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Abstract

This paper considers the efficiency properties of risk-neutral workers' mobility decisions in an equilibrium model with search frictions, but no search externalities, when the rent accruing to a match is split through bargaining. Matches are ex ante homogeneous and their true productivity is learnt after the match is formed. It is shown that the efficiency of worker turnover depends on contract enforceability, and that in the absence of complete enforceability the equilibrium fails to be efficient. This is because without complete enforceability firms cannot credibly offer workers contracts that will guarantee them the entire future of all potential future matches.

Keywords: On-the-Job Search; Learning; Bargaining; Contracts; Enforceability.
JEL codes: J30; J63.

1 Introduction

The existence and the nature of worker turnover[1] has been studied by at least two separate strands of the economic literature. The first focuses on the fact that it is time consuming for workers (firms) to learn about employment opportunities (workers) available, i.e., there are search frictions, to explain that a worker who has just lost her job will typically go

[1]Empirical studies often define worker turnover as follows: worker turnover for a given period at a given establishment is the sum of the number of accessions and separations from the beginning to the end of the period. And aggregate worker turnover is the sum of worker turnover over all establishments.

through an unemployment spell. And once a worker finds a job, this new match will eventually get destroyed, typically because either some productivity shock renders the match no longer productive or the worker has found another match. The Diamond-Mortensen-Pissarides model (DMP) (see Diamond, 1982; Pissarides, 1984; Mortensen and Pissarides, 1994) and the Burdett-Mortensen (1998) model are the canonical models of this branch of the literature. Another strand of the literature, following Jovanovic (1979), focuses on the information frictions that limit the ex ante knowledge that a firm and a worker have about the quality of the match and on the fact that more information is learnt as the tenure of the worker on the job increases. In this case jobs are said to be experience goods.

The existence of search frictions means that when matches are ex ante heterogeneous, workers and firms do not wait for the perfect match to come along. Instead, a firm and a worker will agree to match if its value is acceptable to both of them, which requires the value of the match to each party to be above their own reservation values. However, because firms and workers do not always enter into the best possible match, there is an incentive for them to search for another better match, although it is standard to assume that only workers search while employed.[2] In this case, a job-to-job mobility decision is driven solely by the arrival of information about a new, more profitable match.

When jobs are experience goods, there is another source of worker turnover: since information about a match is discovered over time, the mobility decision is also motivated by what is being learnt about the quality of the current match. If the worker and the firm believe that the match they are in is of low quality, they might decide to severe the match, in which case, absent search frictions, the worker changes job and the firm hires a new worker.

The learning literature generally abstracts from search frictions, which ensures that workers get paid their expected marginal productivity and that their job mobility decisions are efficient (see Jovanovic, 1979, and Felli and Harris, 1996).[3]

On the contrary, random search models are often inefficient. In the search and matching models of the DMP vein the inefficiency is generic and arises because of search externalities (Mortensen, 1978, 1982; Diamond, 1982; Pissarides, 1984). For instance, when a worker

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[2] Mortensen (1978, 1982), Diamond and Maskin (1979) and Kiyotaki and Lagos (2007) are notable exceptions.

[3] Jovanovic (1984) extends his previous (1979) paper by introducing search frictions, but he maintains the assumption that workers get paid their expected marginal productivity.

searches for a job she neither takes into account the negative externality she imposes on the other workers searching, nor the positive externality she creates for firms seeking to fill vacant jobs. Unless the bargaining powers of workers and firms satisfy the Hosios condition,[4] the two externalities will not offset each other, implying that agents are searching inefficiently. Random search models where employment contracts, which in general are simple wage contracts, are posted rather than bargained do not suffer from such search externalities because contact rates are constant. However, although the original Burdett-Mortensen (1998) model with heterogeneous firms is efficient because more productive firms post higher wages, Stevens (2004) shows that when firms post employment contracts rather than wage contracts, the equilibrium need not be efficient. This is because with employment contracts firms can design contracts that ensures that separations between workers and firms are efficient, but the anonymity of contracts posted preclude firms from making sure that their recruitment policies are efficient.

This paper is part of the, still small, literature that aims at bringing together the search and learning literatures. More specifically, this paper is interested in the efficiency properties of the mobility decision of risk-neutral workers in an equilibrium with search frictions, but no search externalities, and when the rent accruing to a match is split through bargaining. The model is a variant of the Burdett-Mortensen framework where firms are endowed with a constant returns to labor production function. However, matches are here ex ante homogeneous, the quality of a match is learnt after the match is formed and employment contracts are not posted but negotiated before a match is formed. Firms and workers bargain over contingent contracts, which can include side-payments like sign-on fees or severance payments. Renegotiations are allowed, but both parties must agree to renegotiate for an employment contract to be modified. In the benchmark model it is further assumed that, although contracts are binding during an employment relationship since the terms of a contract these changes must be approved by both the worker and the firm, the enforceability of a contract is limited because either party can walk away from the match at no cost.

In this environment with limited enforceability, there is a maximum expected value of a match that new employers can credibly offer to a worker, and this maximum expected value depends on the maximum value that the employer can pledge for each possible realization

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[4] See Hosios (1990).

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of the quality of the match. This implies that there is a maximum productivity of a match for a currently employed worker beyond which she will no longer decide to change job. It is shown that with limited enforceability of employment contracts, the mobility decision of workers is inefficient, in that the highest productivity of a match for which workers will stop accepting a new job is lower than the efficient productivity cut-off, for as long as workers do not capture the entire surplus of a match.

The reason for the inefficiency lies, like for models of investments that are subject to hold-up problems (Grout, 1984; Acemoglu, 1997; Masters, 1998; Acemoglu and Shimer, 1999, Malcomson, 1999), in the incompleteness of contracts: an agent, in this case a worker, cannot contract with another agent she has not met yet, in this case a future employer. However, there is no investment in the present paper, and hence the efficiency uncovered is novel. Consider a worker who is deciding whether to change job and who is indifferent between the two contracts offered to her which are such that both her current employer and the poaching firm offer her the entire (expected) surplus of the respective matches. There is a possibility that this new match will be of a low quality, and in this case the worker will wish to find another job with another employer. If the worker could have contracted with that future employer at the moment of her first job change, she would have been able to obtain the entire expected surplus of the match from this firm as well. However, because the worker does not know which firm she will be in contact with in the future, she cannot contract with future employers, and the share of the surplus of her future jobs will be determined by the productivity of her new job. But if the productivity in her new job turns out to be low, she will obtain only a fraction of the surplus of the next match.

It turns out that the incompleteness of contract responsible for the inefficiency can be circumvented if there is full enforceability of contracts in that firms cannot walk away from a contract. This is because in this case a new employer can design a contract that guarantees to the worker it is trying to poach away that she will be able to capture the entire surplus of all future matches she will be a part of. These types of contracts require full enforceability because the value of a match to the firm is negative for a positive measure of values of the productivity draw, implying that for these productivity levels the firm would like to either renegotiate or walk away from the contract.

It has been established by a number of studies that job-to-job flows are large,[5] and the

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[5] For instance, for the US job-to-job transitions are roughly twice as large as flows from unemployment

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results of Nagypál (2007) indicate that learning is economically significant. Furthermore, Cahuc, Postel-Vinay and Robin (2005) estimate, using a French administrative data set and a variant of the Burdett-Mortensen model where wages are determined by bargaining, that the bargaining powers of the all but one group of workers are no greater than 40 per cent and can be as low as 0 per cent for the lowest-skilled groups. Although Cahuc, Postel-Vinay and Robin's model abstract from learning, these results suggest that the welfare consequences of employment contracts enforceability are potentially important.

In addition to the vast separate literatures on search and on learning mentioned earlier, this paper is related to papers by Jovanovic (1984), Moscarini (2005) and Nagypál (2007) who also consider search models where match quality is learnt over time. However, none of these papers is concerned about efficiency of the mobility decisions.

Papers by Burdett and Coles (2003), Stevens (2004) and Dey and Flinn (2005) are also related because they consider more general labor contracts than standard fixed-wage contracts. Postel-Vinay and Robin (2002a,b) and Cahuc, Postel-Vinay and Robin (2005) are also variants of the Burdett-Mortensen model that are interested in match turnover where firms and worker can renegotiate contracts when a worker is contacted by another firm. However, they consider environments where there is no uncertainty about a match quality, and efficiency is trivially obtained.

The models of Mortensen (1978, 1982), Diamond and Maskin (1979) and Kiyotaki and Lagos (2007) study match turnover where both partners to a match can search for another match, whereas in this paper firms do not seek to replace an employed worker.[6] The first two papers show that when an agent is forced to compensate the other(s) party(ies) to the match for her (their) loss(es) when the match is severed, efficiency is obtained. In Kiyotaki and Lagos' model, equilibrium mobility decisions are inefficient, but the inefficiency is due to search externalities arising from the quadratic matching function they assume.

The paper is organized as follows. Section 2 presents the model, and the efficient mobility decision is characterized in section 3. Section 4 characterizes the equilibrium mobility decision in the benchmark model with limited enforceability of contracts. Section 5 then analyzes the source of the inefficiency. Section 6 shows that efficiency can be restored if contracts are fully enforceable and discusses the efficiency result. Section 7 concludes.

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to employment. See Moscarini (2003) for more details.

[6]Firms' production technology displays constant returns to labor and therefore firms are happy to employ all workers whose value is positive to them.

2 The Model

2.1 The Environment

Time is continuous and the horizon is infinite. There is a unit mass of profit-maximizing firms, all of which are endowed with a constant returns to labor production function. The linearity of the technology implies the value of a worker to a firm is independent of the measure, and productivity, of all other workers employed by the firm.

There is a measure $n < 1$ of infinitely-lived workers who discount the future at rate $r > 0$ and maximize the discounted sum of expected flow utility. All workers are ex ante identical, and the flow utility of being unemployed is b while the flow utility of being employed at wage w is $u(w) = w$.

It is assumed that workers, both employed and unemployed, can search costlessly and that search is random. Workers[7] meet firms according to a Poisson process with exogenous parameter $\alpha > 0$. When a worker meets a firm, the productivity the worker will have in that match is ex ante unknown. If they agree to form a match, they then draw a productivity x from a known *cdf* F which has $X = [\underline{x}, \overline{x}]$ for support, with $\underline{x} > b$.[8] It is further assumed that the productivity draw is observed perfectly by the worker and the firm upon formation of a match. Matches between a firm and a worker are exogenously destroyed at an exogenous exponential rate δ. Moreover, when an employed worker meets another firm and decides to leave her current employer for the new employer, one job is destroyed in order to create another job.

In order to close the model we need to lay out the rules pertaining to the determination of a contract governing the relationship between a firm and an employee. First, it is assumed that all contracts are the outcome of a bargaining game between the worker and one or two firms,[9] and bargaining takes place before the productivity of a new match is known. I

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[7]In equilibrium all workers but those employed in jobs with the maximum productivity have a gain from search. Hence, the full measure of workers will be searching in equilibrium.

[8]This is a simplifying assumption. In fact, one could have more generally assumed that $\underline{x} < b$, but the derivations of the results are then more involved without affecting the results on efficiency.

[9]Because the probability of meeting two firms at the same time is zero, an unemployed worker will almost surely negotiate with only one firm at a time. I therefore ignore the possibility of a tripartite bargaining involving an unemployed worker. However, employed workers will be in contact with other firms, in which case the bargaining will involve three parties, ignoring once again measure zero events where a worker is contacted by two or more new firms at the same time.

will be considering an environment where both firms and workers can commit to long-term contracts, and where they can modify the terms of contract governing their relationship if and only if both agree to the modifications. In the benchmark case, however, there is limited enforceability of contracts as it is assumed that both workers and firms can walk away from a contract at no cost. This implies that if a party to the match refuses to renegotiate a contract the other part has two options, either to honor the current contract or to severe the match. This assumption regarding the extent to which workers and firms can commit to a contract will be modified in Section 5 to allow for complete enforceability.

There are two types of contacts between a worker and a new potential employer, since the worker can be either unemployed or employed at that time. There are therefore two types of games to be considered: a bargaining game between a firm and an unemployed worker (Game 1) and a bargaining game between an employed worker, her current employer and a newly contacted firm (Game 2). In either case the game has two stages, and it is assumed that bargaining takes place in artificial time so that there is no delay between the time a contact is made and the time the outcome of the relevant bargaining game is reached.

For Game 1 the bargaining game follows the ensuing protocol.

Game 1:

1. *In the first stage the firm proposes a contract to the worker; the worker either accepts, in which case bargaining is over, or she turns it down, in which case the bargaining process reaches the second round.*

2. *In the second stage the worker gets to propose a contract with probability $\beta \in (0,1)$ and the firm gets to propose with probability $1-\beta$. If both parties agree on the contract proposed, the match is formed. Otherwise bargaining stops and no match is formed.*

If a worker and a firm currently matched wish to renegotiate their contract, the renegotiation takes place following the rules of Game 1.

The rules of Game 2 are the following.

Game 2:

1. *In the first stage both firms simultaneously propose a contract to the worker; the worker either accepts an offer and signs a new contract or keeps her existing contract.*

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2. *In the second, and final, stage the worker gets to propose a contract to each firm with probability $\beta \in (0,1)$ and the firms with probability $1 - \beta$ the firms enter a Bertrand competition for the worker.*[10] *If the worker and one firm agree on the contract proposed, they match. If agreement is reached for both contracts, then the worker picks her preferred contract. Otherwise the worker keeps her previous contract.*

As mentioned earlier, bargaining takes place before the productivity of the worker in a new match is known. Hence, if bargaining is restricted to be over a constant wage, or to be independent of what the actual productivity of the match will be, inefficiencies can arise that originate solely from the restriction on the set of contracts allowed rather than from their enforceability. I thus allow contracts to be contingent on the productivity of the worker, to be tenure-dependent and to include side-payments. However, in the benchmark case some types of side-payments cannot exist in equilibrium: bonuses paid upon termination of a match or penalties for breach of a contract will not exist since both firms and workers can terminate a match and walk away from a contract if they wish to do so. Moreover, for this benchmark case with limited contract enforceability, I will be focusing on contracts that are renegotiation-proof in that neither the firm nor the worker have an incentive to walk away from a match.

At this point a few remarks are in order. The set-up has been chosen so as to focus on the source of the mobility inefficiency, and a number of modelling choices and simplifying assumptions can be relaxed without affecting qualitatively the results obtained.

The linear production technology is reminiscent of the one used by Burdett and Mortensen (1998), and it enables me to assume away externalities. It should also be mentioned that it would be possible to adopt the DMP matching framework and obtain the same inefficiency. But because the matching framework is inherently inefficient due to search externalities, the framework adopted here makes it clear that the source of the inefficiently is contractual, and not due to search externalities.

Similarly, it has been assumed that workers' search efficiency does not depend on their employment status. It would be possible to introduce differentiated arrival rates of meetings for unemployed and employed without changing the main result.

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[10]The result would be unchanged if the stochastic processes giving the right to make an offer were uncorrelated for the two firms.

The form of productivity uncertainty adopted in this paper resembles that of the stochastic job matching model of Pissarides (1984). There is one important difference compared with Pissarides. In his original model the value of the draw is known before the match is formed. However, Pissarides does not allow for on-the-job search and contracts are restricted to be wage contracts, and therefore, assuming that the draw is done before or after the match is formed is innocuous. In the present paper, it is crucial to assume that a worker needs to form a match with a firm in order to be able to learn about her actual productivity in that firm. This captures the idea, in the spirit of Jovanovic (1979, 1984), that jobs are experience goods and therefore that new jobs are uncertain. This uncertainty is both absolute, because the uncertainty reduces with tenure; and relative, since more is known about a job in which one has experience than about a new job.

The learning process in this paper is purposely kept as simple as possible by assuming that the productivity is learnt right after a match is formed.[11] The results I obtain would be left unchanged if instead it were assumed that learning was progressive, but it would complicate the algebra. Furthermore, the results of Pries (2004) indicate that match severance tends to happen early in the tenure of the worker. In fact, he finds that for the US, 80 per cent of match separations happen within the first two years of a match.

There are typically two ways to approach bargaining in search models: using the Nash solution or choosing a specific strategic bargaining game. With on-the-job search there are at least two reasons why the second solution is more appropriate and will therefore be adopted in this paper. First, when an employed worker encounters another firm, he has two potential employers to bargain with. If Nash bargaining were to be used, it would not be clear in such situations what would be the outside options and threat points of the worker and of each of the two firms, and any specific choice might seem arbitrary. Having recourse to a strategic bargaining approach, although the specific game chosen can itself be deemed arbitrary, makes it easier to identify the source of the inefficiency in the mobility decision of the worker, should there be any. Moreover, and perhaps more fundamentally, as Shimer (2006) has shown, the conditions of applicability of the Nash bargaining solution can be

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[11]Wright (1986) makes the same assumption. It would be easy to extend the analysis to introduce a signal of match quality before the decision to match is made. There then would exist a threshold for the signal for each productivity: workers employed in a job with productivity x would change jobs only if the signal obtained is above some value $p(x)$, with $p(x)$ being increasing in x. See Pries and Rogerson (2005) for a model with such signals.

violated when on-the-job search is allowed.

The bargaining games 1 and 2 are similar to the bargaining games adopted by Cahuc, Postel-Vinay and Robin (2005). The game played between an unemployed worker and a firm in their case is of the Rubinstein (1982) alternating-offers type, but the outcomes of their game and mine are identical, and correspond to the Nash solution when workers have a bargaining power of β. Game 2 aims at capturing the fact that when an employed worker is contacted by another firm, her situation improves because she can take advantage of the competition between the two firms.[12]

3 The Efficient Mobility Decision

In this section I present the constrained socially efficient allocation of workers across jobs, where the criterion used for efficiency is the maximization of steady-state output, augmented by the value of leisure for unemployed workers. That is, the efficient allocation is that of a social planner who maximizes

$$(1 - u) \int_{x \in X} x \, dG(x) + ub,$$

$$\text{subject to} \begin{cases} \dot{u} = \delta(1 - u) - \alpha u, \text{ and} \\ \dot{G}(x) = \alpha \left[\frac{u}{1-u} + \int_{z \in X} \sigma_z dG(z) \right] F(x) \\ \qquad - \left[\delta G(x) + \alpha \int_{\underline{x}}^{x} \sigma_z dG(z) \right], \end{cases}$$

where u is the measure of unemployed worker; G is the cumulative distribution function of matched workers' productivities; and σ_z is an indicator function which is equal to 1 if a worker employed in a job with productivity z changes employer when a new firm comes along, and zero otherwise.

Steady-state output, augmented with leisure of unemployed workers, is maximized if and only if the decisions regarding the formation of matches are efficient. There are only two decisions in the model, whether a match that has just been formed and whose productivity has been revealed should be continued; and whether a match between a firm and a worker contacted by another firm should be severed so that the worker can join the new firm.

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[12]Moscarini (2005) abstracts from this competition effect on the share of the surplus a worker obtains when in contact with two firms. However, the results of Cahuc, Postel-Vinay and Robin suggest that this competition effect is important, and that it can be responsible for between 40 to 100 per cent of wage increase for some groups of workers.

Since it is assumed that $\underline{x} > b$, that there is no cost of search, and there is no difference in the efficiency of search while employed and unemployed, we have, trivially, that it is always efficient for a worker who does not have any other option than unemployment to be matched. Hence, the allocation of workers across firms is efficient if and only if the mobility decision between jobs is efficient. And it is efficient if and only if the expected value of the new match is greater than the value of the current match.

Formally, an unemployed worker enjoys a flow value b and he encounters a firm at rate α. The expected capital gains arising from the formation of a new match are $\omega^e - \omega_u$, where $\omega^e = \int_{x \in X} \omega_x dF(x)$ is the expected value of a match and ω_u is the value of an unemployed worker. Hence, we have that ω_u is such that

$$r\omega_u = b + \alpha \left(\omega^e - \omega_u \right). \tag{1}$$

When a worker is working in a job with productivity x and the match is destroyed, ω_x is lost. If the match destruction is exogenous, the worker becomes unemployed and the capital loss is $\omega_x - \omega_u$. If however the match has been destroyed because the worker leaves for another firm, the expected capital gain is then $\omega^e - \omega_x$. Hence, we have that the value of a match with productivity x solves

$$r\omega_x = x + \alpha\sigma_x \left(\omega^e - \omega_x \right) - \delta \left(\omega_x - \omega_u \right). \tag{2}$$

It is clear that the capital gains from a job mobility decision are positive if and only if $\omega^e \geq \omega_x$. It follows that the efficient mobility decision is given by a reservation strategy with cut-off $\widetilde{x}^*$ and the mobility decision follows

$$\sigma_x = \begin{cases} 1 & , & \text{for } x < \widetilde{x}^*; \\ [0,1] & , & \text{for } x = \widetilde{x}^*; \\ 0 & , & \text{for } x > \widetilde{x}^*, \end{cases}$$

This is intuitive. If a worker is currently employed in a low productivity match, say $x = \underline{x} + \epsilon$ for ϵ small, then the probability with which the worker would draw a higher productivity when changing jobs is $1 - G(x)$ which is very close to 1 for a small enough ϵ. Therefore, the risk of drawing a lower productivity than her current one is small, and the expected gains from changing employer will be positive. If, on the contrary, the worker is currently employed in a match with a high productivity, say $x = \overline{x} - \epsilon$, then it is highly likely she will

be less productive in a new job. Therefore, the expected gains from changing jobs will be negative.

Equation (2) yields that the value of a match with productivity x is

$$
\omega_x = \begin{cases} \frac{x+\delta\omega_u+\alpha\omega^e}{r+\delta+\alpha} & , \quad \text{if } x < \widetilde{x}^*, \text{ and} \\ \\ \frac{x+\delta\omega_u}{r+\delta} & , \quad \text{otherwise.} \end{cases}
$$

When a worker is in a job whose productivity x is below the mobility cutoff, the match is destroyed at rate $\alpha + \delta$: New jobs arrive at rate α, in which case the expected new value of employment becomes ω^e; and jobs are exogenously destroyed at rate δ, in which case the new value for the worker is the value of unemployment ω_u. Hence, the expected discounted value of a match with productivity $x < \widetilde{x}^*$ is given by $(x + \delta\omega_u + \alpha\omega^e)/(r + \delta + \alpha)$. When the mobility cutoff is lower than that of the productivity of the match, the match is still severed exogenously at rate δ, but there is no longer any endogenous job destruction. Hence, the value of such a match is $(x + \delta\omega_u)/(r + \delta)$.

Solving for ω^e yields

$$
\omega^e = \frac{\delta\omega_u}{r+\delta} + \Delta(\widetilde{x}^*; \alpha),
$$

where

$$
\Delta(\widetilde{x}^*; \alpha) \equiv \frac{1}{r+\delta+\alpha\left(1-F\left(\widetilde{x}^*\right)\right)} \left[x^e + \frac{\alpha}{r+\delta} \int_{\widetilde{x}^*}^{\overline{x}} x dF(x) \right].
$$

Hence we obtain that

$$
\omega_x = \begin{cases} \frac{x+\Delta(\widetilde{x}^*;\alpha)}{r+\delta+\alpha} + \frac{\delta\omega_u}{r+\delta} & , \quad \text{if } x < \widetilde{x}^*, \text{ and} \\ \\ \frac{x}{r+\delta} + \frac{\delta\omega_u}{r+\delta} & , \quad \text{otherwise.} \end{cases}
$$

Since the efficient mobility cut-off is such that $\omega_{\widetilde{x}^*} = \omega^e$ we have the following proposition.

Proposition 1 *The efficient mobility cut-off $\widetilde{x}^*$ is such that*

$$
\widetilde{x}^* = x^e + \frac{\alpha}{r+\delta} \int_{\widetilde{x}^*}^{\overline{x}} (x - \widetilde{x}^*)\, dF(x), \tag{4}
$$

and it is unique.

Proof. In the Appendix. ∎

It is worth noting for further reference that $\widetilde{x}^* > x^e$. The intuition behind this result is the following. Assume workers are restricted to only one job change in between unemployment spells. The value of the current job is such that $r\omega_x = x - \delta(\omega_x - \omega_u)$, whereas the expected value of the new job is $\omega^e = \int_{\underline{x}}^{\overline{x}} \omega_x dF(x)$, where $r\omega_x = x - \delta(\omega_x - \omega_u)$ for all possible values x of the draw. We therefore have that in this case $\widetilde{x}^* = x^e$. Now allow for more than one job changes in between unemployment spells. If a worker currently employed in a job with productivity x who changes job draws a low level of productivity for her new job, although this decreases the aggregate production level, she will be allowed to search for another job, and therefore she will then have the possibility of having a good draw. Formally, if the worker does not change jobs the expected value of the match is still such that $r\omega_x = x - \delta(\omega_x - \omega_u)$, whereas if she takes the risk of changing jobs the expected discounted value of the new job is $\omega^e = \int_{\underline{x}}^{\overline{x}} \omega_z dF(z)$, where now $r\omega_z = x + \alpha(\omega^e - \omega_z) - \delta(\omega_z - \omega_u)$ for $z \leq \widetilde{x}^*$ and $r\omega_z = x - \delta(\omega_z - \omega_u)$ otherwise. Hence, the possibility of looking for another job if a productivity draw is unsatisfactory makes a new job more attractive than if on-the-job search is ruled out, explaining that the social planner demands workers to change jobs even for some job productivities that are above the expected productivity from a draw.

4   Equilibrium with Limited Enforceability of Contracts

4.1   Value Functions

By standard arguments, U, the value of search to an unemployed worker solves

$$rU = b + \alpha\left(W^e\left(b\right) - U\right), \tag{5}$$

where $W^e\left(b\right)$, the expected value of employment for an unemployed worker, is such that

$$W^e\left(b\right) = \int_{x \in X} W_x^0\left(C_x\left(b\right)\right) dF(x).$$

$C_x\left(b\right)$ denotes the contract that will be implemented if the productivity drawn is x when $\boldsymbol{C}\left(b\right) = \left(C_x\left(b\right)\right)_{x \in X}$ is the menu of contingent contracts that has been negotiated between the firm and the unemployed worker. The initial value of the match to the worker at tenure 0 for productivity x according to $\boldsymbol{C}\left(b\right)$ is $W_x^0\left(C_x\left(b\right)\right)$. Although, as will be shown further below, there might be more than one menu of contracts solving Game 1, all equilibrium

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menus of contracts negotiated between a firm and an unemployed worker must yield the same expected value of a match $W^e(b)$ to the worker.

When a worker employed by a firm i meets another firm j, she must decide whether to quit her current job. This decision will depend on the value of the match she can obtain from her current employer i and from firm j during the negotiations taking place according to the protocol laid out in Game 2. In general, the contracts available to the worker as outcomes of Game 2 can depend on C, the previous contract between the worker and her current employer, on the tenure t under this contract, as well as on the productivity of her current match x. Let $\boldsymbol{C}(x, C, t) = (C_z(x, C, t))_{z \in X}$ denote a menu of contracts available to the worker if she decides to join firm j, and let $\widetilde{C}_x(x, C, t)$ denote a contract available from her current employer. As in the case of negotiations between an unemployed worker and a firm, there can be more than one contract, or menu of contracts, that are outcomes of Game 2 for a given triple (x, C, t). However, all the possible contracting outcomes of Game 2 must yield the same values of employment, $W_x^0(\widetilde{C}_x(x, C, t))$ and $W^e(x, C, t)$, of staying with a firm when the job productivity is x and of joining a new employer respectively.

Naturally a worker changes employers if the contract offered by this new firm yields a higher value to the worker than what her current employer can propose, i.e., if $W^e(x, C, t) \geq W_x^0(\widetilde{C}_x(x, C, t))$. Hence, when a worker is employed in a job with productivity x and with tenure t under a contract C, $W_x^t(C)$, the value function for the worker is such that for all $t \geq 0$,

$$
\begin{aligned}
rW_x^t(C) \;=\; & w^t(C) + \alpha\left(1 - \gamma_x^t(C)\right)\left(W_x^0\left(\widetilde{C}_x(x, C, t)\right) - W_x^t(C)\right) \\
& + \alpha\gamma_x^t(C)\left(W^e(x, C, t) - W_x^t(C)\right) - \delta\left(W_x^t(C) - U\right) + \dot{W}_x^t(C).
\end{aligned}
\tag{6}
$$

The wage the worker receives under the contract C after tenure t on the job is $w^t(C)$, and $\dot{W}_x^t(C)$ is the time derivative of the value function $W_x(C)$. The mobility decision of the worker is given by the indicator function $\gamma_x^t(C)$: it is equal to 0 if the worker stays in her current job under a possibly improved contract, and it is 1 otherwise. It is assumed, without loss of generality, that when a worker is indifferent between the contracts offered by her current employer and a another firm, she chooses to change employer. Note that, given that the wage is allowed to be tenure-dependent, it includes all payments made. For instance, if a contract signed by the worker with her new employer includes a sign-on bonus ϕ and prescribes a flat wage afterwards, i.e., $w^t(C) = w$ for all $t > 0$, then the wage payment at

tenure 0 is $w^0(C) = w + \phi$.

For a firm, the value of a match with productivity x when the contract governing its relationship with the worker is C and the tenure of the worker under contract C is t is denoted $J_x^t(C)$, and is such that for all $t \geq 0$,

$$rJ_x^t(C) = x - w^t(C) - \alpha\left(1 - \gamma_x^t(C)\right)\left(J_x^t(C) - J_x^0\left(\widetilde{C}_x(x,C,t)\right)\right) \tag{7}$$
$$- \left(\alpha\gamma_x^t(C) + \delta\right) J_x^t(C) + \dot{J}_x^t(C).$$

I will denote the value of a match with productivity x governed by a contract C and tenure t by $V_x^t(C) \equiv W_x^t(C) + J_x^t(C)$. Hence, (6) and (7) yield that for all $t \geq 0$,

$$rV_x^t(C) = x + \alpha\left(1 - \gamma_x^t(C)\right)\left(V_x^0\left(\widetilde{C}_x(x,C,t)\right) - V_x^t(C)\right) \tag{8}$$
$$+ \alpha\gamma_x^t(C)\left(W^e(x,C,t) - V_x^t(C)\right) - \delta\left(V_x^t(C) - U\right) + \dot{V}_x^t(C).$$

Because contracts allow for tenure-varying wages, the value of a match to the firm and the worker can change with tenure. Moreover, although the productivity of a match does not change over time, the joint private value of a match can also vary with tenure because the worker's mobility decision can change with tenure. For instance, if a contract prescribes that the value of the match to the worker increases with tenure, then the mobility decision, as well as the value of changing employer, can also vary with tenure. Denote by $\overline{W}^e$ the maximum expected value of a job a new employer can offer a worker. A worker will change employer if and only if she obtains a higher value of a match with a new employer than what her current employer offers. If the contract linking the worker with her current employer is such that the value of the match to the worker increases with tenure, there might be a tenure T such that for all tenure $t < T$, the current employer cannot, or does not want to, offer a contract worth $\overline{W}^e$ to its employee, i.e., the worker changes employer; but after tenure T the worker's value of the matches prescribed by the contract is strictly greater than $\overline{W}^e$, in which case the worker stays in her job. Under this scenario the mobility decision $\gamma_x^t(C)$ depends on tenure, implying the joint value of the match varies with tenure as well.

4.2 Equilibrium Mobility Decision

The productivity of a match is fixed once a worker matches with a firm. Hence, the only possible sources of variation with tenure of the joint value of a match are the indicator

function $\gamma_x^t(C)$, and the values $W^e(x, C, t)$ and $V_x^0(\widetilde{C}_x(x, C, t))$. However, since the productivity of a match is fixed, the joint private value of a match $V_x^0(\widetilde{C}_x(x, C, t))$ can differ from $V_x^t(C)$ if and only if either $\gamma_x^0\left(\widetilde{C}_x(x, C, t)\right)$ or $W^e(\mathbf{C}(x, \widetilde{C}_x(x, C, t), t))$ differ from $\gamma_x^t(C)$ or $W^e(x, C, t)$. This implies that the joint private value of a match can vary with tenure if and only if $\gamma_x^t(C)$ or $W^e(x, C, t)$ change with tenure. A priori, the maximum expected joint private value of a match $\overline{V}_x$ could be different from $V_x^t(C)$ for some t, although we should have $V_x^0(C) = \overline{V}_x$. In fact, given that there are maximum and minimum initial values of a match that can be given to a firm and a worker forming a new match, maximizing the value of a match at the onset could require that the split of the value of the match between the firm and the worker changes with tenure. And this could then imply that the value of the match increases or decreases compared to its initial maximum value.

We will now show that, in equilibrium, this will not be the case. To show this, let us consider the mobility decision of a worker employed in a firm when the match has productivity x, the contract in place is C, and the tenure on this contract is t. There is a maximum expected value that a new potential employer is willing to accept to give to the worker, $\overline{W}^e$, and this value is the maximum expected joint private value of the match $\overline{V}^e$. All contracts that deliver this maximum expected joint private value must be such that, for all possible productivity draws and for all tenures, the value of the match to the firm is not negative, for otherwise it would be better off laying off the worker;[13] And the value of employment to the worker is no less than the value of unemployment, for otherwise the worker would be better off quitting. This implies that $\overline{V}^e = \int_{z \in X} \overline{V}_z dF(z)$. Naturally, the maximum value that the worker's current employer is willing to accept to give to the worker is $\overline{V}_x$. In addition, the worker has a current contract C with her employer, and this contract can prescribe an increase in value for the worker upon contact with another firm. However, the maximum value prescribed must be less than $\overline{V}_x$ if the contract is to be renegotiation proof.

According to the rules of Game 2, the worker gets to propose a contract to the firms with probability β and the firms propose contracts with probability $1 - \beta$. If the worker proposes to the firms, she will ask for $\overline{V}^e$ from the new firm and $\overline{V}_x$ from her current employer. If the firms propose, then the new firm proposes $\min\{\overline{V}^e; \overline{V}_x\}$, whereas the current employer

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[13] If a menu of contract specifies that in some circumstances the value of the job to the firm is negative, the worker knows the firm will not honor the contract, and therefore the menu of contracts is not renegotiation proof.

proposes $\min\{\overline{V}^e; \overline{V}_x\}$. The worker therefore expects to obtain

$$\beta \overline{V}_x + (1 - \beta) \min\{\overline{V}^e; \overline{V}_x\}$$

from her current employer in the second round, while she can guarantee

$$\beta \overline{V}^e + (1 - \beta) \min\{\overline{V}^e; \overline{V}_x\}$$

to herself from the new firm. Hence, if follows that in the first round, if $\overline{V}^e \geq \overline{V}_x$, the current employer offers $\overline{V}_x$ and the new firm offers $\beta \overline{V}^e + (1 - \beta)\overline{V}_x$; Whereas if $\overline{V}^e < \overline{V}_x$, the current employer offers $\beta \overline{V}_x + (1 - \beta)\overline{V}^e$ and the new firm offers $\overline{V}^e$. In this latter case it can be that the initial contract C linking the firm to its current employer prescribes that the worker should receive more than $\beta \overline{V}_x + (1 - \beta)\overline{V}^e$, but in any case this must be less than $\overline{V}_x$. Hence, the mobility decision of the worker in a match with productivity x under contract C after tenure t is

$$\gamma_x^t(C) = \begin{cases} 1, & \text{if } \overline{V}^e \geq \overline{V}_x, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

We first note that the mobility decision indicator function $\gamma_x^t(C)$ does not vary with tenure, nor with the particular contract C governing the employment relationship between a firm and a worker. Moreover, if the worker changes job when she is offered a new job, the expected value of the new match is $\beta \overline{V}^e + (1 - \beta)\overline{V}_x$, which is also independent of the actual contract, and tenure under this contract, linking the worker to her previous employer. Hence, for any contract C belonging to the equilibrium set of contracts, the joint private value of a match solves

$$rV_x = x + \alpha \gamma_x \left(W^e(x) - V_x \right) - \delta \left(V_x - U \right),$$

where $W^e(x) \equiv \beta V^e + (1 - \beta)V_x$. All references to tenure or a specific contract have been dropped, and it is recognized that all matches at all times have a joint private value equal to the maximum joint private value that I simply denote by V_x when the productivity is x. Hence,

$$rV_x = x + \beta \alpha \gamma_x \left(V^e - V_x \right) - \delta \left(V_x - U \right), \tag{10}$$

and, denoting by x^* the equilibrium mobility cutoff, i.e., the productivity value such that $V_{x^*} = V^e$, the equilibrium mobility decision is given by

$$\gamma_x = \begin{cases} 1, & \text{for } x \leq x^*, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

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Although the mobility decision indicator function and the expected value of a new job are uniquely determined for a worker employed in a job with productivity x, the set of equilibrium contracts yielding these is not a singleton. For instance, both tenure-dependent and -independent contracts are possible in equilibrium. The only restrictions these contracts must satisfy, in addition to their initial values being equal to $V_x + \beta (V^e - V_x)$, is that for all possible productivity levels and any tenure $t \geq 0$ the value of the match to the worker is no less than the value of unemployment, for otherwise he would be better off quitting the job; and the value of the match to the firm is non-negative, for otherwise the firm would lay off the worker. Hence, any menu of contracts $\boldsymbol{C}(x)$ such that $W^e(x) = V_x + \beta (V^e - V_x)$ and $W_z^t(C_z(x)) \in [U, V_z]$ for all $z \in X$ and all tenure $t \geq 0$ is an equilibrium contract. This is reminiscent of Stevens (2004), who, in a set-up also based on Burdett and Mortensen (1998) but where contracts are posted by firms rather than being bargained over, obtains that there is more than one contract delivering the same allocation of workers and the same initial value of a job to both firms and workers. One example of an admissible contract, which is the type of contract that standard random search models focus on, is a flat-wage contract, where for all z, all t,

$$W_z^t (C_z(x)) = V_x + \beta (V_z - V_x).$$

Another type of contract, which is the type Stevens (2004) focuses on is a fee-contract. For such contracts a worker joining a new firm pays a sign-on fee, which can be contingent on his productivity, and the worker receives the entire product of the match thereafter.

We now turn our attention to the bargaining game played by an unemployed worker and a firm, Game 1. In the second stage of the game, when the worker gets to propose a contract she will propose a contract that gives her an expected value of V^e. The firm, when it gets to propose a contract, offers a contract whose value to the worker equals the value of unemployment U. Hence, the worker expects to obtain $\beta V^e + (1 - \beta)U$ if the negotiations reach the second round, and the firm consequently offers her in the first round a menu of contracts $\boldsymbol{C}(b)$ such that the expected value of matching is given by

$$W^e(b) = U + \beta(V^e - U). \tag{12}$$

Naturally, $\boldsymbol{C}(b)$ must also be such that $W_x^t(C_x(b)) \in [U, V_x]$ for all $x \in X$ and all tenure $t \geq 0$ to be a renegotiation-proof contract. Once again, although the initial expected value

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of a match is unique for both the worker and the firm, the set of equilibrium contracts is not a singleton.

Given (5) and (12), the value of unemployment can be rewritten as

$$rU = b + \beta\alpha\left(V^e - U\right). \tag{13}$$

The joint private value of a match satisfies (10) with the mobility decision given by (11). Therefore, it follows that

$$V_x = \begin{cases} \frac{x+\delta U+\beta\alpha V^e}{r+\delta+\beta\alpha} & , \quad \text{for } x \leq x^*, \text{ and} \\[2ex] \frac{x+\delta U}{r+\delta} & , \quad \text{otherwise.} \end{cases}$$

The interpretation of the private joint value of the match V_x is similar to the one for the social value, ω_x, except that in the market economy, when a worker changes jobs she captures a fraction β of the extra value generated from this job change. Solving for V^e yields

$$V^e = \frac{\delta U}{r+\delta} + \Delta(x^*; \beta\alpha),$$

and hence we obtain that

$$V_x = \begin{cases} \frac{x+\Delta(x^*;\beta\alpha)}{r+\delta+\alpha} + \frac{\delta U}{r+\delta} & , \quad \text{if } x < x^*, \text{ and} \\[2ex] \frac{x}{r+\delta} + \frac{\delta U}{r+\delta} & , \quad \text{otherwise.} \end{cases}$$

Proposition 2 states that the equilibrium mobility cut-off x^* exists and is unique, and that as long as worker do not have all the bargaining power the equilibrium mobility cutoff is always lower than the efficient one.

Proposition 2 *(i) The equilibrium mobility cut-off x^* is such that*

$$x^* = x^e + \frac{\beta\alpha}{r+\delta} \int_{x^*}^{\overline{x}} (x-x^*)\,dF(x), \tag{14}$$

and it is unique.

(ii) For all $\beta \in [0,1]$, $x^* \leq \widetilde{x}^*$, with strict inequality for $\beta < 1$.

Proof. In the Appendix. ∎

5 Understanding the Inefficiency

The inefficiency of the equilibrium mobility decision of workers comes from the interaction between (i) the incompleteness of contracts, and (ii) the asymmetry in the positions an incumbent firm and a poaching firm are in because of the asymmetric information on the quality of the two matches with the worker. When a firm negotiates with a worker, they split the expected surplus of the match according to their respective bargaining powers, and a firm can, if necessary, pledge to give the worker the entire (expected) surplus of the match in order to keep (recruit) her. However, there is an asymmetry between a firm negotiating with a current employee and a firm trying to poach the worker from another firm. If the worker decides not to quit her current employer, it is because the contract with her current employer is better than what she can obtain from a new job, and she will therefore never quit her job. Hence, the joint private value of the match formed with her current employer is such that

$$rV_x = x - \delta(V_x - U).$$

If the worker instead decides to change job, the expected joint value of the new match solves

$$rV^e = x^e + \int_{x \in X} [\gamma_x (W^e(x) - V_x) - \delta(V_x - U)] \, dF(x).$$

The asymmetry arises from the fact that if the productivity draw is below the mobility cut-off x^*, the worker will be looking to switch jobs again. And in this case, even if the firm the worker just joined gave her the entire surplus of the match for each possible productivity draw, when $x < x^*$ the worker's value of her current match will be $V_x < V^e$. Hence, when she switches jobs again, the expected value she will be able to secure from the subsequent match is $W^e(x) = V_x + \beta(V^e - V_x)$, which is strictly less than V^e for all $x < x^*$. That is, she will capture only a fraction β of the additional surplus $(V^e - V_x)$ derived from the creation of this subsequent match. This implies that when a worker changes jobs, although the firm trying to poach her can pledge the entire expected surplus of this new match, because the firm cannot include in her menu of contracts guarantees regarding the split of the surplus with subsequent employers, should the draw of productivity be bad, this surplus does not include the entire expected surplus of possible subsequent matches when $x < x^*$. The incompleteness of contracts comes from the fact that a worker cannot contract with potential future employers since she does not who that employer will be. If workers were able to write down contracts with their future potential employers at the time of a job change,

they would be able to obtain the entire expected surplus of a match from these potential future employers, and efficiency would be obtained. This type of contract incompleteness is related to the contract incompleteness responsible for hold-up problems in models with investment.

5.1 Efficiency Without Uncertainty

To illustrate the fact that the inefficiency requires both incompleteness of contracts and uncertainty regarding new jobs' productivity, let us consider the case where the productivity a worker will have in a new job is known to her prior to deciding whether to accept a new job. In this case, the value of unemployment U still solves (9): when a worker encounters a firm and the productivity of the match if it is formed is known to be x, the firm will propose $\beta V_x + (1 - \beta)U$ to the worker in the first round of Game 1, and the worker accepts. The value of being employed in a job with productivity x under contract C with tenure t follows

$$
\begin{aligned}
rW_x^t(C) \;=\; & w^t(C) + \dot{W}_x^t(C) + \alpha \int_{z \in X} \gamma_{x,z} \left(W_z^0(C_z(x)) - W_x^t(C) \right) dF(z) \\
& + \alpha \int_{z \in X} \left(1 - \gamma_{x,z} \right) \left(W_x^0 \left(\widetilde{C}_x(z,C) \right) - W_x^t(C) \right) dF(z) - \delta \left(W_x^t(C) - U \right).
\end{aligned}
$$

Following the line of argument used earlier, we know that when the worker is in contact with another firm, the decision of whether to change jobs, summarized in $\gamma_{x,z}$, depends on the productivity of her current job x and of the new match offered to her, z; And it does not depend on her tenure nor on the specific contract C she was working under. When the worker does not change jobs, the value of staying with the current employer can increase, i.e., $W_x^0(\widetilde{C}_x(z,C)) > W_x^t(C)$, but that depends on the contract that is currently linking her to her employer and the productivity of the new job she was proposed. Here the value of the match to the worker can still change with tenure because the wage is allowed to be tenure-dependent. We can easily derive a similar equation for the value of a match to the firm and obtain that the joint private value of a match with productivity x follows

$$
rV_x = x + \alpha \int_{z \in X} \gamma_{x,z} \left(W_z^0(C_z(x)) - V_x \right) dF(z) - \delta \left(V_x - U \right).
$$

It is quite straightforward to establish that following the protocol of Game 2, a worker will change job if and only if $z \geq x$, which corresponds to the efficient mobility decision. And in this case the worker's new value of a match is $W_z^0(C_z(x)) = V_x + \beta(V_z - V_x)$. Hence, despite the fact that a worker does obtain only a fraction β of the additional surplus

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created by her job change, the mobility decision is efficient. Efficiency is obtained in this case because two firms competing for a worker are in a symmetric situation. In fact, the surplus of either match depends on the value of future matches in higher productivity jobs, and in either case the worker will obtain a fraction β of the additional surplus created by her job changes.

6 Efficiency With Complete Enforceability of Contracts

Despite the incompleteness of contracts, it is possible to obtain efficiency of the mobility decision without altering the random search assumption.[14] Efficiency requires extending the ability of firms to commit to honoring a contract to situations where they would have liked to severe the match because its value to them is negative. More precisely, if contracts are fully binding, firms and workers will be able to negotiate contracts that render the equilibrium mobility decision efficient because contracts guarantee that a worker will secure the entire surplus of future matches should she decide to change jobs.

6.1 Efficient Contracts

Consider a worker currently unemployed and who is bargaining with a firm. Consider the menu of contract $\mathbf{C}(b)$ such that $W_x^t(C_x(b)) = V^e$ for all possible productivity levels x below the mobility cut-off for all tenure $t > 0$, and such that $W^e(b) = U + \beta (V^e - U)$. The expected value of the menu of contracts to the firm is $(1 - \beta) V^e$. This menu of contracts, to be implemented, requires that a firm be able to commit to a contract, no matter whether it regrets it ex post. In fact, $\mathbf{C}(b)$ prescribes that the worker gets V^e for all values of productivities $x \leq x^*$ after the match is created, and we have that the value of the match to the firm is $J_x^t(C_x(b)) = V_x - W_x^t(C_x(b)) = V_x - V^e \leq 0$, for all tenure $t > 0$, with strict inequality for $x < x^*$. In this latter case, the firm would like to be able to renege on its contract and lay off the worker.

$\mathbf{C}(b)$ is such that the mobility decision of the worker is privately efficient since the worker will quit the job when contacted by another firm if and only if the productivity draw is $x \leq x^*$. In fact, the firm and the worker can renegotiate the contract later on if the

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[14]Acemoglu and Shimer (1999) show that firms' investment decisions is efficient in search models only if workers can direct their search.

worker is contacted by another firm. Since $W_x^t(C_x(b)) = V^e$ and $J_x^t(C_x(b)) = V_x - V^e \leq 0$ for all tenure $t > 0$ when $x \leq x^*$, the current employer in this case does not wish to retain the worker if she is contacted by another firm, whereas a newly contacted firm can offer V^e as expected value of a new match to the worker. However, if $x > x^*$, even if the value of employment to the worker with her current employer at the time she is contacted by another firm is below V^e, her current employer will offer the worker at stage 1 of Game 2 a contract that prevents her from quitting: it will offer $\beta V_x + (1 - \beta) V^e$.

Moreover, $\mathbf{C}(b)$ maximizes the joint private value of the match by putting the worker in the position of extracting all the additional surplus she will create by changing jobs if the productivity draw in the new job were to be below x^*. With this contract, the joint private value of the match for a productivity draw x is such that

$$rV_x = x + \alpha \gamma_x (V^e - V_x) - \delta(V_x - U), \tag{15}$$

with

$$\gamma_x = \begin{cases} 1, & \text{if } x \leq x^*, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Menus of contracts $\mathbf{C}(b)$ such that $W_x^t(C_x(b)) = V^e$ for all $x \leq x^*$, for all tenure $t > 0$, and such that $W^e(b) = U + \beta(V^e - U)$ maximize the expected joint private value of a match, and therefore all equilibrium menus of contracts negotiated between a firm and an unemployed worker following the protocol of Game 1 must have these same properties.

We have established above that with complete contract enforceability the value of employment for workers who negotiated their contracts while unemployed must be such that for all $x \leq x^*$ they receive $W_x^t(C_x) = V^e$ for all $t > 0$. Hence, these workers will capture the entire additional surplus created by their job mobility when the productivity of their current jobs is lower than the mobility productivity cut-off. Naturally, to be able to poach away a worker, a firm must offer at the first stage of Game 2 a contract that maximizes the joint private value of the match. This implies that the contract offered to a worker currently employed in a job with productivity x must also be such that for all productivity draws $z \leq x^*$, the worker's value of employment is $W_z^t(C_z(x)) = V^e$ for all $t > 0$ and $W^e(x) = V^e$ for all x. Hence, the joint private value of a match created by a firm and a previously employed worker when the productivity draw is x also follows (15), with the mobility decision given by (16).

In summary, when a firm meets a currently employed worker whose job's productivity is x, if $x > x^*$ the current employer offers the worker a contract worth $V_x + \beta (V^e - V_x)$ in the first stage of game 2 whereas the poaching firm offers V^e, and the worker stays with her current employer. If $x \leq x^*$, the current employer will offer the worker V_x in the first stage of Game 2 and the poaching firm will offer V^e, and since the worker is indifferent between her current contract and the contract offered by the poaching firm, she joins the new firm.

Hence, we have that when firms can fully commit to a contract, the joint private value of a match with productivity x is given by

$$V_x = \begin{cases} \frac{x+\delta U+\alpha V^e}{r+\delta+\alpha}, & \text{for } x \leq x^*, \text{ and} \\ \frac{x+\delta U}{r+\delta}, & \text{for } x > x^*. \end{cases}$$

This implies that the expected joint private value of a match is

$$V^e = \frac{\delta}{r+\delta}U + \Delta(x^*;\alpha),$$

and therefore the equilibrium mobility cut-off in this case is such that

$$x^* = x^e + \frac{\alpha}{r+\delta} \int_{x^*}^{\overline{x}} (x - x^*)dF(x). \tag{17}$$

Equation (17) is identical to (4) which characterizes the efficient mobility productivity cut-off, and since this equation has a unique solution, we obtain that $x^* = \widetilde{x}^*$. This is summarized in the following proposition.

Proposition 3 *When contracts are fully enforceable, the equilibrium mobility cut-off x^* is equal to the efficient mobility cut-off $\widetilde{x}^*$ for all values of β.*

There are several ways to design menus of contracts for unemployed and employed workers that will deliver an efficient mobility decision. Once such way is to give to the worker the value $W_x^t(C_x(b)) = V^e$ for all tenure $t \geq 0$ and all $x \leq \widetilde{x}^*$. However, in order to obtain $\int_{x \in X} W_x^0(C_x(b)) dF(x) = U + \beta (V^e - U)$ for unemployed workers, this implies that

$$\int_{x^*}^{\overline{x}} W_x^0(C_x(b)) dF(x) = (\beta - F(x^*)) V^e + (1 - \beta) U, \tag{18}$$

which gives that the expected initial value of employment, conditional on drawing $x \leq x^*$ is

$$\frac{(\beta - F(x^*)) V^e + (1 - \beta) U}{F(x^*)}.$$

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This expected initial value of employment might thus be less than the value of unemployment U. Such menus of contracts might therefore require that in some realization of the productivity draw the worker continue working for the firm, although he would prefer to be unemployed. However, it might not be desirable for a firm to employ a worker under such condition. For instance, if we were to introduce effort in this paper, in this case the worker is not likely to exert effort.

There is another way to design menus of contracts that deliver efficiency and at the same time do not require workers to work when they would like to quit. These menus of contracts have a fee-paying structure à la Stevens (2004), where the worker pays a fee φ upon acceptance of the job. The payment of an up-front fee by the worker enables the firm to secure a fraction $1 - \beta$ of the expected surplus when the worker is unemployed, and at the same time guarantee the workers a value of the match of V^e if the productivity draw is $x \leq \widetilde{x}^*$, and a value at least equal to the value of unemployment if $x > \widetilde{x}^*$.

Let us consider the case where offering an unemployed worker the value $W_x^t(C_x(b)) = V^e$ for all tenure $t \geq 0$ and all $x \leq \widetilde{x}^*$ implies that the value of employment for $x > x^*$ has to be negative for a positive measure of the productivity draw. And let us consider the case where firms charge an up-front fee which is not contingent on the productivity draw. From equation (18), we have that the minimum non-contingent fee that delivers that, for all $x > \widetilde{x}^*$, the value of employment is such that $W_x^t(C_x(b)) = U$ for all tenure $t \geq 0$, is such that

$$\varphi = U - [(\beta - F(x^*)) V^e + (1 - \beta) U],$$

which simplifies to

$$\varphi = (F(x^*) - \beta)(V^e - U).$$

6.2 The Limits to Efficiency

As noted by Stevens (2004), fee-contracts require workers to pay an up-front fee, which might not be feasible for a host of reasons, like the existence of credit or legal constraints. Stevens thus studies the structure of optimal contracts when fees are not allowed and there is a minimum wage. She shows that the constrained-optimal contract in this case is such that the worker receives the minimum wage up to some tenure T, after which she receives the entire product of the match. In the environment with bargaining of this paper Stevens'

analysis can be followed to show that the tenure T during which a worker receives the minimum wage depends on the initial value of the match given to the worker.

If a worker is initially unemployed, the contract proposed by a firm must be such that $W^e(b) = U + \beta(V^e - U)$ and $W_x^t(C_x(b)) \geq U$ for all x, all $t \geq 0$. However, suppose a contract such that $W_x^t(C_x(b)) = V^e$ for all $x \leq x^*$, all $t \geq 0$, implies that $W_x^t(C_x(b)) < U$ for some positive measure of productivities $x > x^*$, some interval for tenure $[0, \widetilde{t}) \geq 0$. Then, to obtain $W_x^t(C_x(b)) \geq U$ for all x, all $t \geq 0$, a contract must prescribe that for some positive measure of productivities $x \leq x^*$, $W_x^t(C_x(b)) < V^e$ for some interval for tenure $[0, \widehat{t})$. The optimal contract is then such that for all x and $z \leq x^*$, $W_x^t(C_x(b)) = W_z^t(C_z(b)) < V^e$ for the interval for tenure $[0, T(b))$, the value of employment increases continuously until tenure $T(b)$, and $W_x^t(C_x(b)) = V^e$ for all $t \geq T(b)$, all $x \leq x^*$.

If a worker employed in a job with productivity $x \leq x^*$ becomes in contact with another firm before the tenure after which she will receive V^e, then the tenure for the match during which she will receive a value of employment less than V^e will vary with the value of employment at the time she was hired. This is because, if, for instance, we consider workers employed in their first job since their last unemployment spell, the value of employment for all $x \leq x^*$ will be strictly less than V^e for all $t < T(b)$ and $\gamma_x^t(C) = 1$ for all t, with

$$rW_x^t(C) = w_{\min} + \alpha\left(W^e(x, C, t) - W_x^t(C)\right) - \delta\left(W_x^t(C) - U\right) + \dot{W}_x^t(C),$$

where $w_{\min}$ is the minimum wage a firm is allowed to pay a worker, and

$$W^e(x, C, t) = \max\{W_x^t(C), V_x^t(C)\} + \beta V^e < V^e.$$

This is because if $W_x^t(C) \geq V_x^t(C)$ the worker's bargaining position in Game 2 is determined by the value of her current match under her current contract, $W_x^t(C)$. If, on the contrary, $V_x^t(C) > W_x^t(C)$, then match has a positive value for the current employer. In this case the current employer will be offering an improved contract to the worker in Game 2. Since $W_x^t(C)$ and, as will be shown shortly, $V_x^t(C)$ increase with the tenure of the worker on the job, we have that $W^e(x, C, t)$ increases with t. It follows that when the worker changes jobs, the tenure during which she receives less than V^e depends negatively on $\max\{W_x^t(C), V_x^t(C)\}$, and therefore depends negatively on the tenure on her previous job.

Importantly, it follows that if it is not possible, or desirable, for firms to employ workers when their value of employment is less than the value of unemployment, and if fee-contracts

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are not feasible, the mobility decisions of workers will not be efficient. In fact, the joint private value of a match with productivity $x \leq x^*$ when the tenure of a match is $t < T(b)$ for a worker hired when unemployed depends on the tenure, and is such that

$$rV_x^t(C) = x + \alpha \left(W^e(x, C, t) - V_x^t(C) \right) - \delta \left(V_x^t(C) - U \right) + \dot{V}_x^t(C).$$

A similar argument applies for workers who have recently changed jobs, and whose value of employment $W_x^t(C)$ is still strictly less than V^e. Hence, workers employed in jobs with productivity $x \leq x^*$ with tenure less than the tenure beyond which they will receive V^e from their employers will not be able to capture the entire surplus of a future match if they meet another firm. It should be noted, however, that the degree of inefficiency is lesser than when firms can walk away from a contract freely.

7 Conclusion

This paper has identified a new inefficiency in search models when jobs are experience goods and employment contracts are the outcomes of bargaining games. In the absence of full enforceability of contracts, firms cannot credibly offer to currently employed workers looking for another job contracts that will guarantee them the entire surplus of all future matches should the new match be of low productivity. Furthermore, even if firms can fully commit to contracts they have agreed to, if efficient contracts prescribe that workers work even if the value of employment to them for some draws of the productivity falls below the value of unemployment, and that either such contracts are illegal or cannot be enforced, then efficiency would fail to be obtained.

Given that workers can always choose not to exert effort and that effort is difficult to verify by a third party, including a court of justice, it is difficult to imagine that such contracts are indeed enforceable to workers. Moreover, even if it is more easily conceivable that workers can obtain from a court of justice that a firm honor a contract, there are issues with the determination of the dismissal costs to be paid to the worker in case of a breach of a contract. For instance, it might not be possible to guarantee that severance payments paid by a firm to a worker it dismissed reflect entirely the loss of wealth suffered by the worker. In such cases, firms might prefer to dismiss a worker because the loss due to the severance payment is less than the value of the loss the firm makes by keeping the worker. Moreover, firms might ostracize workers whose value of the match to them is negative in order to

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get the workers to resign rather than dismiss them. Hence, it would appear that complete enforceability of contracts might fail, and therefore worker turnover would be inefficient. Hence, the title of the paper.

APPENDIX

Proof of Proposition 1: Since $\widetilde{x}^*$ is such that $\omega_{\widetilde{x}^*} = \omega^e$ we have

$$\frac{\widetilde{x}^*}{r+\delta} + \frac{\delta\omega_u}{r+\delta} = \frac{\delta\omega_u}{r+\delta} + \Delta(\widetilde{x}^*; \alpha),$$

or

$$\widetilde{x}^* = \frac{r+\delta}{r+\delta+\alpha\left(1-F\left(\widetilde{x}^*\right)\right)} \left[x^e + \frac{\alpha}{r+\delta} \int_{\widetilde{x}^*}^{\overline{x}} x dF(x)\right],$$

which can be rewritten as

$$\widetilde{x}^* = x^e + \frac{\alpha}{r+\delta} \int_{\widetilde{x}^*}^{\overline{x}} \left(x - \widetilde{x}^*\right) dF(x).$$

Defining $\Phi(z) = z - x^e - \alpha/(r+\delta) \int_z^{\overline{x}} (x-z)\, dF(x)$, we have that $\Phi(\underline{x}) = \underline{x} - x^e - \alpha/(r+\delta) \int_{x\in X} (x-z)\, dF(x) < 0$ and $\Phi(\overline{x}) = \overline{x} - x^e > 0$ and therefore since Φ is continuous on X we have proven existence by the Implicit Function Theorem. The fact that

$$\frac{\partial\Phi(z)}{\partial z} = 1 + \frac{\alpha(1-F(z))}{r+\delta} > 0$$

yields uniqueness. QED

Proof of Proposition 2: (i) Since x^* is such that $V_{x^*} = V^e$ we have

$$\frac{x^*}{r+\delta} + \frac{\delta U}{r+\delta} = \frac{\delta U}{r+\delta} + \Delta(x^*; \beta\alpha),$$

or

$$x^* = \frac{r+\delta}{r+\delta+\beta\alpha\left(1-F\left(x^*\right)\right)} \left[x^e + \frac{\beta\alpha}{r+\delta} \int_{x^*}^{\overline{x}} x dF(x)\right],$$

which can be rewritten as

$$x^* = x^e + \frac{\beta\alpha}{r+\delta} \int_{x^*}^{\overline{x}} \left(x - x^*\right) dF(x).$$

Defining $\widetilde{\Phi}(z) = z - x^e - \beta\alpha/(r+\delta) \int_z^{\overline{x}} (x-z)\, dF(x)$, we have that $\widetilde{\Phi}(\underline{x}) = \underline{x} - x^e - \beta\alpha/(r+\delta) \int_z^{\overline{x}} (x-z)\, dF(x) < 0$ and $\widetilde{\Phi}(\overline{x}) = \overline{x} - x^e > 0$ and therefore since $\widetilde{\Phi}$ is continuous on X we have proven existence by the Implicit Function Theorem. The fact that

$$\frac{\partial\widetilde{\Phi}(z)}{\partial z} = 1 + \frac{\beta\alpha(1-F(z))}{r+\delta} > 0$$

yields uniqueness. QED

(ii) The result follows from the fact $\widetilde{\Phi}\left(\underline{x}\right) < \Phi\left(\underline{x}\right)$ and $\partial\widetilde{\Phi}\left(z\right)/\partial z < \partial\Phi\left(z\right)/\partial z$. QED

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