Future Fiscal and Budgetary Shocks**

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## Abstract

We study the effects of future tax and budgetary shocks in a non-monetary and possibly non-Ricardian economy. An (unanticipated) temporary labor tax cut to be effective on a given future date—a delayed “debt bomb”—causes at once a drop in the (unit) value placed on the firms’ business asset, the customer, with the result that share prices, the hourly wage, and employment drop in tandem. This paradox of reduced activity through announcement of future “stimulus” does not hinge on an upward jump of long interest rates. A future tax-rate cut lacking a “sunset” provision has the same negative effects.

**Keywords:** Future shocks, business assets, employment

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## 1. Introduction

The impetus for this paper was the enactment in summer 2001 of the Economic Growth and Tax Relief Reconciliation Act (EGTRRA), familiarly known as the first Bush tax cut. This bill was another structural tax cut, one interpretable as aimed at boosting rates of investment, thus economic growth, and, it was said, thereby shrinking the medium-term natural unemployment rate.<sup>1</sup> The bill also had the backloading feature, one present to a lesser degree in the multi-stage Reagan tax cut enacted in 1981, of scheduling the largest rate reductions in the future years from 2005 to 2010—yet proponents of the bill said that its backloaded cuts would have announcement effects exerting an expansionary impulse on employment and investment in the present. The bill also had the novelty of providing “sunset” of the tax cuts in 2011. This legislation has left economists unsure and divided about its effects. We suggest that the first step toward clarity, whether or not resolution of our divisions, is to investigate the theoretical effects of such an (unanticipated) fiscal shock on the natural, or equilibrium, path of the unemployment rate—or its neoclassical counterpart, hours worked per employee. Accordingly we conduct in this paper an intertemporal-equilibrium analysis of such a fiscal shock in a model that abstracts from monetary channels.

This perspective on the economics of taxation is a marked departure from the postwar tax literature. For Keynesians fiscal policy was all about the deployment of tax rates to moderate swings in business activity—in both supply-side and Keynesian models (Lerner [14]; Mundell [15]). Theorists of a more neoclassical persuasion focused on shifts of tax rates appropriate to shifts in circumstances presumed to be permanent. Early investigations, often recalling Ramsey, explored the “neoclassical principles” for the design of fiscal policy (Samuelson [21, 22]). One of the last in this genre argued that the (flat) tax rate was best set at the level needed for an unchanging public debt (Barro [2]).

But, with the Reagan era, times changed. Tax rates have now been used as a strategic instrument to preempt expansion of welfare entitlements in the expectation that doing so will succeed in lowering future tax rates as well or at any rate lessening their rise. In this new world, the tax rates are more like shocks than responses to shocks and the government’s entitlement spending may be endogenous rather than parametric. With this paradigm shift in the way fiscal policy is conceived, the Ramseyan framework needs some changes. However, it is no longer clear that such a framework is well-suited

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<sup>1</sup>True, those who saw employment as still well above their estimates of its natural level believed that unemployment was on its way up. But even if employment had been seen as below estimates of its natural level, the bill’s proponents would have defended the bill as a fillip to growth and a boost to employment.

to capture the immediate impacts on rates of investment in business assets that sudden prospects of future tax-rate decreases may have. For example, we would like our model to contain the prices of the one or more business assets in which the existing firms invest. It would be nice to see firms!

We use here a model of the closed economy in which output is sold on a customer market and, for simplicity, the labor variable is hours worked. A key feature is the shadow price or (unit) asset value of an incremental customer ( $q$ ): a decrease operates to reduce the demand wage and thus decreases hours worked. This channel that we introduce—through which present asset price changes, which occur in response to the sudden prospect of changes in future labor tax rates, produce shifts in the labor demand curve—is distinct from, and operates on top of, the neoclassical intertemporal substitution channel (also present in our model). The intertemporal substitution channel, so to speak, shifts only the labor supply curve.<sup>2</sup>

Our basic analysis incorporates imperfect competition in the product market and non-Ricardian Equivalence. Rotemberg and Woodford [20] argued convincingly that a model featuring imperfect competition in the product market is required in order to explain how aggregate demand changes, such as increases in government purchases, can increase output while at the same time raise the real wage. They assumed a Ricardian economy.<sup>3</sup> Does introducing non-Ricardian Equivalence to an environment of imperfect competition make any difference to the effects of future fiscal shocks? We show that it makes a big difference. In section 3, we study a future “debt bomb” that is announced at  $t_0$  to occur at  $t_1$ —a “time bomb” of exploding public debt, such as the present enactment of a labor tax cut to become effective at a future date and with a sunset provision soon thereafter—offset by cuts in entitlements.<sup>4</sup> In a Ricardian world, this future fiscal shock has no effect on share prices, employment and output as households’ total wealth remains unchanged. However, in a non-Ricardian world, we show that this shock depresses the (unit) value of the business asset, the price of shares, hourly wage, hours worked and thus the GDP. In effect, the prospective burden of the public debt in

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<sup>2</sup>The classic reference here is Barro and King [3].

<sup>3</sup>Schmitt-Grohé and Uribe [23] incorporate imperfect competition into a stochastic dynamic Ricardian model through the Dixit-Stiglitz framework and introduce nominal price stickiness. They find that, under plausible assumptions about the degree of price stickiness, there is a presumption for the monetary authority to aim for price stability. As for optimal tax policy, they find support for tax-smoothing even though their framework differs from that of Barro [2]).

<sup>4</sup>In fact, we suppose that there is an infinitesimally small interval over which the tax-rate cut occurs, producing a big government deficit. We leave to section 4 to study the case where the tax-rate cut is made permanent.

the non-Ricardian economy acts to depress present economic activity. In section 4, we investigate the effects of a small tax-rate cut that is delayed and permanent, with entitlement spending adjusting gradually to retain solvency. We obtain the result of employment contraction in the present even in the case of a delayed permanent tax cut under the assumption of non-Ricardian Equivalence. In contrast, a delayed permanent tax-rate cut that is fiscally sustainable is expansionary in a Ricardian world.

Another strand of the literature assumes a perfectly competitive product market structure. Within this strand, one approach assumes a Ricardian world and studies the effects on output and employment of introducing distortive wage taxes to finance government spending (see Baxter and King [5] and Chari, Christiano and Kehoe [10]).<sup>5</sup> Suppose that the future debt bomb is offset by subsequently higher labor taxes. While public debt, taken by itself, is neutral in a Ricardian economy, raising distortive wage taxes in the future to finance the debt bomb causes individuals to feel poorer today and so to work more in the present. This is the intertemporal substitution channel, and leads to a rightward shift of the labor supply curve. Another approach, such as that of Barry and Devereux [4], retains the competitive framework but introduces non-Ricardian Equivalence by adopting the Blanchardian setup. With non-Ricardian Equivalence but a competitive framework, individuals feel richer on account of the debt bomb but poorer on account of impending higher taxes required to re-balance the budget. Consequently, the labor supply curve could either shift left or right, leaving the effect on current activity ambiguous under such a shock with perfect competition and non-Ricardian Equivalence. In contrast, in our model with imperfect competition and non-Ricardian Equivalence, present employment unambiguously contracts. The increase in the tax rate required to re-balance the budget has the classic supply-side effect of reducing hours worked through increasing the tax wedge at given  $q$ . In addition, as the increase in tax rate reduces current earnings on business assets from future time  $t_1$  onwards, there is a consequent decline in  $q$  at  $t_1$ . In anticipation, present  $q$  drops. With non-Ricardian Equivalence, present  $q$  drops both on account of the higher future short rates of interest caused by the higher stock of public debt as well as due to the supply-side effect of higher distortive wage taxes. The lower  $q$  shifts the labor supply curve to the right via intertemporal substitution but also shifts the labor demand curve to the left via higher Phelps-Winter markups but the net effect, we show, is a drop of employment and output.

An early paper by Phelps and Shell [17] analyzed the relationships that exist among capital per

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<sup>5</sup>Chari, Christiano and Kehoe [10] argue, in the context of a stochastic dynamic Ricardian model under perfect competition, that optimal labor tax rates should be constant.

worker, public indebtedness, and the income tax rate in the context of the Solow model. Phelps [16] introduced a customer market model with dynamic and variable markups to examine a public debt shock in a non-Ricardian setting.<sup>6</sup> However, it does not incorporate the distortionary effect of the wage income tax that plays a crucial role in this paper and it neither analyzes the effects of a backloaded tax cut nor studies the endogenous evolution of the debt-income ratio in a fully specified general-equilibrium system. Based upon their empirical study of the U.S. economy, Beaudry and Portier [6] make a case for the importance of expectational shocks in explaining business fluctuations. They suggest that these shocks take the form of news regarding shifts in future technological possibilities. Our paper shows that present concerns about future fiscal and budgetary overhangs that do not directly affect technological possibilities might nonetheless also depress present asset prices and contract employment through the adverse shifts of labor demand.

The rest of the paper is organized as follows. After the exposition of the basic model in section 2, the paper studies in section 3 the effects of a future debt bomb that is offset, first, by cuts in entitlements and, second, by subsequently higher labor taxes. Section 4 studies a delayed tax cut without the sunset provision. Section 5 concludes.

## 2. The Model

Our model describes a closed economy with no retirement.<sup>7</sup> We follow the treatment by Blanchard [7] in which worker-savers toil throughout life, save by buying annuities invested in the shares of the firms, and *die off* exponentially.<sup>8</sup> In order to provide a business asset to back the shares of firms, and in order to give a role to the variation of price-marginal cost markups, we use the customer-market model set up by Phelps and Winter [18] and placed in a general-equilibrium setting by Calvo and Phelps [9] and Greenwald and Stiglitz [11]. Owing to frictions in the transmission of price information, the competition of firms for market share will fail to wipe out all pure profit, and so leave the optimal price charged by firms hanging above the average and marginal cost. Production for the customer market, which is the only commercial market supplied by firms using only labor to

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<sup>6</sup>Since the employment effect of an asset price change that this paper emphasizes results from varying the price-cost markup, physical capital does not play an important part in our story.

<sup>7</sup>The working paper version of this paper (Hoon and Phelps [13]) also works out the case of mandatory retirement.

<sup>8</sup>This model nests the special case of the Ricardian economy exhibiting Ricardian Equivalence obtained by setting a parameter representing the probability of death ( $\theta$ ) to zero.

produce a single homogeneous good, is carried out by a large (constant) number of atomistic firms in identical (or symmetrical) circumstances. The size of the population and the stock of customers are equal to a positive constant, which we normalize to one. Hence the number of customers per firm is a demographic parameter in our closed economy.

Agents derive utility from consumption and leisure, have finite lives and face an instantaneous probability of death  $\theta$  that is constant throughout life. Let  $c(s, t)$  denote consumption at time  $t$  of an agent born at time  $s$ ,  $l(s, t)$  the number of hours worked,  $w(s, t)$  non-human wealth, and  $h(s, t)$  human wealth. Also let  $y^g(s, t)$  be welfare entitlement received and  $v^h(s, t)$  be the after-tax real hourly wage (both measured in units of output, our numeraire good), where  $v^h$  is related to the hourly labor cost to the firm,  $v^f$ , by  $v^f \equiv (1 + \tau)v^h$ ,  $\tau$  being the proportional wage income tax rate. We make the assumption that workers of all age cohorts have the same productivity, face the same tax rate and receive the same entitlement so  $v^h(s, t) = v^h(t)$  and  $y^g(s, t) = y^g(t)$  for all  $s$ . We let  $r(t)$  denote the real instantaneous short-term interest rate,  $\rho(> 0)$  the pure rate of time preference, and  $\bar{L}$  the total time available per worker.

The agent maximizes

$$\int_t^\infty [\log c(s, \kappa) + B \log(\bar{L} - l(s, \kappa))] \exp^{-(\theta + \rho)(\kappa - t)} d\kappa, \quad B \equiv \text{parameter} > 0$$

subject to

$$\frac{dw(s, t)}{dt} = [r(t) + \theta]w(s, t) + v^h(t)l(s, t) + y^g(t) - c(s, t)$$

and a transversality condition that prevents agents from going indefinitely into debt. The solution to the agent's problem is given by

$$\begin{aligned} c(s, t) &= (\theta + \rho)[h(s, t) + w(s, t)], \\ \frac{\bar{L} - l(s, t)}{c(s, t)} &= \frac{B}{v^h(t)}, \end{aligned}$$

where human wealth is given by

$$h(s, t) = \int_t^\infty [l(s, \kappa)v^h(\kappa) + y^g(\kappa)] \exp^{-\int_t^\kappa [r(\nu) + \theta]d\nu} d\kappa.$$

Aggregating across all individuals, dropping the time index  $t$  and denoting per capita aggregate variables by capital letters, we obtain

$$C = (\theta + \rho)[H + W], \tag{1}$$

$$\frac{BC}{\bar{L} - L} = v^h, \tag{2}$$

$$\dot{H} = (r + \theta)H - (Lv^h + y^g), \quad (3)$$

$$\dot{W} = rW + Lv^h + y^g - C, \quad (4)$$

where a dot over a variable denotes its time derivative. Notice that because of the presence of a perfect annuity market with annuity companies investing in the firms, the private return faced by individuals (who all purchase annuities) is higher (at  $r + \theta$ ) than the rate ( $r$ ) at which aggregate non-human wealth accumulates. It is the difference in the discount rates in (3) and (4) that results in the non-neutrality of debt and deficits.

The government's budget constraint can, in general, be expressed as

$$\dot{D} = rD + G + y^g - \tau Lv^h, \quad (5)$$

where  $D$  is the per capita level of government debt,  $G$  is the per capita amount of government purchases, and tax revenue collected is entirely from wage income taxation. For simplicity, we will throughout set  $G = 0$ . Assuming that, in equilibrium, agents have zero holdings of private bonds,  $W \equiv V + D$ , where  $V$  is the total value of shares held by individuals. Taking the time derivative of (1), and using (3) and (4), we obtain, after re-arrangement of terms,

$$\frac{\dot{C}}{C} = (r - \rho) - \frac{\theta(\theta + \rho)[V + D]}{C}. \quad (6)$$

We now turn to the firms. We assume that each identically situated symmetric firm faces a technology that converts one unit of labor into one unit of output. Taking the wage rate,  $v^f$ , as given, each firm  $i$  has to choose the price at which to sell to its current customers or, equivalently, the output to supply per customer to its consumers. Raising its price causes a decrease, and lowering the price an increase, in the quantity demanded by its current customers according to a per-customer demand relationship,  $D(p^i/p, C^s)$ . For simplicity, we assume that  $D(\cdot)$  is homogeneous of degree one in total sales,  $C^s$ , and so we write  $C^{si} = \eta(p^i/p)C^s$ ;  $\eta'(p^i/p) < 0$ ;  $\eta(1) = 1$ . Each firm chooses the path of its real price or, equivalently, the path of its supply per customer to its consumers, to maximize the present discounted value of its cash flows. The maximum at the  $i$ th firm is the value of the firm,  $V^i$ , which depends upon  $x^i$ :

$$V_0^i \equiv \max \int_0^\infty \left[ \left( \frac{p_t^i}{p_t} \right) - v_t^f \right] \eta \left( \frac{p_t^i}{p_t} \right) C_t^s x_t^i \exp^{-\int_0^t r_\nu d\nu} dt.$$

The maximization is subject to the differential equation giving the motion of the stock of customers of the  $i$ th firm as a function of its relative, or real, price given by (7) below and an initial  $x_0^i$ :

$$\dot{x}^i = g\left(\frac{p^i}{p}\right)x^i; \quad g' < 0, g'' \leq 0; \quad g(1) = 0. \quad (7)$$



The first-order condition for optimal  $p^i$  is

$$\eta\left(\frac{p^i}{p}\right)\frac{C^s x^i}{p} + \left[\left(\frac{p^i}{p}\right) - v^f\right] \eta'\left(\frac{p^i}{p}\right)\frac{C^s x^i}{p} + q_m^i g'\left(\frac{p^i}{p}\right)\frac{x^i}{p} = 0, \quad (8)$$

where  $q_m^i$  is the shadow price, or worth, of an additional customer. Another two other necessary first-order conditions (which are also sufficient under our assumptions) from solving the optimal control problem are:

$$\dot{q}_m^i = [r - g\left(\frac{p^i}{p}\right)]q_m^i - \left[\left(\frac{p^i}{p}\right) - v^f\right] \eta\left(\frac{p^i}{p}\right)C^s, \quad (9)$$

$$\lim_{t \rightarrow \infty} \exp^{-\int_0^t r_\nu d\nu} q_{mt}^i x_t^i = 0. \quad (10)$$

We note that “marginal q” denoted  $q_m^i$  is equal to “average q,” which we denote as  $q_a^i \equiv V^i/x^i$ , so  $q_m^i = q_a^i \equiv q^i$ .<sup>9</sup>

Now we move on to consider the economy’s general equilibrium. First, we take note that in the closed economy, the aggregate stock of customers is a fixed constant given by the size of the population, which we have normalized to one. Hence, in the closed economy,  $x = 1$ . Next, equating  $p^i$  to  $p$  and setting  $q^i = q$  in (8) for a symmetric equilibrium gives:

$$\left[1 + \frac{\eta(1)}{\eta'(1)} - v^f\right] = -\left(\frac{q}{C^s}\right) \left(\frac{g'(1)}{\eta'(1)}\right); \quad \eta(1) = 1, \quad \eta'(1) < 0, \quad g'(1) < 0. \quad (11)$$

The expression in the square brackets in (11) is the algebraic excess of marginal revenue over marginal cost, a negative value in customer-market models as the firm supplies more than called for by the static monopolist’s formula for maximum current profit, giving up some of the current profit for the sake of its longer-term interests. Defining the average (gross) markup as  $m \equiv 1/v^f$  since  $p^i = p$  in  $(p^i/p)/v^f$  in a symmetric equilibrium, we can re-arrange (11) to obtain:

$$\left(\frac{m-1}{m}\right) = -\left(\frac{1}{\eta'(1)}\right) \left(\eta(1) + g'(1)\left(\frac{q}{C^s}\right)\right). \quad (12)$$

Equation (12) shows that the optimal markup depends negatively on what may be called, Tobin’s Q,  $q/C^s$ —the ratio of the present discounted value of acquiring an additional customer relative to the payoff from current consumption. We write  $m = \phi(q/C^s)$ , with  $\phi'(q/C^s) < 0$ . An increase in  $q$  relative to  $C^s$  means that profits from future customers are high relative to payoff from current

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<sup>9</sup>The proof is as follows: Taking the time derivative of the product  $q_{mt}^i x_t^i$ , we obtain  $d(q_{mt}^i x_t^i)/dt = q_{mt}^i [dx_t^i/dt] + x_t^i [dq_{mt}^i/dt] = r_t q_{mt}^i x_t^i - [(p_t^i/p_t) - v_t^f] \eta(p_t^i/p_t) C_t^s x_t^i$ , after using (7) and (9). Integrating, and using (10), we obtain  $q_{mt}^i x_t^i = \int_t^\infty [(p_\kappa^i/p_\kappa) - v_\kappa^f] \eta(p_\kappa^i/p_\kappa) C_\kappa^s x_\kappa^i \exp^{-\int_t^\kappa r_\nu d\nu} d\kappa \equiv V_t^i$ .

consumption so that each firm reduces its price (equivalently its markup) in order to increase its customer base.

There is yet another way of expressing (11), which puts a focus on the labor market, that will be useful for developing intuition for the results we obtain in this paper. Noting our simple production technology,  $C^s = L$ , we can re-express (11) as saying that the representative firm's real demand wage,  $v_{\text{demand}}^f$ , is negatively related to employment,  $L$ , and positively related to the shadow value of an additional customer,  $q$ :

$$v_{\text{demand}}^f = 1 + \frac{\eta(1)}{\eta'(1)} + \left(\frac{q}{L}\right) \left(\frac{g'(1)}{\eta'(1)}\right). \quad (13)$$

In a Marshallian employment-real wage diagram, (13) gives a downward-sloping labor demand curve, with  $q$  acting as a labor demand-shifter. An increase in  $q$  leads firms to reduce their monopoly power and thus to increase the demand wage.

The other schedule in this plane is the aggregate labor supply curve, which comes from (2). Noting the identity  $v^h \equiv (1 + \tau)^{-1}v^f$ , (2) can be re-expressed as

$$v_{\text{supply}}^f = \frac{(1 + \tau)BC}{\bar{L} - L}, \quad (14)$$

which says that the real supply wage,  $v_{\text{supply}}^f$  is positively related to employment,  $L$ , given the current tax rate and consumption. In the Marshallian employment-real wage diagram, (14) gives an upward-sloping labor supply curve, with the parameter representing the value of leisure ( $B$ ), the tax rate ( $\tau$ ), and the level of consumption demand ( $C$ ) acting as labor supply shifters.

Putting together labor demand and supply, we see that a decrease in current  $\tau$  reduces the tax wedge and consequently expands  $L$ . To understand the role played by  $C$ , we substitute (1) and the definition of non-human wealth into (14) to get

$$v_{\text{supply}}^f = \frac{(1 + \tau)(\theta + \rho)B[q + D + H]}{\bar{L} - L}, \quad (15)$$

where  $H$  is human wealth. We see from (15) that an increase in  $q$  raises the worker's non-human wealth and as a result raises the supply wage. Since an increase in  $q$  increases labor demand but also decreases labor supply, can we determine what the net effect on employment is? To get the answer, we draw upon the condition that, in the closed economy, equilibrium requires that total consumption demand must be equal to the economy's supply of the consumer good, an equilibration that is brought about through an adjustment in human wealth,  $H$ . Since  $C^s = L$ , we can impose the goods market-clearing condition,  $C = C^s = L$ , in (14), and then equate the demand wage to the

supply wage to obtain

$$\left[1 + \frac{\eta(1)}{\eta'(1)}\right] + \left(\frac{q}{\bar{L}}\right) \left(\frac{g'(1)}{\eta'(1)}\right) = \frac{B(1 + \tau)L}{\bar{L} - L}, \quad (16)$$

where  $L$ , we find, is unambiguously increasing in  $q$ : an increase in  $q$  induces firms to lower their markups, thus to raise the demand wage, and that effect dominates the wealth effect on labor supply.

The result that an increase in  $q$  unambiguously raises employment also holds with an individual utility function introduced by Greenwood, Hercowitz and Huffman [12]:  $\log[c(s, t) + G(\bar{L} - l(s, t))]$ ;  $G'(\cdot) > 0, G''(\cdot) < 0$ , which allows exact aggregation in the non-Ricardian economy to give the aggregate optimum condition,  $G'(\bar{L} - L) = v^h$ .<sup>10</sup> An increase in  $q$ , which increases  $v^h$  for an unchanged  $\tau$  via (13), unambiguously raises  $L$ . In a Ricardian economy exhibiting a representative utility function having the properties that over the long run the real wage and consumption grow at the same secular rate whereas labor supply exhibits no secular trend:  $\{[CG(\bar{L} - L)]^{1-\sigma} - 1\}/(1 - \sigma)$ ;  $G'(\cdot) > 0, G''(\cdot) < 0$ , optimum choice of  $C$  and  $L$  leads to  $G'(\bar{L} - L)/G(\bar{L} - L) = v^h/C$  so labor supply ( $L$ ) is increasing in  $v^h/C$ ; setting  $C = C^s = L$  and noting that  $v^h$  is increasing in  $q$ , given  $\tau$ ,  $L$  is therefore increasing in  $q$ . If individuals in the non-Ricardian economy exhibit such a form of utility function, namely,  $\{[c(s, t)G(\bar{L} - l(s, t))]^{1-\sigma} - 1\}/(1 - \sigma)$ , we are not able to obtain an exact aggregation that makes aggregate labor supply ( $L$ ) a function of aggregate consumption ( $C$ ) except when  $\sigma = 1$ . When  $\sigma = 1$ , individual utility function becomes  $\log c(s, t) + B \log(\bar{L} - l(s, t))$ , which is the case we have treated in this section, giving us the aggregate optimum condition expressed in (2). Prescott [19] has adduced supporting empirical evidence for taking  $\sigma = 1$ .

**Lemma 1.** We obtain  $C^s = L = \Omega(q; \tau)$ , with  $0 < e_q \equiv d \ln C^s / d \ln q < 1$ , where  $e_q$  denotes the elasticity of  $C^s$  with respect to  $q$ , and the partial derivative  $\Omega_\tau < 0$ . Additionally,  $v^f = V^f(q; \tau)$ , with the partial derivatives  $V_q^f > 0$  and  $V_\tau^f > 0$ . Equivalently, the equilibrium markup,  $m$ , can be expressed as  $m = (V^f(q; \tau))^{-1} = \psi(q; \tau)$  with the partial derivatives  $\psi_q < 0$  and  $\psi_\tau < 0$ .

It will sometimes be useful to adopt another reduced-form function for output supply or equilibrium employment. For this alternative formulation, we note from setting  $L = C^s$  in (16) that since the demand wage is increasing in  $\hat{q} \equiv q/C^s$ , and the supply wage is increasing in  $L$ , therefore,  $L$  is

<sup>10</sup>From the individual optimal condition that  $G'(\bar{L} - l(s, t)) = v^h$ , we note that since  $G'$  is monotone increasing in  $l(s, t)$  for each individual born at time  $s$ , every individual chooses the same amount of labor to supply at the common take-home wage,  $v^h$ . Hence,  $l(s, t) = L$  for all  $s$ .

increasing in  $\hat{q}$ .

**Lemma 2.** Output supply, equal to employment, can alternatively be expressed as  $C^s = L = \Psi(\hat{q}; \tau)$  with  $\Psi_{\hat{q}} > 0$  and  $\Psi_{\tau} < 0$ .

In a symmetric situation across firms, (9) simplifies to

$$r = \frac{[1 - V^f(q; \tau)]\Omega(q; \tau)}{q} + \frac{\dot{q}}{q} + g(1); \quad g(1) = 0, \quad (17)$$

after using  $v^f = V^f(q; \tau)$  and  $C^s = \Omega(q; \tau)$ . Finally, equating consumption demand to supply in (6), and noting that non-human wealth is held in the form of shares and public debt and that  $\dot{C}^s/C^s = e_q(\dot{q}/q)$ , we obtain an expression for the consumer's required rate of interest,  $r$ :

$$r = \rho + \frac{\theta(\theta + \rho)(q + D)}{\Omega(q; \tau)} + e_q \left( \frac{\dot{q}}{q} \right); \quad 0 < e_q < 1. \quad (18)$$

If we define the long-term (real) interest rate as the yield on consols paying a constant coupon flow of unity, and let  $R$  be their yield and hence  $R^{-1}$  be their price, arbitrage between short and long bonds gives the condition  $R = r + (\dot{R}/R)$ . Equating the consumer's required rate of interest in (18) to the market rate of return in (17), and noting that  $g(1) = 0$ , we obtain, for given fiscal parameters, an expression for the size of capital gains (or loss):

$$\frac{\dot{q}}{q} = (1 - e_q)^{-1} \left[ \rho + \frac{\theta(\theta + \rho)(q + D)}{\Omega(q; \tau)} - \frac{[1 - V^f(q; \tau)]\Omega(q; \tau)}{q} \right]. \quad (19)$$

From (19), we observe that the capital gain ( $\dot{q}/q$ ) is increasing in  $q$ ,  $D$  and  $\tau$ . If we now substitute for the capital gain term in either (17) or (18) using (19), we obtain

$$r = \left( \frac{1}{1 - e_q} \right) \left[ \rho + \frac{\theta(\theta + \rho)(q + D)}{\Omega(q; \tau)} - e_q \frac{(1 - V^f(q; \tau))\Omega(q; \tau)}{q} \right], \quad (20)$$

which makes the equilibrium interest rate or market rate of return an increasing function of  $q$ ,  $D$  and  $\tau$ .

**Lemma 3.** The natural interest rate function is given by the following:  $r = \Upsilon(q; D, \tau)$  with the partial derivatives  $\Upsilon_q > 0$ ,  $\Upsilon_D > 0$  and  $\Upsilon_{\tau} > 0$ .

We now study the economy's equilibrium state given  $D > 0$ ,  $y^g > 0$ , and  $\tau > 0$ .

**Lemma 4.** Given  $D$ ,  $y^g$ , and  $\tau$ , the rational expectations equilibrium is given by a unique value of  $q$ , denoted  $q_{ss}$ , that makes the RHS of (19) equal to zero.

**Proof.** Since the elasticity of  $C^s$  with respect to  $q$  is less than one, the RHS of (19) is increasing in  $q$ . Applying the transversality condition,  $\lim_{t \rightarrow \infty} [\exp^{-\int_0^t r_s ds} q_t] = 0$ , the unique perfect foresight path of  $q$  requires that it be stationary at the value that makes  $\dot{q} = 0$ .<sup>11</sup>

### 3. Effects of a Future Debt Bomb

Let us suppose that at  $t_0$ , it is announced that at  $t_1$ , there will be a temporary tax cut, one that produces a big government deficit over an infinitesimally small time interval, that we dub a debt bomb.<sup>12</sup> As a result of the temporary tax cut, the stock of public debt is suddenly increased by the amount  $\Delta \equiv \text{parameter} > 0$ . We will explore the effects of the debt bomb under two alternative modes of financing, one where the debt bomb is accommodated by cuts in entitlement spending at  $t_1$  onwards and the other where subsequent wage income tax rates are raised to re-balance the budget. To analyze the effects on asset prices, interest rates, and employment, it is convenient to refer to Figure 1, which depicts the stationary loci of the following pair of equations:

$$\dot{D} = \Upsilon(q; D, \tau)D + y^g - T(q; \tau), \quad (21)$$

$$\frac{\dot{q}}{q} = \Upsilon(q; D, \tau) - \frac{[1 - V^f(q; \tau)]}{q/\Omega(q; \tau)}, \quad (22)$$

where (21) is obtained by using  $r = \Upsilon(q; D, \tau)$  and  $T \equiv \tau(1 + \tau)^{-1}V^f(q; \tau)\Omega(q; \tau)$  in (5), and (22) is obtained by using  $r = \Upsilon(q; D, \tau)$  in (17).

The stationary  $q$  locus is downward sloping as an increase in  $D$  raises the interest rate, which requires a lower  $q$  to raise the earnings-price ratio. Given  $D$ , an increase in  $q$  above the stationary locus leads to capital gains while a decrease leads to capital loss. As Lemma 4 established, use of the transversality condition implies that a rational expectations solution is a unique stationary  $q$  at given  $D$ . What the negative slope of the stationary locus implies is that the unique stationary  $q$  value is decreasing in the stock of public debt.

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<sup>11</sup>A problem can arise in the stationary state of this model, as pointed out by Ascari and Rankin [1] in a different context, that the demand for leisure for some very wealthy individuals might exceed their time endowment,  $\bar{L}$ . We can avoid this problem, first, by considering expectational shocks that get the economy out of the stationary state and that cause  $r$  to fall when asset prices decline, possibly below the rate of time preference (see (20)), and thus lead individual economic agents to decumulate wealth outside the stationary state and, second, by introducing mandatory retirement into the model as in Hoon and Phelps [13]. Ascari and Rankin [1] instead propose to use the utility function introduced by Greenwood, Hercowitz and Huffman [12], which makes the demand for leisure independent of wealth.

<sup>12</sup>The opposite polar case of a permanent tax-rate decrease is studied in the following section.

**Lemma 5.** The unique stationary  $q$ , denoted  $q_{ss}$ , is decreasing in the stock of public debt,  $D$ .

The stationary  $D$  locus can be either positively or negatively sloped. In the empirically relevant case where a depressed stock market leads to rising debt-GDP ratios, as the implied collapse in labor demand leads both to a reduction in total hours worked as well as hourly pay so tax revenue falls at given tax rates and the size of the deficit grows despite government interest cost savings, the stationary  $D$  locus is positively sloped. Appendix A.1 establishes that both roots associated with the dynamic system given by (21) and (22) are positive whether or not the debt-GDP ratio rises when equity prices fall.

The economy is initially at point A with  $(D_{ss}^0, q_{ss}^0)$ . Working backwards, let us ask, “What is the value of  $q$  in the new stationary state after the debt bomb?” Given the dynamic instability of the system, the new stationary state must be attained precisely at  $t_1$ . With the explosion of the public debt at  $t_1$  (the stock of public debt is suddenly augmented by  $\Delta$ ), Lemma 5 tells us that the new stationary value of  $q$  is lower. The anticipation of the reduced future  $q$ , however, causes an immediate drop in present  $q$ . The path taken by the economy from  $t_0$  onwards is illustrated in Figure 1. Upon suddenly receiving the news of a future debt bomb, therefore, asset prices fall immediately from  $q_{ss}^0$  to  $q^B$ , the value of  $q$  that corresponds to point B in Figure 1, and the expected rate of change of  $q$ , i.e., the expected capital gains term, goes from zero to a negative value as market participants form a rational expectation of further asset price declines.

As asset prices drop and continue a path of further decline until future time  $t_1$ , the government starts to lose tax revenue even before the temporary tax cut is implemented so the stock of public debt begins to grow from  $t_0$  onwards. At  $t_1$ , the stock of public debt is suddenly augmented by the amount  $\Delta$  as a result of the debt bomb. In order to retain solvency, we first suppose that the whole stream of entitlement spending is reduced from  $t_1$  onwards to accommodate the debt bomb. (The reduction of  $y^g$  from  $t_1$  onwards shifts the stationary  $D$  locus rightwards to pass through point D in Figure 1.) In essence, public debt is now substituted for social wealth (the present discounted value of the whole stream of entitlements). In a non-Ricardian setup, the stream of government entitlements is discounted at  $r + \theta$  but non-human wealth accumulates at  $r$ . Consequently, the substitution of public debt for social wealth makes consumers feel wealthier. The stimulus to consumption demand raises the whole path of the short real rate of interest from  $t_1$  onwards and thus depresses asset price at  $t_1$ . In anticipation, present asset price is also reduced. We obtain the following proposition:

**Proposition 1.** Suppose that the economy is initially in a stationary state with  $(D_{ss}^0, q_{ss}^0)$ . At  $t_0$ , there is a sudden announcement that at  $t_1$ , there will be a temporary tax cut causing a debt bomb. The government budget is re-balanced by a subsequent cut in the constant stream of entitlement spending. The asset price immediately drops and continues to fall until it reaches a permanently depressed level. Employment immediately drops and steadily worsens from then on until it reaches a lower plateau at  $t_1$ .

In the alternative financing scheme, the government raises the wage income tax rate to re-balance the budget after the initial splash of public debt. (This also has the effect of shifting the stationary  $D$  locus to the right.) This increase in the tax rate has the classic supply-side effect of reducing hours worked through increasing the tax wedge at given  $q$ . In addition, as the increase in tax rate reduces current earnings on business assets from future time  $t_1$  onwards, there is a consequent decline in  $q$  at  $t_1$ . In anticipation, present  $q$  drops. We obtain the following proposition:

**Proposition 2.** Suppose that the economy is initially in a stationary state with  $(D_{ss}^0, q_{ss}^0)$ . At  $t_0$ , there is a sudden announcement that at  $t_1$ , there will be a temporary tax cut causing a debt bomb. The government budget is re-balanced by a subsequent permanent increase in the wage income tax rate to service the debt. The asset price immediately drops and continues to fall until it reaches a permanently depressed level. Employment immediately drops and steadily worsens from then on until, at  $t_1$ , there is another abrupt drop to reach a lower plateau as the tax rate is increased.<sup>13</sup>

It is also of some interest to ask what are the effects of the future debt bomb when Ricardian Equivalence holds. This case is obtained in the formal model simply by setting the parameter that represents the probability of death,  $\theta$ , to zero. We can readily establish the following proposition:

**Proposition 3.** Suppose that Ricardian Equivalence holds, and that the economy is initially in a stationary state. At  $t_0$ , there is a sudden announcement that at  $t_1$ , there will be a temporary tax cut causing a debt bomb. If the government budget is re-balanced by a subsequent cut in entitlement spending, there is no effect on employment, asset price and interest rate. If, however, the government budget is re-balanced by a subsequent increase in the wage income tax rate to service the debt, the

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<sup>13</sup>There is a discontinuous drop in output, and hence employment, at  $t_1$  because  $q$  does not jump at  $t_1$  but the wage income tax rate is increased at that point to finance the interest on increased debt. The negative supply-side effect leads to the further decline in employment at  $t_1$ .

asset price immediately drops and continues to fall until it reaches a permanently depressed level. Employment immediately drops and steadily worsens from then on until, at  $t_1$ , there is another abrupt drop to reach a lower plateau as the tax rate is increased.

How can we infer the whole path of the Wicksellian natural rate of interest in response to a future debt bomb in both the Ricardian and non-Ricardian cases accompanied by either a cut in entitlement spending or an increase in the tax rate to retain solvency? At  $t_0$  when the announcement is made, the value of  $q$  drops except in the case of Ricardian Equivalence accompanied by a cut in entitlement spending. (In that case, there is no change in  $q$  as public debt serves as a perfect substitute for social wealth.) The fall in share price increases the earnings-price ratio, which (taken alone) raises the market rate of return but this is more than offset by the anticipation of capital loss so the short real rate of interest, in fact, falls as we can confirm by inspecting (20). Further inspection of (20) shows us that the further decline in  $q$  causes the short real rate of interest to fall further although the gradual build-up of the stock of public debt tends to attenuate the fall when Ricardian Equivalence does not hold. Thus it is very possible that the short real rate of interest will remain low for some time between announcement and implementation. Since the short rate,  $r$ , initially drops and may continue to fall (until  $t_1$ ) in response to the sudden news of a future debt bomb, the depressed stock market could be accompanied by an initial decline in the long rate,  $R$ . Hence the paradox of employment contraction does not result from nor imply any immediate increase of the long real interest rate, contrary to the view held by some financial commentators.<sup>14</sup>

#### **4. Future Permanent Tax Cuts with Gradual Welfare Payment Adjustment**

We now turn to study the effects of future tax cuts without the sunset provision. This immediately raises the question of whether the proposed fiscal changes are sustainable in the sense that the debt-income ratio will not explode when account is taken of all the general-equilibrium effects resulting from the proposed tax cuts (see Blanchard, et al. [8] for the concept of the sustainability of fiscal policy). We ensure fiscal sustainability by requiring that, for a given tax rate, the primary (non-interest) surplus is made a sufficiently responsive positive function of the debt-income ratio. Conversely, the primary deficit is reduced sufficiently as the debt-income ratio rises in order to retain solvency. In Hoon and Phelps [13], we show that it is not possible to launch a fiscally sustainable permanent tax cut while keeping welfare spending constant as a share of GDP. The extent to which

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<sup>14</sup>See the editorial entitled, “The Demise of Rubinomics,” in the Wall Street Journal (August 28, 2002).



policy-makers must reduce the primary (non-interest) deficit, such as through cutting government entitlement programs, depends upon how much a decrease in asset prices decreases the tax revenue to GDP ratio compared to how much it lowers government interest debt burden.

Making the assumption of gradual welfare payment adjustment in response to growing budgetary deficits, it turns out to be convenient to examine a dynamic system where asset price,  $q$ , and per capita debt,  $D$ , are normalized by GDP per business asset. Defining then  $\hat{D} \equiv D/C^s$ ,  $\hat{y}^g \equiv y^g/C^s$ , and  $\hat{q} \equiv q/C^s$ , where  $\hat{q}$  has the interpretation of real asset price normalized by GDP per business asset (which is here a customer) or the stock market capitalization as a ratio to GDP, and  $\hat{D}$  gives the debt-GDP ratio, we modify (5) and (19), respectively, to yield

$$\dot{\hat{D}} = \mu(\hat{q}, \hat{D})\hat{D} + \Phi(\hat{D}) - \hat{T}(\hat{q}; \tau), \quad (23)$$

$$\frac{\dot{\hat{q}}}{\hat{q}} = \mu(\hat{q}, \hat{D}) - \frac{[1 - (\phi(\hat{q}))^{-1}]}{\hat{q}}, \quad (24)$$

where  $\hat{y}^s = \Phi(\hat{D})$ ;  $\Phi'(\hat{D}) < 0$  and  $r - e_q[\dot{q}/q] = \rho + \theta(\theta + \rho)[\hat{q} + \hat{D}] \equiv \mu(\hat{q}, \hat{D})$ , which gives the interest rate net of GDP growth. We see that it is increasing in  $\hat{q}$  and  $\hat{D}$  so  $\mu_{\hat{q}} > 0$  and  $\mu_{\hat{D}} > 0$ . Note also that the tax revenue to GDP ratio is given by  $\tau(1 + \tau)^{-1}V^f(q; \tau) = \tau(1 + \tau)^{-1}(\phi(\hat{q}))^{-1} \equiv \hat{T}(\hat{q}; \tau)$ ;  $\hat{T}_{\hat{q}} > 0$ ,  $\hat{T}_{\tau} > 0$ . To obtain (23) and (24), we have also used the relationships  $\dot{\hat{q}}/\hat{q} \equiv [1 - e_q](\dot{q}/q)$ , and  $m \equiv (v^f)^{-1} = \phi(\hat{q})$ .

In Appendix A.2, we show that, for fiscal sustainability, the extent to which entitlement spending as a ratio to GDP has to shrink as the debt-GDP ratio rises depends on the extent to which tax revenue to GDP ratio declines relative to interest cost savings when asset prices decline. In the empirically relevant case where a depressed stock market leads to rising debt-income ratios, as the implied collapse in labor demand leads both to a decline in employment as well as wage earnings so tax revenue falls at given tax rates and enlarges the fiscal deficit despite interest cost savings, we find that in order to achieve saddle-path stability, and hence achieve fiscal sustainability in response to a tax cut, both stationary loci must be negatively sloped in such a way that

$$\left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{\hat{D}}=0} \right| > \left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{\hat{q}}=0} \right|.$$

We show this case in Figure 2.

We now establish the following proposition concerning a future permanent tax cut in the case of sustainable fiscal policy under the assumption that depressed asset prices lead to a rising debt-GDP ratio (at given tax rates) as the loss in tax revenue exceeds any interest cost savings.

**Proposition 4.** Suppose that the economy is initially in a stationary state with  $(\hat{D}_{ss}^0, \hat{q}_{ss}^0)$ . At

$t_0 = 0$ , there is an announcement that at  $t_1 = T$ , the wage income tax rate will be cut permanently from  $\tau^0$  to  $\tau^1$ ,  $\tau^1 < \tau^0$ . This leaves the real asset price normalized by GDP per business asset ( $\hat{q}$ ) permanently depressed notwithstanding the positive supply-side effect so employment could either contract or expand in the long run. Asset prices fall between announcement and implementation with the result that employment and output contract between  $t_0$  and  $t_1$  and the debt-GDP ratio steadily rises throughout.

We observe from (24) that the stationary  $\hat{q}$  locus does not shift in response to a tax cut. On the other hand, the stationary  $\hat{D}$  locus shifts up since, at a given debt-income ratio and a given size of welfare spending relative to GDP, higher asset prices are required to generate additional tax revenues to offset the direct loss of tax revenue owing to the tax cut. The result of the curve shift is that the new stationary level of  $\hat{q}_{ss}$  is lower and  $\hat{D}_{ss}$  is higher at  $(\hat{D}_{ss}^1, \hat{q}_{ss}^1)$ . Intuitively, the pile up of debt resulting from the tax cut leads to higher short real rates of interest in the new stationary state. As a result, the new stationary  $\hat{q}_{ss}$  must be reduced to generate a higher market rate of return to match the higher interest rate. To infer what happens to employment,  $L$ , we refer to Lemma 2, where we have the result that  $L$  is increasing in  $\hat{q}$  through its influence on markups and decreasing in  $\tau$  through its supply side influence. Although employment in the new stationary state expands on account of reduced tax rates, it contracts on account of depressed stock market capitalization as a ratio to GDP, a depression that is brought about by a swollen debt-income ratio and resulting higher future short real interest rates.

How does the market respond today in anticipation of the prospective permanent tax cut? We leave to Appendix A.3 to prove that for small changes in  $\tau$ , the non-Ricardian consequences of higher future short rates overwhelm the positive supply-side effect of a lower wage income tax rate. As a result, stock market capitalization as a ratio to GDP ( $\hat{q}$ ) drops in anticipation of the future permanent tax cut and current economic activity declines. We illustrate the economy's response to the future permanent tax cut in Figure 3. We show that the prospective tax cut leads to an immediate decline in  $\hat{q}$  from point  $A$  to point  $B$  at  $t_0$ , and continues to fall further from point  $B$  to point  $C$ , which it reaches at  $t_1$ , the time of implementation.<sup>15</sup> Hence, employment contracts between announcement and implementation. At  $t_1$ , both the asset price as well as the level of debt cannot jump but the tax cut itself leads to an increase in output supply through the positive supply-side effect, which causes

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<sup>15</sup>Note that at  $t_0$ , the level of debt does not jump while the fall in asset price leads to a fall in output so the debt-GDP ratio rises. Consequently, point  $B$  lies south-east of point  $A$ .

$\hat{q} \equiv q/\Omega(q; \tau)$  and  $\hat{D} \equiv D/\Omega(q; \tau)$  to drop equiproportionately so moving from point  $C$  to point  $D$  along the ray  $OX$ . From that point onwards, the economy travels along the negatively-sloped saddle path to reach a new stationary state with higher debt-GDP ratio and lower market capitalization as a ratio to GDP at point  $E$ .

When we set  $\theta = 0$  so that Ricardian Equivalence holds, (23) and (24) are replaced by

$$\dot{\hat{D}} = \rho \hat{D} + \Phi(\hat{D}) - \hat{T}(\hat{q}; \tau), \quad (25)$$

$$\frac{\dot{\hat{q}}}{\hat{q}} = \rho - \frac{[1 - (\phi(\hat{q}))^{-1}]}{\hat{q}}. \quad (26)$$

The stationary  $\hat{q}$  locus is now horizontal in the  $(\hat{D}, \hat{q})$  plane. Figure 4 shows the path taken by the economy when there is a sudden expectation at  $t_0$  that there will be a permanent tax cut to be implemented at future time  $t_1$ . Since public debt is neutral,  $q$  at  $t_1$  must be expected to rise, whereupon present  $q$  (and so  $\hat{q}$ ) rises. Hence the paradoxical result of a dampening of present economic activity of the future stimulus requires non-Ricardian Equivalence.

## 5. Concluding Remarks

We have shown that under imperfect competition, a future debt bomb financed by future cuts in entitlement spending is neutral for asset prices and employment in a Ricardian world but contractionary in a non-Ricardian world. We also showed that a sudden expectation of future labor tax cuts without a sunset provision, with entitlement spending gradually adjusting to ensure fiscal sustainability, is expansionary in a Ricardian world but (for small tax changes) unambiguously contractionary in the time period before implementation in a non-Ricardian world.

How well does our model help to explain recent US experience starting from the internet boom from 1995/96? If future prospects of an upward lift to productivity made possible by technological advance in information and communications technology caused the boom, their materialization with a string of outsized productivity gains starting in 2001 should have stabilized the stock market at late-1990s levels and brought employment down for a soft landing at its pre-boom 1996 level. But that was not exactly what occurred. Instead, the stock market suffered a deep decline until regaining in 2003 and 2004 its 1998 level; correspondingly, the unemployment rate, participation rate and work week all overshot considerably their 1996 marks before achieving a mixed recovery by 2004: unemployment back to its 1996 level but participation and hours still markedly below their 1996 levels. The explanation that the analysis here offers (on top of others such as overinvestment and 9/11) is that the tax cut in 2001 with its backloading feature along with mounting awareness

of the bulge in retirement benefits threatening tax rates even above pre-cut levels caused a drop in the stock market starting in 2001, which reduced the demand wage and hours per worker. The low short rates of real interest that we have observed since 2001 are a further consequence. From our structuralist perspective, it comes as no great surprise that the buildup in 2001-2003 of fiscal stimulus and central bank interest rate cuts did not succeed by 2004 in staving off the economy's fall back to pre-boom levels or worse. The reduction of certain marginal tax rates may have had structural impacts every bit as effective as econometric estimates have led us to believe yet the former impacts may have failed to outweigh appreciably, if at all, their perverse impact in worsening the already alarming prospects for future fiscal deficits.

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### Appendix: Proofs

A.1. Technically, the trace of the  $2 \times 2$  matrix associated with the linearized dynamic system given below is positive, and the determinant is also positive:

$$[\dot{D} \quad \dot{q}]' = A[D - D_{ss} \quad q - q_{ss}]',$$

where  $[\dots]'$  denotes a column vector, the system (21) and (22) is linearized around the stationary-state values,  $D_{ss}$  and  $q_{ss}$ , and the  $2 \times 2$  matrix  $A$  contains the following elements:

$$\begin{aligned} a_{11} &\equiv \Upsilon + \Upsilon_D D_{ss}, \\ a_{12} &\equiv \Upsilon_q D_{ss} - T_q, \\ a_{21} &\equiv \Upsilon_D q_{ss}, \\ a_{22} &\equiv q_{ss} \left[ \Upsilon_q + \frac{[1 - V^f]}{(q_{ss}/\Omega)^2} \frac{d(q_{ss}/\Omega)}{dq} + \frac{V_q^f}{(q_{ss}/\Omega)} \right]. \end{aligned}$$

We can readily check that  $a_{11} > 0$ ,  $a_{21} > 0$  and  $a_{22} > 0$  while  $a_{12}$  can either be positive or negative. (Consequently, the trace of  $A$  ( $\text{Tr}(A)$ ) is clearly positive.) The sign of  $a_{12}$  depends upon the relative influence of a change in  $q$  on the tax revenue on the one hand, and the interest cost on the other hand. If a rise in the asset price raises the tax revenue by more than it raises the interest cost so a booming stock market leads to declining debt or, conversely, a depressed stock market leads to

rising debt, then  $a_{12}$  is negative, and the determinant of  $A$  ( $\text{Det}(A)$ ), equal to  $a_{11}a_{22} - a_{21}a_{12}$ , is clearly positive. In the alternative case when  $a_{12}$  is positive, we can check that  $(a_{11}/a_{12}) > (a_{21}/a_{22})$  so once again the determinant of  $A$  is positive. Therefore, the system represented by (21) and (22) is globally unstable. If the eigenvalues associated with the system represented by (21) and (22) are denoted  $\lambda_1$  and  $\lambda_2$ , we have that  $\lambda_1\lambda_2 = a_{11}a_{22} - a_{12}a_{21} > 0$ .

A.2. With entitlement spending as a ratio to GDP made a negative function of the debt-income ratio, we have  $a_{11} \equiv \mu + \mu_{\hat{D}}\hat{D}_{ss} + \Phi_{\hat{D}}$  in the linearized system of (23) and (24). In the empirically relevant case where a depressed stock market leads to rising debt-income ratios so  $a_{12} < 0$ , we find that in order to achieve saddle-path stability, it will be necessary though not sufficient for  $\hat{y}^s$  to fall in response to an increase in  $\hat{D}$  so that  $a_{11}$  is negative. In other words, a unit increase in the debt-income ratio necessitates a cut in entitlement spending as a ratio to GDP that more than offsets the rise in interest costs so that the debt-income ratio actually declines. The necessary and sufficient condition for saddle-path stability, and hence fiscal sustainability in response to a tax cut with gradual welfare payment adjustment, in the case when  $a_{12} < 0$  is for  $-(a_{11}) > -a_{12}(a_{21}/a_{22}) > 0$ . Noting that we can write down the respective slopes of the  $\dot{\hat{D}} = 0$  and  $\dot{\hat{q}} = 0$  loci as:

$$\begin{aligned} \left. \frac{d\hat{q}}{d\hat{D}} \right|_{\dot{\hat{D}}=0} &= \frac{-[\mu_{\hat{D}} + (\mu/\hat{D}_{ss}) + \hat{D}_{ss}^{-1}\Phi_{\hat{D}}]}{\mu_{\hat{q}} - (\hat{T}_{\hat{q}}/\hat{D}_{ss})} \equiv \frac{-a_{11}}{a_{12}}, \\ \left. \frac{d\hat{q}}{d\hat{D}} \right|_{\dot{\hat{q}}=0} &= \frac{-\mu_{\hat{D}}}{\mu_{\hat{q}} + \hat{q}_{ss}^{-1}[\mu - (\phi(\hat{q}_{ss}))^{-2}\phi'(\hat{q}_{ss})]} \equiv \frac{-a_{21}}{a_{22}} < 0, \end{aligned}$$

we obtain saddle-path stability only if both stationary loci are negatively sloped in such a way that

$$\left| \left. \frac{d\hat{q}}{d\hat{D}} \right|_{\dot{\hat{D}}=0} \right| > \left| \left. \frac{d\hat{q}}{d\hat{D}} \right|_{\dot{\hat{q}}=0} \right|.$$

If a decline in stock market capitalization as a ratio to GDP leads to bigger cost savings for the government (as a result of a huge drop in interest debt service burden) than its loss of tax revenue (relative to GDP) so  $a_{12} > 0$ , then the condition for fiscal sustainability is immediately satisfied by a fiscal rule that makes  $\hat{y}^s$  fall sufficiently in response to a rise in  $\hat{D}$  to make  $a_{11}$  negative. (Referring to (23) and (24), this condition says that when  $a_{12} > 0$  and  $a_{11} < 0$ , we are assured of saddle-path stability and the stationary locus for  $\dot{\hat{D}} = 0$  is positively sloped in the  $(\hat{D}, \hat{q})$  plane.) If  $a_{12} > 0$ , the condition that  $a_{11} < 0$  is sufficient for fiscal sustainability but it is not necessary. If declining asset prices lead to a smaller loss in tax revenue (relative to GDP) than the government can save from a decline in interest costs so the debt-income ratio actually falls, then, in order to attain fiscal sustainability, big cuts in entitlement spending may not be required when the debt-income ratio rises

so  $a_{11}$  remains positive so long as the condition  $0 < a_{11} < a_{12}(a_{21}/a_{22})$  is satisfied. Referring to (23) and (24), this condition says that when  $a_{12} > 0$  and  $a_{11} > 0$ , we obtain saddle-path stability if both stationary loci are negatively sloped in such a way that

$$\left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{D}=0} \right| < \left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{q}=0} \right|.$$

In summary, there are three cases where we obtain saddle-path stability. If a drop in  $\hat{q}$  leads to a greater loss in tax revenue (relative to GDP) than cost savings from a lower interest debt service burden so  $a_{12} < 0$ , the only way to achieve saddle-path stability is to cut entitlement spending as a ratio to GDP sharply enough to make not only  $a_{11} < 0$  but also to satisfy the condition:  $-a_{11} > -a_{12}(a_{21}/a_{22}) > 0$ . However, if a drop in  $\hat{q}$  leads to greater interest cost savings for the government than the amount of tax revenue lost (relative to GDP), saddle-path stability is guaranteed for a government that cuts entitlement spending as a ratio to GDP sufficiently to make  $a_{11} < 0$ . In this case, we have  $a_{12} > 0$  and  $a_{11} < 0$  so  $\text{Det}(A)$  is unambiguously negative. If  $a_{12} > 0$ , the government can, in fact, attain fiscal sustainability without sharp cuts to entitlement spending as a ratio to GDP so long as  $0 < a_{11} < a_{12}(a_{21}/a_{22})$ . Letting  $\lambda_1 = [\text{Tr}(A) - \sqrt{\text{Tr}(A)^2 - 4\text{Det}(A)}]/2$  be the negative root, the slope of the saddle path is given by  $(\lambda_1 - a_{11})/a_{12} = a_{21}/(\lambda_1 - a_{22})$ , which is unambiguously negative in all the three cases summarized here. It is readily checked that the qualitative results regarding the effects on asset prices and employment of the tax shocks we study are similar in all three cases. The differences occur in the short-term movement of the debt-income ratio in response to asset price changes.

A.3. To establish that an announcement made at  $t_0 = 0$  that the tax rate will be permanently reduced from  $\tau^0$  to  $\tau^1$ ,  $\tau^1 < \tau^0$ , from time  $t_1 = T$  onwards causes  $\hat{q}$  at  $t_0$  to drop, we proceed as follows. We let the eigenvalues associated with the system represented in Figure 3 and (23) and (24) be denoted  $\lambda_1$  and  $\lambda_2$ . The fact that the system is saddle-path stable means that the product  $\lambda_1\lambda_2 = a_{11}a_{22} - a_{12}a_{21} < 0$ . We shall assume  $\lambda_1 < 0$  and  $\lambda_2 > 0$ . Over the period  $0 < t \leq T$ , before the tax cut occurs, the solutions for  $\hat{D}_t$  and  $\hat{q}_t$  are of the form

$$\begin{aligned} \hat{D}_t &= \hat{D}_{ss}^0 + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}, \\ \hat{q}_t &= \hat{q}_{ss}^0 + \left( \frac{\lambda_1 - a_{11}}{a_{12}} \right) A_1 e^{\lambda_1 t} + \left( \frac{\lambda_2 - a_{11}}{a_{12}} \right) A_2 e^{\lambda_2 t}. \end{aligned}$$

We note that because  $\lambda_i$  are eigenvalues,

$$\frac{\lambda_i - a_{11}}{a_{12}} = \frac{a_{21}}{\lambda_i - a_{22}} < 0; \quad i = 1, 2.$$

For the period  $t > T$ , after the tax cut has occurred, the solutions for  $\hat{D}_t$  and  $\hat{q}_t$  are

$$\begin{aligned}\hat{D}_t &= \hat{D}_{ss}^1 + A'_1 e^{\lambda_1 t}, \\ \hat{q}_t &= \hat{q}_{ss}^1 + \left( \frac{\lambda_1 - a_{11}}{a_{12}} \right) A'_1 e^{\lambda_1 t}.\end{aligned}$$

Hence after time  $T$ ,  $\hat{D}_t$  and  $\hat{q}_t$  must follow the saddle path leading to  $\hat{D}_{ss}^1$  and  $\hat{q}_{ss}^1$ , respectively.

Assuming that at time 0,  $\hat{D}$  is at  $\hat{D}_{ss}^0$  implies  $A_1 + A_2 = 0$ . At time  $T$ , since  $q_T$  and  $D_T$  cannot jump, and noting that  $\hat{q} \equiv q/\Omega(q; \tau)$  and  $\hat{D} \equiv D/\Omega(q; \tau)$  so  $\hat{q}$  and  $\hat{D}$  change equiproportionately at time  $T$ , we obtain

$$\begin{aligned}[\Omega(q_T; \tau^0)A_1 - \Omega(q_T; \tau^1)A'_1]e^{\lambda_1 T} + \Omega(q_T; \tau^0)A_2 e^{\lambda_2 T} &= \beta_D, \\ \left( \frac{\lambda_1 - a_{11}}{a_{12}} \right) [\Omega(q_T; \tau^0)A_1 - \Omega(q_T; \tau^1)A'_1]e^{\lambda_1 T} + \left( \frac{\lambda_2 - a_{11}}{a_{12}} \right) \Omega(q_T; \tau^0)A_2 e^{\lambda_2 T} &= \beta_q,\end{aligned}$$

where

$$\begin{aligned}\beta_D &\equiv \hat{D}_{ss}^0 \Omega(q_T; \tau^0) \left[ \frac{\hat{D}_{ss}^1 \Omega(q_T; \tau^1)}{\hat{D}_{ss}^0 \Omega(q_T; \tau^0)} - 1 \right], \\ \beta_q &\equiv \hat{q}_{ss}^0 \Omega(q_T; \tau^0) \left[ \frac{\hat{q}_{ss}^1 \Omega(q_T; \tau^1)}{\hat{q}_{ss}^0 \Omega(q_T; \tau^0)} - 1 \right].\end{aligned}$$

Solving out  $A_1$  and  $A'_1$ , we obtain

$$A_2 = \left[ \Omega(q_T; \tau^0) e^{\lambda_2 T} \right]^{-1} \left[ \frac{\beta_q - \left( \frac{\lambda_1 - a_{11}}{a_{12}} \right) \beta_D}{\left( \frac{\lambda_2 - \lambda_1}{a_{12}} \right)} \right].$$

The initial response of  $\hat{q}$  at time 0 is given by

$$\hat{q}_0 - \hat{q}_{ss}^0 = \left( \frac{\lambda_2 - \lambda_1}{a_{12}} \right) A_2 = \left[ \frac{e^{-\lambda_2 T}}{\Omega(q_T; \tau^0)} \right] \left[ \beta_q - \left( \frac{\lambda_1 - a_{11}}{a_{12}} \right) \beta_D \right].$$

We note that  $(\lambda_1 - a_{11})/a_{12} = a_{21}/(\lambda_1 - a_{22})$ , which gives the slope of the saddle-path, is negative and that  $\beta_D > 0$  but the sign of  $\beta_q$  is ambiguous. However, we now show that for small changes in tax rates,  $\beta_q - [(\lambda_1 - a_{11})/a_{12}] \beta_D < 0$ , a condition that is satisfied if and only if

$$\left[ \frac{\frac{\Omega(q_T; \tau^1) \hat{q}_{ss}^1}{\Omega(q_T; \tau^0)} - \hat{q}_{ss}^0}{\frac{\Omega(q_T; \tau^1) \hat{D}_{ss}^1}{\Omega(q_T; \tau^0)} - \hat{D}_{ss}^0} \right] - \left( \frac{\lambda_1 - a_{11}}{a_{12}} \right) < 0.$$

For small changes evaluated around the original stationary state  $(\hat{D}_{ss}^0, \hat{q}_{ss}^0)$ , the square bracketed term converges to the value giving the gradient of the  $\dot{q} = 0$  locus at  $(\hat{D}_{ss}^0, \hat{q}_{ss}^0)$ , which is equal to  $-a_{21}/a_{22}$ . Since, as we can observe from Figure 3, the slope of the saddle-path has the smallest absolute value, that is,

$$\left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{D}=0} \right| > \left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{q}=0} \right| > - \left( \frac{\lambda_1 - a_{11}}{a_{12}} \right),$$

we have that

$$\frac{a_{21}}{a_{22}} > - \left( \frac{\lambda_1 - a_{11}}{a_{12}} \right).$$

Consequently, we establish that  $\hat{q}_0$  drops below  $\hat{q}_{ss}^0$  in response to the tax cut. Moreover, we find that the extent of the initial drop of  $\hat{q}$  is inversely related to how far away in time the tax cut will be implemented,  $T$ .

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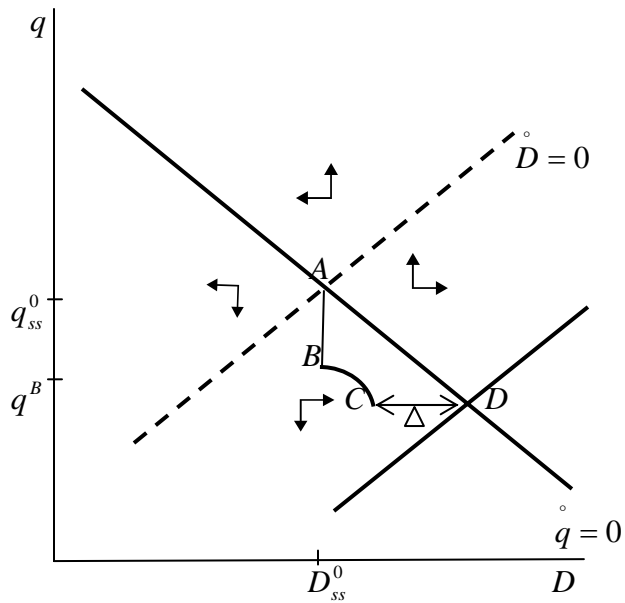


Fig.1. A future debt bomb financed by subsequent cuts in entitlement spending

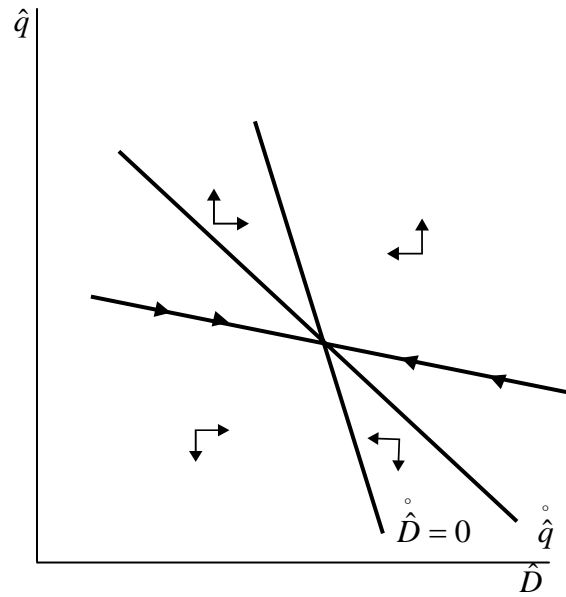


Fig.2. Saddle-path stability

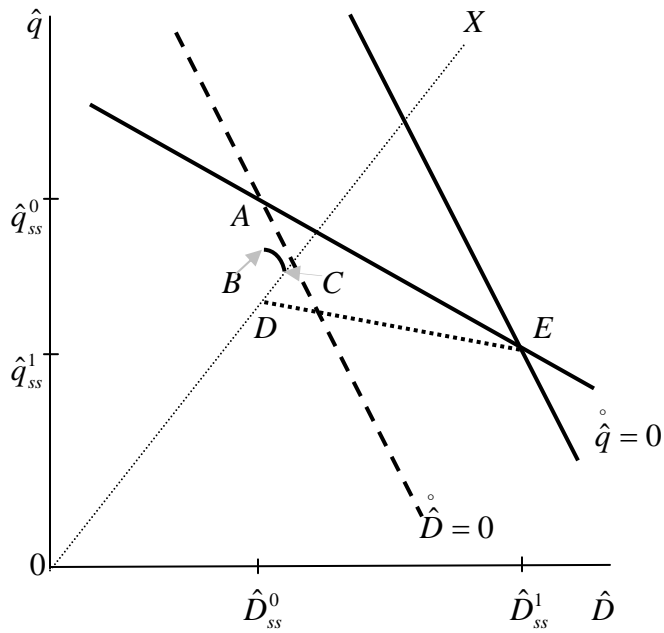


Fig.3. Future tax cut without sunset in non-Ricardian case

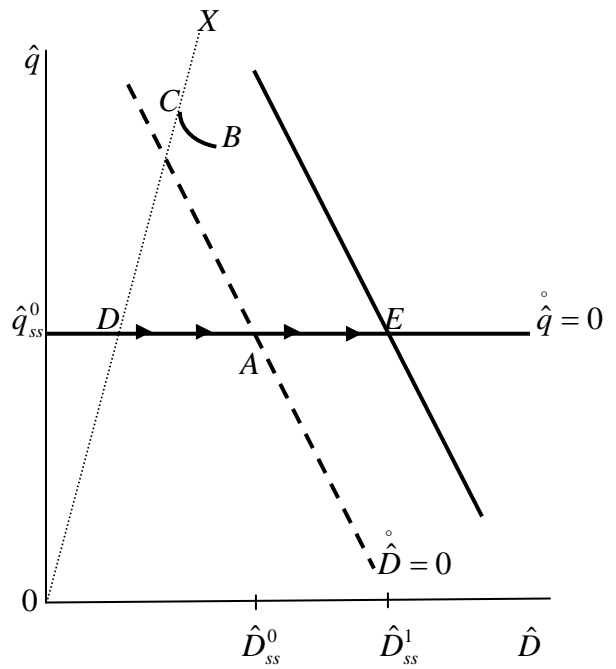


Fig.4. Future tax cut without sunset in Ricardian case