SMU ECONOMICS & STATISTICS
WORKING PAPER SERIES

Financial Variables as Predictors of Real Output Growth

Anthony Tay

March 2007

Paper No. 14-2007

Financial Variables as Predictors of Real Output Growth

Anthony S. Tay
School of Economics and Social Sciences
Singapore Management University
90 Stamford Road, Singapore 178903
email: anthonytay@smu.edu.sg

This Version: March 2007

Preliminary and Incomplete

Abstract

We investigate two methods for using daily stock returns to forecast, and update forecasts of, quarterly real output growth. Both methods aggregate daily returns in some manner to form a single stock market variable. We consider (i) augmenting the quarterly AR(1) model for real output growth with daily returns using a nonparametric Mixed Data Sampling (MIDAS) setting, and (ii) augmenting the quarterly AR(1) model with the most recent r -day returns as an additional predictor. We discover that adding *low* frequency stock returns (up to annual returns, depending on forecast horizon) to a quarterly AR(1) model improves forecasts of output growth

Keywords: Forecasting, Mixed Frequencies, Functional linear regression.

Acknowledgements: The author is grateful for financial support from the SMU Research Grant 05-C208-SMU-008 "Optimal Use of Economic Data Sampled at Different Frequencies". The author also thanks Peter Hansen for the use of his SPA programmes. Zhang Xuefeng and Fred Yan-Qing Goh provided valuable research assistance.

1. Introduction

The fact that financial variables are available at high frequencies, and quickly, raises two related questions pertinent to their use in forecasting macroeconomic variables, which are only available in much lower frequencies, and much less timely. First, is it gainful to use the financial variables at the higher frequencies, instead of aggregating down to the frequency of the macroeconomic variables? Second, can high-frequency financial data be used to update macroeconomic forecasts as and when the financial data becomes available? We explore these questions, beginning here with the particular case where daily stock market returns are used to forecast, and update forecasts of, quarterly real output growth.

The extant out-of-sample evidence on stock returns as a predictor of output growth is actually rather weak, despite the traditional view of stock returns as a leading indicator of economic activity (see Stock and Watson 2003, for a comprehensive survey of the use of asset prices as predictors for output and inflation). Results from a quick out-of-sample forecasting exercise using our data show exactly this: starting with the sample 1964Q1 to 1989Q4, we recursively estimate the two models, a quarterly AR(1) for real output growth, and a quarterly AR(1) for real output growth, with lagged quarterly returns on the S&P 500 index as an additional predictor. We use these models to generate one-step ahead forecasts of real output from 1990Q1 to 2005Q4. In every estimation sample, lagged returns is statistically significant at the 10%, if not the 5% level of significance. Yet lagged returns do not improve out-of-sample forecasts of real output, and in fact produces slightly worse forecasts: the ratio of the out-of-sample forecast root mean square error (FRMSE) from the supplemented AR(1) model to the FRMSE of the AR(1) model is 1.0017. Repeating the exercise with current returns replacing lagged returns produces similar results: the out-of-sample FRMSE for the augmented AR(1) model is larger than the FRMSE for the unaugmented AR(1) model. In these regressions, we used the standard practice of aggregating the higher-frequency variable down to the frequency of the macro-variable. In this paper, we ask if alternative approaches might improve the out-of-sample predictive ability of models using stock returns.

The question of how to use high frequency data for forecasting macroeconomic variables is essentially a question of how to weight the daily observations in forming an aggregate predictor, that is, the question is how best to filter the data. By using quarterly stock returns, the forecaster is essentially filtering the data, by adding up consecutive lots of 66 daily returns. This smooths out daily fluctuations in stock returns and emphasizes the lower frequency features of the data. Some smoothing is obviously desirable: individual daily observations of stock returns is unlikely to be useful for forecasting quarterly real output growth. But what is the optimal way to filter the data? There is no compelling reason to believe that an equal-weighted average of 66 daily returns is the best filter of the data. Perhaps there is too much smoothing, perhaps there is too little. Should more weight be placed on more recent observations? It may very well be that the quarterly frequency for stock returns is optimal for predicting quarterly output, but there is no reason why it must be so. Finding the optimal frequency is ultimately an empirical question, which is the focus on in our investigation. We investigate two models for using daily stock returns to forecast, and update forecasts of, quarterly real output growth. Both models use daily returns aggregated in some manner to form a single stock market predictor. We consider (i) augmenting the quarterly AR(1) model for real output growth with daily returns using a nonparametric Mixed Data Sampling (MIDAS) setting, and (ii) augmenting the quarterly AR(1) model with the most recent r -day returns as an additional predictor.

Many recent papers have considered the issue of mixing data of different frequencies for the purpose of forecasting. Ghysels, Santa Clara, and Valkanov (2004a) investigate if equity returns at very high frequency (data observed at 5 minute intervals) are useful for forecasting future daily equity return realized volatility, defined as increments in quadratic variation. They find that models that make direct use of 5 minute data do not produce better volatility forecasts than models that use daily information as predictors. Clements and Galvao (2006) consider monthly predictors of quarterly output growth and inflation. Both papers use the MIDAS models introduced in Ghysels, Santa-Clara, Valkanov (2004b) and Ghysels, Santa-Clara, Sinko, and Valkanov (2004) in their investigations. Stock returns are among the many predictors studied in

Clements and Galvao (2006), but there are important differences between that work, and the work that is presented here. First, we use a nonparametric framework based on functional regression models (Ramsay and Silverman, 2004). Furthermore, our treatment of lagged quarterly output is different. Second, Clements and Galvao (2006) consider the optimal combination of monthly stock returns whereas we consider the optimal combination of daily returns. Third, in addition to the MIDAS setting, we also consider simple regression models that include stock returns at various frequencies. Finally, and most importantly, our conclusions and recommendations regarding using stock returns as a predictor for quarterly real output growth are substantially different from theirs.

In the next section, we describe the data, the setup of our study, and the models we use. Results are discussed in Section 3. Section 4 concludes.

2. Data, Study Setup, Models

Throughout this paper, we measure quarterly real output growth using quarterly percentage change in seasonally adjusted annualized chained-weighted GDP. Stock returns are daily returns on the S&P500 stock index. We use data from the start of 1964 to the end of 2005, which gives 168 quarterly observations, and 10,958 daily observations (we use five-day weeks). Data from 1962Q1 to 1963Q4 are available in our sample, but are reserved for accomodating lags in our forecasting models.

2.1 Study Setup

We evaluate two classes of models for forecasting US quarterly real output growth. Both will be described in detail shortly, but for now we will simply note that both use lagged quarterly output growth and daily stock returns as predictors; the difference between the two models is in how the mix of frequencies is handled. The performances of these two models to forecast real output growth at various horizons are compared against a benchmark model that uses only the latest available lagged output growth. We consider forecasting at horizons of $h = 0$, 20, 40, 70, 90, 110, and 130 days, corresponding approximately to monthly horizons of zero to six months.

The choice of forecast horizons covers both our desire to determine how best to combine daily return data into a quarterly forecasting model, and our desire to see if daily return data within a quarter can be helpful in updating our forecasts of real output growth for that quarter. The zero-month horizon forecast, sometimes called a 'nowcast', is a forecast of real GDP growth in a particular quarter, made at the end of the quarter. This is, of course, not a trivial exercise because GDP data for any one quarter is available only about a month after the end of the quarter.

Describing models that combine quarterly and daily data requires fairly intricate notation; we explain here the system used in the paper. We use τ as a time index, with $\tau = 1$ to $10{,}985$ denoting the ends of days 1 through 10,985. We use t_k, $k = 1$ to 168, to indicate the end-of-quarters: t_1 is the day-index of the last day of 1964Q1, t_2 is the day-index of the last day of 1964Q2, and so on. We have $t_1 = 65$, $t_2 = 130$, $t_3 = 196$, …, $t_{176} = 10{,}985$. Each quarter contains 64 to 66 days, not counting weekends. We use the notation $x(\tau)$ to denote the one-day return over day τ, and $x(\tau, r)$ to denote the r-day return measured at time τ. Thus, $x(\tau,1) \equiv x(\tau)$, and the total return on the index over day τ and the previous $r-1$ days is

$$x(\tau, r) = \sum_{i=0}^{r-1} x(\tau - i, 1) \qquad (1)$$

Observed values of output growth are denoted by $y(t_k)$, $k = 1, 2, ..., 168$.

2.2 Models

The first model considered is a non-parametric implementation of the MIDAS model. In this implementation, we treat τ as running continuously from 0 to 10,985, and $x(\tau)$ as noisy observations of a continuous-time process. This is incorporated into a h-day ahead forecasting model for quarterly real output growth as

$$y(t_k) = \beta_0 + \beta_1 y^* + \int_0^L \beta(s) \tilde{x}_{k,h}(s)\, ds + \varepsilon(t_k) \qquad (2)$$

where $\tilde{x}_{k,h} \equiv x(t_k - h - s)$, and y^* is an appropriate lag of quarterly real output growth. We take $L = 270$, i.e., we include just over a year's worth of daily returns data in each of our forecasting models. Because of the timing of the release of quarterly real output growth, the latest available lagged quarterly real output growth y^* will be $y(t_{k-1})$ for horizons $h = 0$, 20, and 40, $y(t_{k-2})$ for horizons $h = 70$, 90, and 110, and $y(t_{k-3})$ for horizon $h = 130$.

Before proceeding, we note that no attempt is being made in model (2) to capture the structural aspects of the stock market – real output relationship. It is merely a projection of real output growth on available lagged output growth and stock returns data, for the purpose of forecasting. It is a forecasting model, and will be evaluated as such.

Model (2) is an example of a functional regression model. Although there have been rather few applications of such models in economics (e.g. Ramsay and Ramsey, 2002), such models are becoming popular in the statistics literature. Methods for working with such models, and examples of its application, are discussed in detail in Ramsay and Silverman (2006), and Ramsay and Silverman (2002). Software is available for estimating general functional regressions; we use Prof. James Ramsay's MATLAB implementation, downloaded from http://ego.psych.mcgill.ca/misc/fda/. The idea behind the estimation method is to represent $\beta(s)$ and $\tilde{x}_{k+1,h}(s)$ in terms of basis functions

$$\beta(s) = \sum_{i=1}^{m} b_i \theta_i(s) \quad \text{and} \quad \tilde{x}_{k,h}(s) = \sum_{j}^{n} c_{k,h,j} \psi_{k,h,j}(s)$$

where $\theta_i(s)$, $i = 1,...,m$, and $\psi_{k,h,j}(s)$, $j = 1,...,n$, are the basis functions used to represent $\beta(s)$ and $\tilde{x}_{k,h}(s)$ respectively, and where b_i and $c_{k,h,j}$ are the corresponding weights on these basis functions. We use B-spline bases of order 4 for both $\beta(s)$ and $\tilde{x}_{k,h}(s)$, with knots placed at every five lags. Model (2) is estimated by minimizing the residual sum of squares subject to a roughness penalty. Suppose we are estimating (2) over a sample of T observations. In vector notation, we can write $\mathbf{x}(s) = \mathbf{C}\boldsymbol{\psi}(s)$ and $\beta(s) = \boldsymbol{\theta}(s)'\mathbf{b}$, where $\boldsymbol{\theta}(s)$ and $\mathbf{b}$ are the $n \times 1$ vectors of the basis functions $\theta_i(s)$ and their weights b_i respectively, $\boldsymbol{\psi}(s)$ is the vector of basis functions $\psi_j(s)$, and $\mathbf{C}$ is the $T \times m$ matrix of the coefficients $c_{k,h,j}$ on these basis functions. Then

$$\begin{aligned}
\mathbf{y} &= \beta_0 + \beta_1 \mathbf{y}^* + \int_0^L \mathbf{x}(s)\beta(s)\, ds + \boldsymbol{\varepsilon} \\
&= \beta_0 + \beta_1 \mathbf{y}^* + \int_0^L \mathbf{C}\boldsymbol{\psi}(s)\boldsymbol{\theta}(s)'\mathbf{b}\, ds + \boldsymbol{\varepsilon} \\
&= \beta_0 + \beta_1 \mathbf{y}^* + \mathbf{C}\mathbf{J}_{\psi\theta}\mathbf{b} + \boldsymbol{\varepsilon} \\
&= \mathbf{Z}\boldsymbol{\zeta}
\end{aligned}$$

where $\mathbf{y}$ and $\mathbf{y}^*$ are $T \times 1$ vectors of observations of quarterly real output $y(t_k)$ and its appropriate lag respectively, $\mathbf{J}_{\psi\theta} = \int_0^L \boldsymbol{\psi}(s)\boldsymbol{\theta}(s)' \, ds$, $\mathbf{Z} = [\mathbf{1} \quad \mathbf{y}^* \quad \mathbf{C}\mathbf{J}_{\psi\theta}]$, and $\boldsymbol{\zeta}$ is the vector $[\beta_0 \quad \beta_1 \quad \mathbf{b}]'$. The ensure sufficient smoothnest in $\beta(s)$, the coefficient vector $\boldsymbol{\zeta}$ is estimated by minimizing the penalized sum of squares

$$\boldsymbol{\zeta}'\mathbf{Z}'\mathbf{Z}\boldsymbol{\zeta} + \lambda \mathbf{b}'\mathbf{R}\mathbf{b} = \boldsymbol{\zeta}'\mathbf{Z}'\mathbf{Z}\boldsymbol{\zeta} + \lambda \boldsymbol{\zeta}'\mathbf{R}_0\boldsymbol{\zeta}$$

where $\mathbf{R} = \int_0^T [D^2\theta(s)][D^2\theta(x)'] \, ds$, and where $\mathbf{R}_0$ is the matrix $\mathbf{R}$ augmented with two leading rows and columns of zeros. Then the minimizing value of $\boldsymbol{\zeta}$ is

$$\hat{\boldsymbol{\zeta}} = (\mathbf{Z}'\mathbf{Z} + \lambda \mathbf{R}_0)^{-1}\mathbf{Z}'\mathbf{y}$$

with variance $\mathrm{var}[\hat{\boldsymbol{\zeta}}] = \sigma_\varepsilon^2 (\mathbf{Z}'\mathbf{Z} + \lambda \mathbf{R}_0)^{-1}\mathbf{Z}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z} + \lambda \mathbf{R}_0)^{-1}$, where σ_ε^2 can be estimated from the residuals. The smoothing parameter λ is chosen by cross-validation over the estimation period, where we search over $\lambda = 10^2$ to $\lambda = 10^7$. We choose a new smoothing parameter each time the estimation period is rolled forward.

The second model we consider is a linear regression forecasting model of the form

$$y(t_k) = \beta_0 + \beta_1 y^* + \beta_2 x(t_k - h, r) + \nu(t_k)\,, \tag{3}$$

that is, where we use lagged r-day returns on the stock market together with an appropriate lag of real output. This model is a restricted version of the functional forecasting model (2), forcing $\beta(s)$ to be zero over some days, and constant over other days. This is a natural alternative to (2) for several reasons. First, taking r-day returns is a very natural, easily interpretable way of combining r one-day returns. Second, model (3) is easy to apply, and directly addresses the question of whether there is an optimal frequency of stock returns for forecasting quarterly real output growth. We consider the performance of model (3) for $r = 1$, 5, 10, 15, … 270, over each of the horizons $h = 0$, 20, 40, 70, 90, 110, 130. That is, at each horizon, we evaluate 55 versions of (3), corresponding to 55 values of r, the level of aggregation of stock returns.

2.3 Forecast Evaluation

We evaluate both models on the basis of out-of-sample forecast performance, with necessary parameters estimated using a rolling scheme. In the first instance, the models are estimated over the estimation period 1964Q1 – 1989Q4 (104 quarters). Then forecasts from our two models and the benchmark model are computed for 1990Q1. The estimation period is then rolled forward by one quarter, to 1964Q2 – 1990Q1, and forecasts computed for quarterly real output in 1990Q2. This is repeated until forecasts for 2005Q4 are made, giving 64 forecasts from each model.

The quarterly real output growth data is plotted in Figure 1, with the initial estimation sample period and the forecast sample period marked out. We will evaluate our forecasts over the full forecast sample period, as well as over each of three forecast subperiods: 1990Q1 to 1994Q4 (20 quarters), 1995Q1 to 2000Q4 (24 quarters), and 2001Q1 to 2005Q4 (20 quarters). The purpose for evaluating the forecasts over three subperiods is because each of the three subperiods contain different features of real output growth. The first subperiod starts just before the NBER-dated contraction of 1990Q3 – 1991Q1, and the subsequent recovery. Real output growth is fairly stable over the second subperiod, with minor fluctuations around a rate of about 4 percent. Real output growth in the third subperiod falls in early 2001, increases sharply in 2003 to 7.5 percent, and then falls back to levels averaging approximately three percent. We take the view, in evaluating forecasts, that a good forecast ought to capture the important features of the target variable. Thus, we are more interested in the forecast performance over the first and third subperiods. In the second period, the target variable fluctuates steadily over a significant length of time. Any two reasonable forecasting models should predict the same, and so there would little to choose between them over this subperiod. Furthermore, and importantly, a *bad* forecasting model that *always* predicts minor fluctuations over a constant rate will also appear to be a reasonable forecasting model. A comparison between the good and the bad forecasting models will reveal little over such a subperiod.

We will evaluate, at each forecast horizon h, the 56 models (the functional forecasting model (2), plus the 55 regression forecasts (3)) by comparing their Mean Squared Error (MSE) and Mean Absolute Error (MAE) against those of the appropriate benchmark model

$$y(t_k) = \beta_0 + \beta_1 y^* + \eta(t_k) \tag{4}$$

where, as before, y^* is $y(t_{k-1})$, $y(t_{k-2})$, or $y(t_{k-3})$, depending on the horizon h.

Statistical significance is evaluated using the test for Superior Predictive Accuracy (SPA) developed in Hansen (2005), for comparing multiple alternative forecasts against a benchmark forecast. The SPA test is closely related to the 'reality check' test of White (2000), and like the latter is designed to account for the mining over alternative forecasting models. The SPA test, however, controls for poor alternatives, and is therefore less sensitive to the inclusion of these alternatives, and has been demonstrated to be more powerful in simulations and application (Hansen, 2005; Hansen and Lunde, 2005). Both the MSE and MAE criteria are used in the comparison.

4. Results and Discussion

The estimated coefficient functions for the functional forecasting model for $h = 0$, 70, and 130 are displayed in Figures 2(a), (b), and (c) respectively. Only the coefficient functions estimated over the initial estimation period are shown. The estimated coefficient function for the functional forecasting model with $h = 0$ shows that most of the weight is placed on returns in the previous two quarters ($s =$ 70 to 200 approximately), and that these are statistically significant. Little weight is placed on returns in the current quarter (s = 0 to 70, approx.). Some weight is placed on returns in the fourth quarter ($s > 200$) but these are statistically insignificant. The estimated weight functions as the estimation sample rolls forward are similar. This weight function is fairly similar to what is obtained when quarterly real output growth is regressed onto lagged output growth, with current and three lags of quarterly stock returns. Over the period 1964Q1 – 1989Q4, this regression gives

$$\underset{}{y_t} = \underset{[3.857]}{0.02} + \underset{[1.937]}{0.191}\, y_{t-1} + \underset{[0.203]}{0.009}\, x_t + \underset{[1.684]}{0.076}\, x_{t-1} + \underset{[2.781]}{0.128}\, x_{t-2} + \underset{[0.747]}{0.036}\, x_{t-3} .$$

where we use standard notation for regressions where all variables are measured in the same frequency, with x_t denoting quarterly stock returns.

For $h = 70$ (3-month horizon), the estimated coefficient function for the functional forecasting model places weights that decline with the number of lags (Figure 2b). The weights on the first three quarters of included returns are significant. For $h = 130$ (six months horizon), statistically significant weight is placed only on the first quarter of included daily returns only. Again, these estimated weight functions are broadly similar to what is obtained from corresponding all-quarterly-data regressions. The estimated weight functions suggest that it is stock returns data within one year of the end of the target quarter that is relevant for forecasting.

In Figure 3, the performance of the functional forecasting model is visually compared with forecasts from the baseline model (4). Three groups of plots are presented: Figure 3(a) compares the 0, 20, and 40 day horizon forecasts against forecasts from (4) when $y^* = y(t_{k-1})$; Figure 3(b) compares the 70, 90, and 110 day horizon forecasts against forecasts from the benchmark model when $y^* = y(t_{k-2})$; Figure 3(c) compares the 130 day horizon forecasts against forecasts from (4) when $y^* = y(t_{k-3})$. Different benchmark models are used for forecasts at different horizons to account for the fact that at longer horizons, most recent lags of real output growth are not yet available. The top-left panels of Figure 3(a) and (b), and the left panel of Figure 3(c), all labeled 'benchmark', show the forecasts from (4) (dots) plotted over the line graph representing the realizations of quarterly real output growth. The other panels plot forecasts from (2) at the different horizons, against actual data. The baseline forecasts are generally much smoother than the actual data, but visually the forecasts do not appear to pick out the important features of the data. The benchmark model for the six-month horizon model is particularly weak (figure 3c). The functional models' forecasts do a better job at tracking the data in the first subperiod 1990Q1 – 1994Q4, and especially in the last forecast subperiod, 2001Q1 – 2005Q4.

The MSE and MAE comparisons of the functional model forecasts with the benchmark forecasts are given in Table 1. The numbers in the first panel of the table are ratios of the functional models' out-of sample forecast MSE to the corresponding benchmark models' forecast MSE. The second panel shows the corresponding ratios of MAEs. The numbers are in agreement with the visual comparisons. Some improvement in MSE and MAE is seen in the first forecast subperiod, but these are very small. There is generally no improvement in the second forecast subperiod, but as pointed out earlier, little can be drawn from this subperiod. More noteworthy is the fact that large improvements in out-of-sample MSE and MAE are obtained the last forecast subperiods, with reductions in MSE of approximately 20 to over 30 percent. The fact that such large improvement are observed in the last subperiod compared with the first subperiod is consistent with the observation elsewhere that the usefulness of financial assets for predicting output growth varies over time (Stock and Watson, 2003).

The corresponding figures and statistics for the regression forecast models are given in figures 4(a), (b), and (c), and table 2. Although at each horizon, we evaluate forecasts from 55 models (corresponding to 55 values of r), we only report the statistics for the best performing model at each horizon. Visual comparisons in figure 4 generally gives the same conclusions as obtained in figure 3. There are important differences between the numbers reported in table 2 and table 1. In table 2, we see that no improvement is observed in the first subperiod, whereas some improvements in MSE and MAE is observed in the second subperiod. These are small, however. Very large reductions in MSE and MAE are observed for the third forecast subperiod. In terms of MSE, the reductions over baseline range from just under 37 percent to over 50 percent.

An interesting point to note is that the improvements are even larger than those obtained from the functional forecasting model, even though the restrictions imposed in the regression models (3) are generally false in-sample. It is also interesting to note which models perform best at the various horizons. At the 0-month horizon, the best model is one that uses one-year returns on the stock market. With each one-month increase in the forecast horizon, up to the 4 month forecast horizon, the length of the return included in the best model falls by approximately one month. This supports the assertion made earlier that it is returns over the one-year period prior to the end of the target quarter that is relevant for forecasting output.

We now carry out a slightly different, more formal, comparison of the forecasts, using the SPA test of Hansen (2005). We carry out one test per forecast horizon. For each horizon, we ask if one of the 56 models (1 functional forecasting model, and 55 regression forecasting models) produces forecasts that are significantly better than the baseline model appropriate to the horizon. Fixing the horizon, suppose that the forecasts from the 56 alternative models are $\hat{Y}_{mt}$, where $m=1,...,56$, and the evaluation period is $t=1,...,n$, and the baseline model forecasts are $\hat{Y}_{0,t}$. Then the hypothesis is $\boldsymbol{\mu} \le \mathbf{0}$ where

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_{56} \end{pmatrix} = E \begin{pmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_{56}(t) \end{pmatrix}$$

and $X_{m,t} = L(Y_t, \hat{Y}_{0t}) - L(Y_t, \hat{Y}_{mt})$. The function $L(.,.)$ is the loss criteria. We use mean square loss $L(Y_t, \hat{Y}_{mt}) = (Y_t - \hat{Y}_{mt})^2$, and absolute loss $L(Y_t, \hat{Y}_{mt}) = \left| Y_t - \hat{Y}_{mt} \right|$. The test statistic is

$$T_n^{sm} = \max_m \frac{n^{1/2} \bar{X}_m}{\hat{\sigma}_m}.$$

An estimate of $\hat{\sigma}_m^2 = \widehat{\text{var}}(n^{1/2} \bar{X}_m)$ and a consistent p-value for this test, as well as an upper and a lower bound for the p-value, are obtained using a bootstrap implementation described in Hansen (2005). We use 1000 bootstrap replications throughout. The bootstrap implementation also requires the choice of a parameter q to account for dependency in the relative loss series. We report results for $q = 0.5$ and $q = 0.25$.

The results of this test are reported in Table 3. Results are reported for both MSE and MAE loss metrics, for $q = 0.25$ and $q = 0.5$, for the three forecast subsamples. The number that appears under ‘Best’ refers to the number of days over which returns are aggregated in the best performing model. If ‘fncl’ appears in this column, this indicates that the best performing model is the functional forecasting model. We observe that as the forecast horizon increases by a month, the number of days over which returns are aggregated in the best performing model falls by about a month, very similar to an earlier observation. The exception is the 6-month horizon. We note, however, that the ‘second best’ performing model in this case is the model with 125-day returns. Three “p-values” are reported for each loss function. The p-values reported under SPA*c* are the asymptotically consistent p-values for the test statistic. The numbers reported under SPA*l* and

SPA_u are the lower and upper bounds for the p-values. The results show that the benchmark model is outperformed in almost all cases in the third forecast subsample, at either the 5% or 10% level of significance. The exceptions are the 2-months-ahead forecasts with $q = 0.5$, and the 5-months-ahead forecasts, where weak statistical significance is obtained only for $q = 0.25$ with MSE as the loss criterion. The 2- and 5- month forecast horizons are the horizons at which a new observation of lagged real output growth becomes available, which may explain the fall in statistical significance at these horizons. Nonetheless, the results in table 2 suggest that the gains from including stock returns at an appropriately aggregated level into the forecasting model is substantial. The results also highlight the value of using latest available observations (of low frequency) stock returns to update forecasts of real output.

4. Conclusion

Financial data are in general available at much higher frequencies than macroeconomic data. We asked how high frequency stock market data might best be used when the objective is to forecast real output growth. In particular, we asked how daily data should be combined in a model for forecasting quarterly real output growth. We explored two different methods, one where daily data enters into the quarterly forecasting model as a functional predictor, and another where daily stock return data is simply aggregated into multi-day returns and put into a linear regression forecasting model.

All the results appear to suggest that stock returns are useful predictors for real output growth, particularly in recent years: (i) our forecasting models that include stock returns have succeeded in tracking the main features of real output growth; (ii) stock returns are useful for updating forecasts of real output growth. Our functional forecasting models suggest that it is returns over the year leading up to the end of the target quarter that contains information useful for forecasting output growth. Although estimating the weights in-sample leads to forecasts that are better than a baseline forecast model using only lagged output growth, results from our linear regression models that assume equal weight over the daily returns perform even better, even though restriction of equal weights is generally rejected in-sample.

The results suggest a simple way of using the latest available stock return data to forecast, or to update forecasts of, real output growth, and that is to use aggregated stock returns as a predictor, including only data in the year leading up to the end of the target quarter: to predict real output growth 2 months prior to the end of the target quarter, use aggregate returns over the latest 10 months; to predict real output growth 10 months prior to the end of the target quarter, use the latest available 2-month returns. More generally, the results in the paper suggest that the practice of coercing all variables in a forecasting model to have the same frequency might not be an optimal precedure. Mixing frequencies can lead to better forecasting models.

References

Clements, M.P., and Galvão, A.B. (2006), "Macroeconomic Forecasting with Mixed Frequency Data: Forecasting US Output Growth and Inflation", manuscript.

Ghysels, E., Santa-Clara, P., Sinko, A., and Valkanov, R. (2004), "MIDAS Regressions: Further Results and New Directions", manuscript.

Ghysels, E., Santa-Clara, P., and Valkanov, R. (2004a), "Predicting Volatility: Getting the Most out of Return Data Sampled at Different Frequencies", manuscript.

Ghysels, E., Santa-Clara, P., and Valkanov, R. (2004b), "The MIDAS Touch: Mixed Data Sampling Regression Models", forthcoming in *Journal of Econometrics*.

Hansen, P.R. (2005) "A Test for Superior Predictive Ability", *Journal of Business and Economic Statistics*. Vol. 23, pp. 365-380.

Hansen, P.R. and Lunde, A. (2005) "A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH(1,1)?". *Journal of Applied Econometrics*, Vol. 20, pp. 873-889.

Mariano, R.S. and Murasawa, Y. (2003) "A New Coincident Index of Business Cycles Based on Monthly and Quarterly Series", *Journal of Applied Econometrics*, 18, 427-433.

Ramsay, J. O. and Ramsey, J. B., (2002), "Functional data analysis of the dynamics of the monthly index of nondurable goods production", *Journal of Econometrics*, 107, 327-344.

Ramsay, J. O. and Silverman, B. W., (2002), *Applied Functional Data Analysis*, Springer Series in Statistics, Springer, New York.

Ramsay, J. O. and Silverman, B. W., (2005), *Functional Data Analysis*, 2nd Ed., Springer-Verlag, New York.

White, H. (2000), "A Reality Check for Data Snooping", *Econometrica*, 68, 1097-1126.

Table 1 Performance of Functional Forecasts vs Benchmark Forecasts

Horizon	0 mth	1 mth	2 mth	3 mth	4 mth	5 mth	6 mth
MSFE Ratio							
Full Sample	0.910	1.026	0.940	0.908	0.875	0.871	0.899
1990Q1:1994Q4	0.906	0.839	0.930	0.857	0.898	0.908	1.099
1995Q1:2000Q4	1.021	1.346	1.053	1.146	0.934	0.890	0.757
2001Q1:2005Q4	0.728	0.744	0.763	0.676	0.768	0.791	0.783
MAFE Ratio							
Full Sample	0.963	1.060	1.021	0.992	0.959	0.936	0.985
1990Q1:1994Q4	0.933	0.926	0.958	0.945	0.985	0.922	1.070
1995Q1:2000Q4	1.067	1.297	1.130	1.131	1.047	0.990	0.975
2001Q1:2005Q4	0.842	0.868	0.937	0.864	0.807	0.879	0.887

Note: Numbers in the upper panel are ratios of mean squared forecast error (MSFE) from the functional forecasting models (2) to the MSFE of the corresponding benchmark models (4). The numbers in the lower panel are the corresponding ratios of mean absolute forecast errors (MAFE)

Table 2 Performance of Regression Forecasts vs Benchmark Forecasts

Horizon	0 mth	1 mth	2 mth	3 mth	4 mth	5 mth	6 mth
Best model based on full sample MSE ratio	270	255	230	195	175	170	150
MSE Ratio							
Full Sample	0.872	0.870	0.882	0.820	0.798	0.824	0.772
1990Q1:1994Q4	1.078	1.041	0.979	0.903	0.912	0.884	0.877
1995Q1:2000Q4	0.878	0.873	0.926	1.002	0.881	0.852	0.700
2001Q1:2005Q4	0.572	0.624	0.670	0.466	0.528	0.704	0.709
MAE Ratio							
Full Sample	0.958	0.966	0.982	0.917	0.897	0.925	0.917
1990Q1:1994Q4	1.060	1.027	0.998	0.998	0.998	0.962	0.975
1995Q1:2000Q4	0.988	0.984	1.026	0.996	0.948	0.955	0.899
2001Q1:2005Q4	0.776	0.855	0.892	0.703	0.696	0.838	0.863

Note: Numbers in the upper panel are ratios of mean squared forecast error (MSFE) from the regression forecasting models (3) to the MSFE of the corresponding benchmark models (4). The numbers in the lower panel are the corresponding ratios of mean absolute forecast errors (MAFE)

Table 3 Tests for Superior Predictive Ability

		0 Month Horizon				1 Month Horizon				2 Month Horizon				3 Month Horizon			
Metric	q	Best	SPA_l	SPA_c	SPA_u	Best	SPA_l	SPA_c	SPA_u	Best	SPA_l	SPA_c	SPA_u	Best	SPA_l	SPA_c	SPA_u
1990Q1:1994Q4																	
MSE	0.5	func	0.444	0.656	0.701	func	0.246	0.357	0.357	155	0.529	0.692	0.745	125	0.274	0.300	0.300
	0.25	func	0.277	0.412	0.449	func	0.150	0.188	0.193	155	0.367	0.499	0.526	125	0.287	0.326	0.326
MAE	0.5	func	0.141	0.184	0.194	func	0.162	0.211	0.214	100	0.386	0.482	0.482	75	0.306	0.344	0.344
	0.25	func	0.083	0.105	0.111	func	0.043	0.063	0.064	100	0.203	0.234	0.274	75	0.168	0.189	0.189
1995Q1:2000Q4																	
MSE	0.5	270	0.387	0.578	0.639	255	0.275	0.449	0.513	1	0.441	0.635	0.717	200	0.609	0.825	0.847
	0.25	270	0.264	0.403	0.466	255	0.181	0.296	0.361	1	0.381	0.520	0.653	200	0.569	0.761	0.794
MAE	0.5	270	0.884	0.997	0.997	225	0.763	0.953	0.964	1	0.102	0.256	0.319	175	0.674	0.863	0.869
	0.25	270	0.879	0.996	0.998	225	0.728	0.938	0.969	1	0.085	0.210	0.316	175	0.680	0.856	0.857
2001Q1:2005Q4																	
MSE	0.5	255	**0.041**	**0.041**	**0.041**	235	**0.038**	**0.038**	**0.038**	225	0.116	0.128	0.137	185	**0.042**	**0.042**	**0.042**
	0.25	255	**0.020**	**0.020**	**0.020**	235	**0.015**	**0.015**	**0.015**	225	**0.068**	**0.068**	**0.073**	185	**0.034**	**0.034**	**0.034**
MAE	0.5	255	**0.037**	**0.038**	**0.038**	235	**0.005**	**0.005**	**0.005**	225	0.150	0.181	0.200	195	**0.069**	**0.069**	**0.071**
	0.25	255	**0.023**	**0.026**	**0.026**	235	**0.001**	**0.001**	**0.001**	225	**0.101**	**0.121**	**0.137**	195	**0.024**	**0.024**	**0.024**

Note: The number that appears under 'Best' refers to the number of days over which returns are aggregated in the best performing model in the model class (3). If 'fncl' appears in this column, this indicates that the best performing model is the functional forecasting model (2).

Table 3 cotd. Tests for Superior Predictive Ability

		4 Month Horizon				5 Month Horizon				6 Month Horizon			
Metric	q	Best	SPA_l	SPA_c	SPA_u	Best	SPA_l	SPA_c	SPA_u	Best	SPA_l	SPA_c	SPA_u
1990Q1:1994Q4													
MSE	0.5	100	0.254	0.340	0.340	85	**0.042**	**0.043**	**0.043**	65	0.286	0.432	0.432
	0.25	100	0.232	0.305	0.305	85	**0.040**	**0.040**	**0.040**	65	0.307	0.413	0.421
MAE	0.5	230	0.178	0.211	0.211	85	**0.001**	**0.001**	**0.001**	65	0.317	0.345	0.357
	0.25	230	0.109	0.125	0.125	85	**0.000**	**0.000**	**0.000**	65	0.273	0.300	0.308
1995Q1:2000Q4													
MSE	0.5	185	0.230	0.293	0.293	210	0.285	0.483	0.485	190	**0.045**	**0.048**	**0.048**
	0.25	185	0.088	0.116	0.116	210	0.118	0.212	0.214	190	**0.016**	**0.017**	**0.017**
MAE	0.5	180	0.454	0.681	0.683	185	0.531	0.697	0.744	165	0.320	0.371	0.388
	0.25	180	0.388	0.553	0.587	185	0.472	0.655	0.694	165	0.288	0.328	0.337
2001Q1:2005Q4													
MSE	0.5	165	**0.076**	**0.076**	**0.076**	145	0.117	0.117	0.137	15[1]	**0.045**	**0.045**	**0.045**
	0.25	165	**0.038**	**0.038**	**0.038**	145	**0.084**	**0.084**	**0.093**	15[1]	**0.020**	**0.020**	**0.020**
MAE	0.5	165	**0.001**	**0.001**	**0.001**	145	0.183	0.234	0.243	15[1]	**0.047**	**0.047**	**0.047**
	0.25	165	**0.000**	**0.000**	**0.000**	145	0.149	0.192	0.197	15[1]	**0.020**	**0.020**	**0.020**

Note: The number that appears under 'Best' refers to the number of days over which returns are aggregated in the best performing model in the model class (3). If 'fncl' appears in this column, this indicates that the best performing model is the functional forecasting model (2).

[1] The second best model is the model (3) with 125-day return.

Figure 1 Quarterly Real Output Growth 1964Q1 to 2005Q4

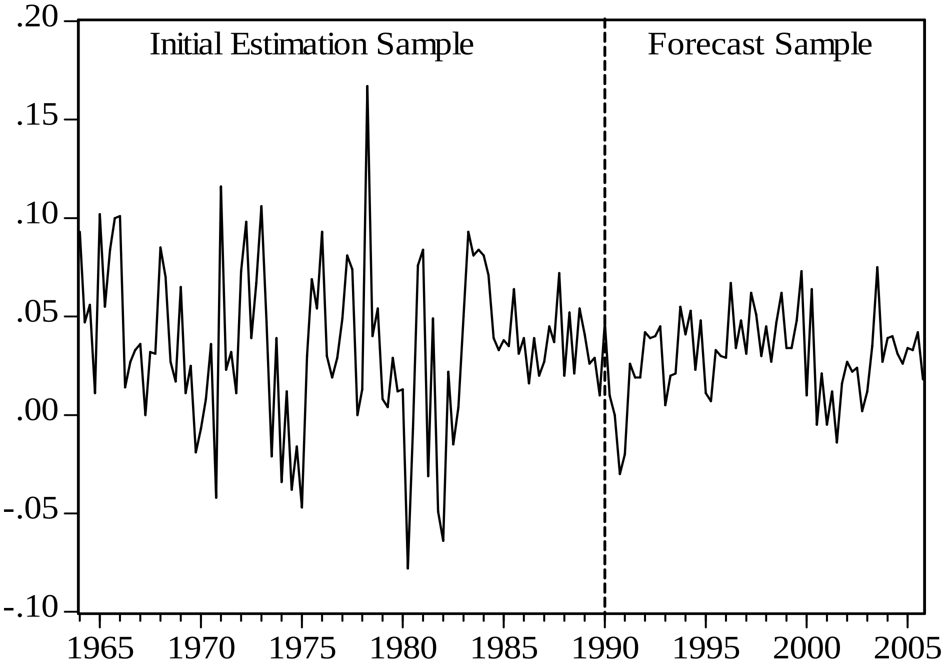
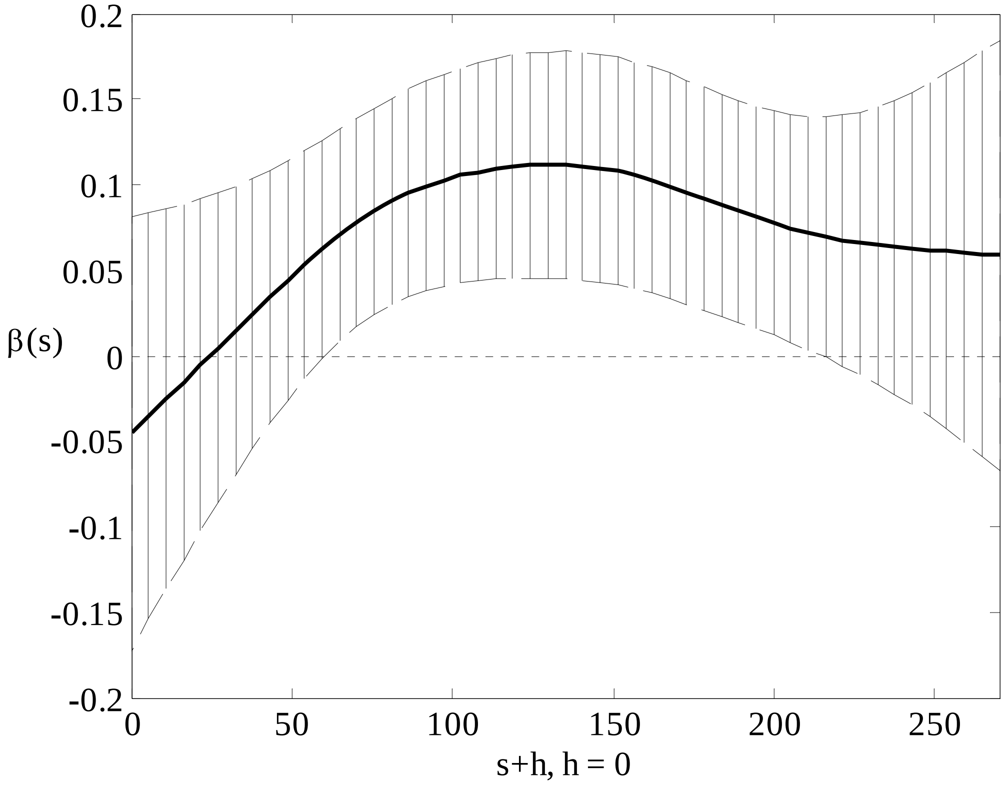
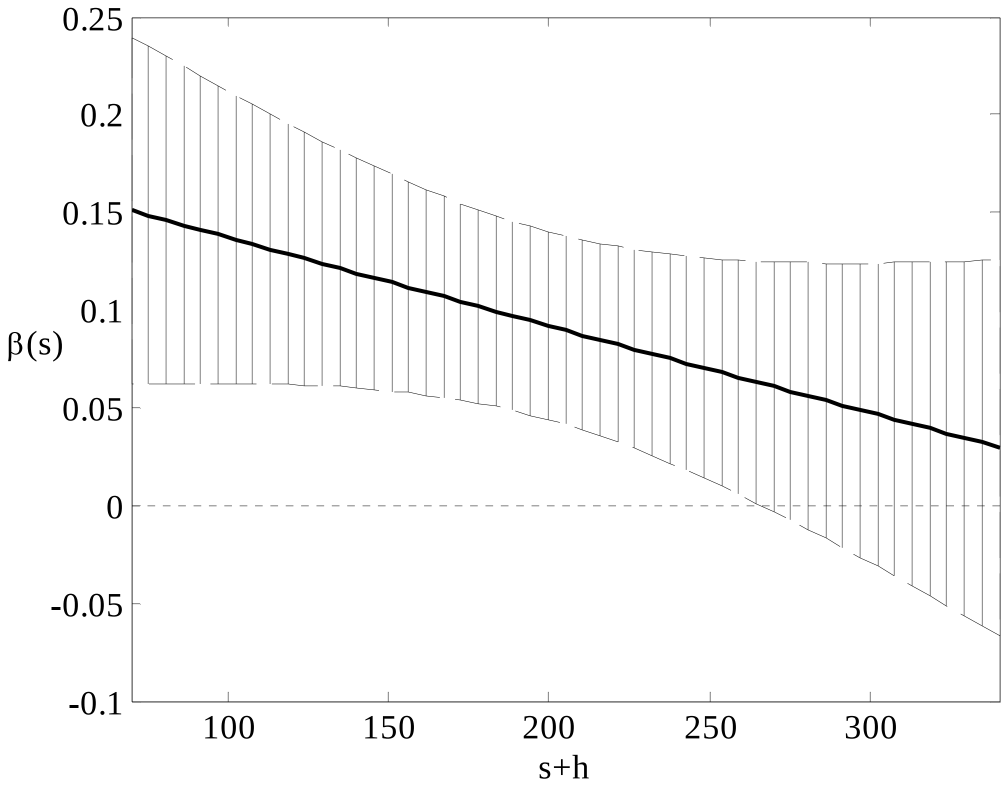


Figure 2(a) Estimated Coefficient Function for Daily Stock Returns (0 month horizon)



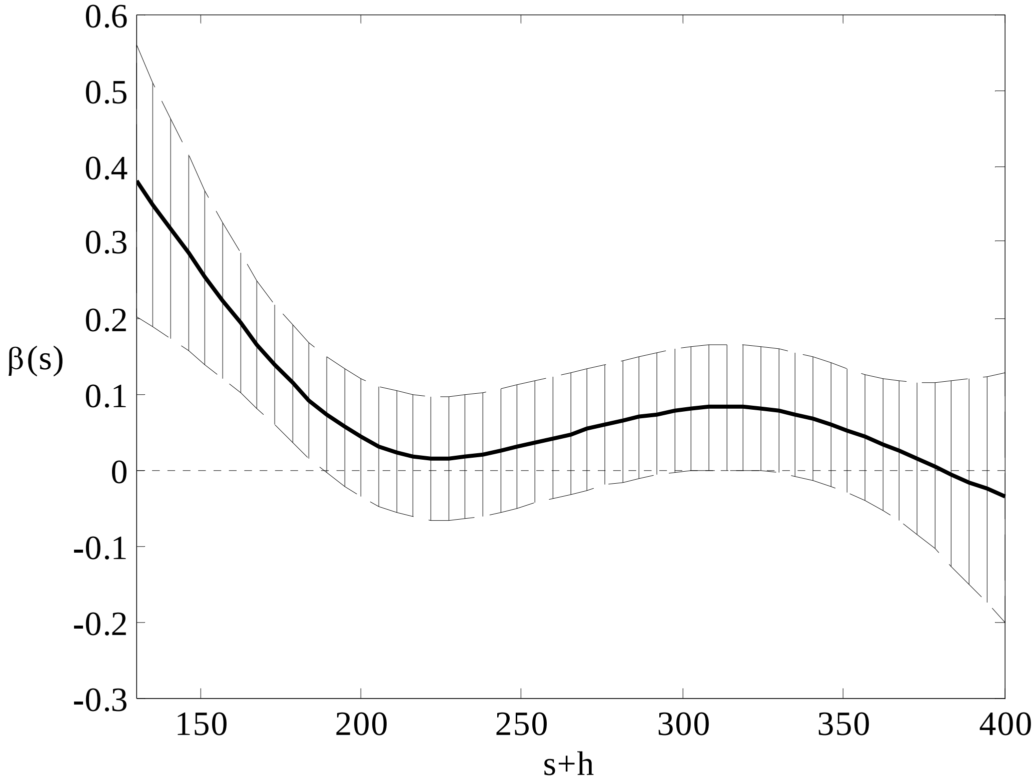
Note: $\beta(s)$ is the estimated coefficient function on daily stock returns for model (2), with s from 0 to 270, h = 0.

Figure 2(b) Estimated Coefficient Function for Daily Stock Returns (3 month horizon)



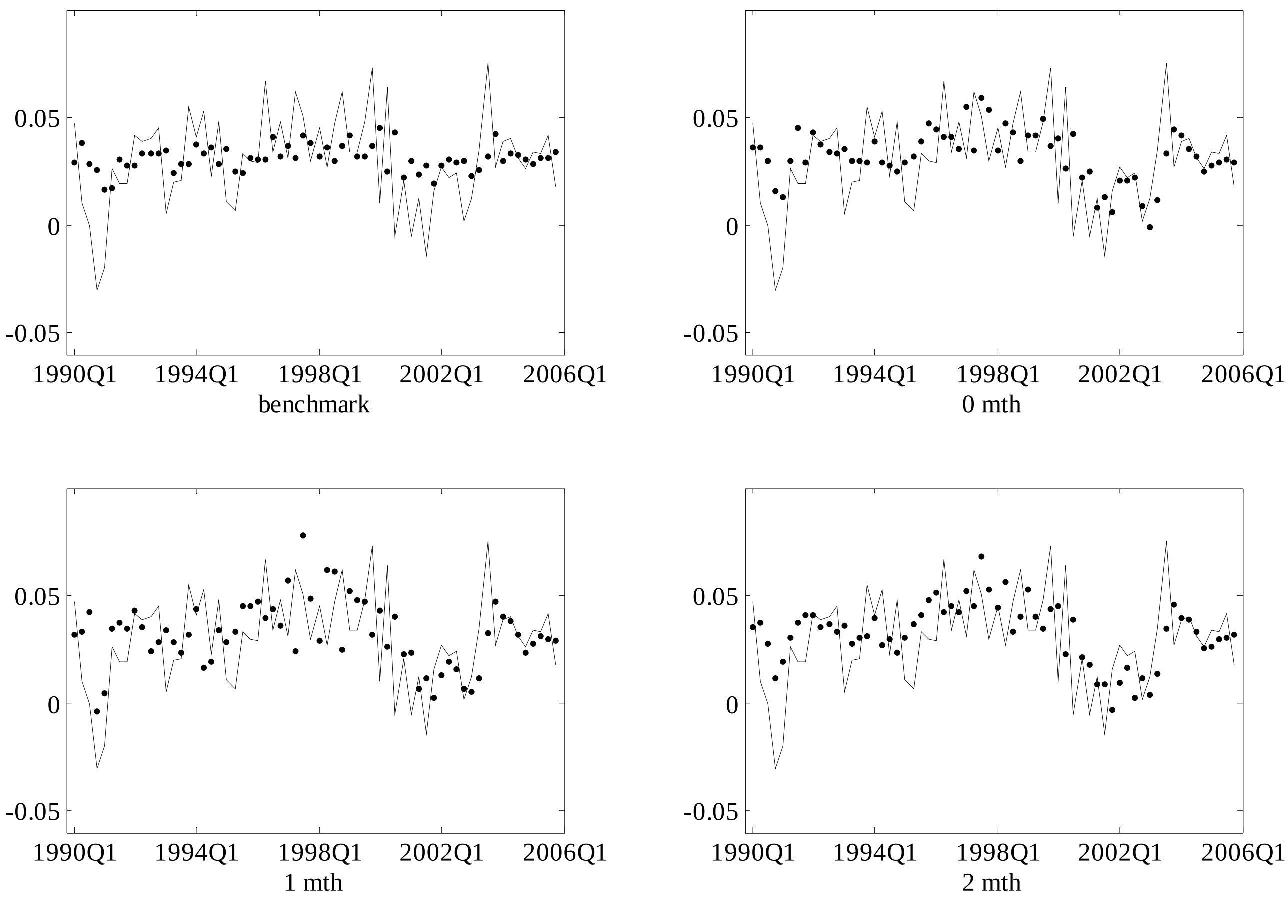
Note: $\beta(s)$ is the estimated coefficient function on daily stock returns for model (2), with s from 0 to 270, h = 70.

Figure 2(c) Estimated Coefficient Function for Daily Stock Returns (6 month horizon)



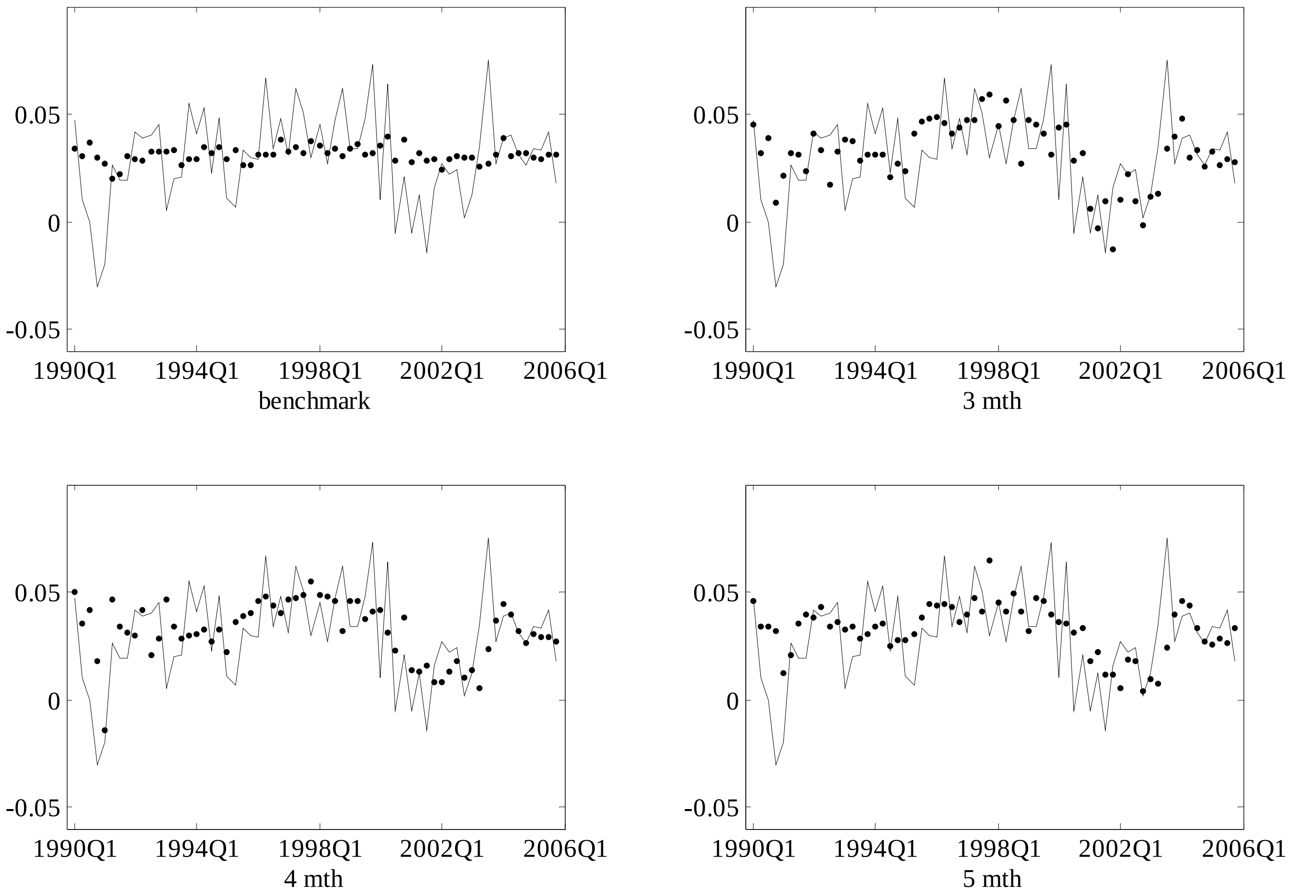
Note: $\beta(s)$ is the estimated coefficient function on daily stock returns for model (2), with s from 0 to 270, h = 130.

Figure 3(a) Realized Output with Functional Forecasts, Various Horizons



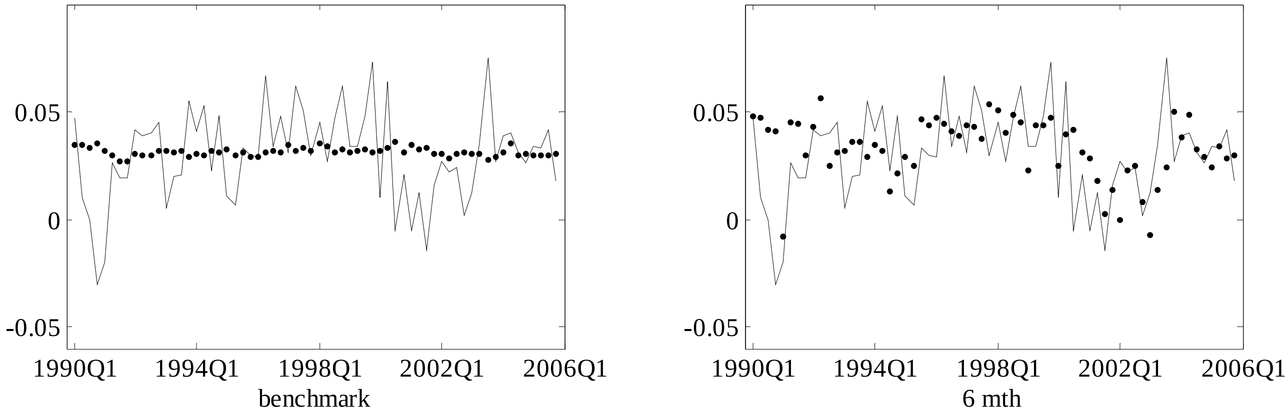
Note: Time series of forecasts (dots) from various models plotted against realizations of real output growth (solid line). The figure labeled 'benchmark' show forecasts from model (4) with $y^* = y(t_{k-1})$. The other figures show forecasts from model (2) for h = 0, 20, and 40 (0 month, 1 month, and 2 month horizons, resp.).

Figure 3(b) Realized Output with Functional Forecasts, Various Horizons



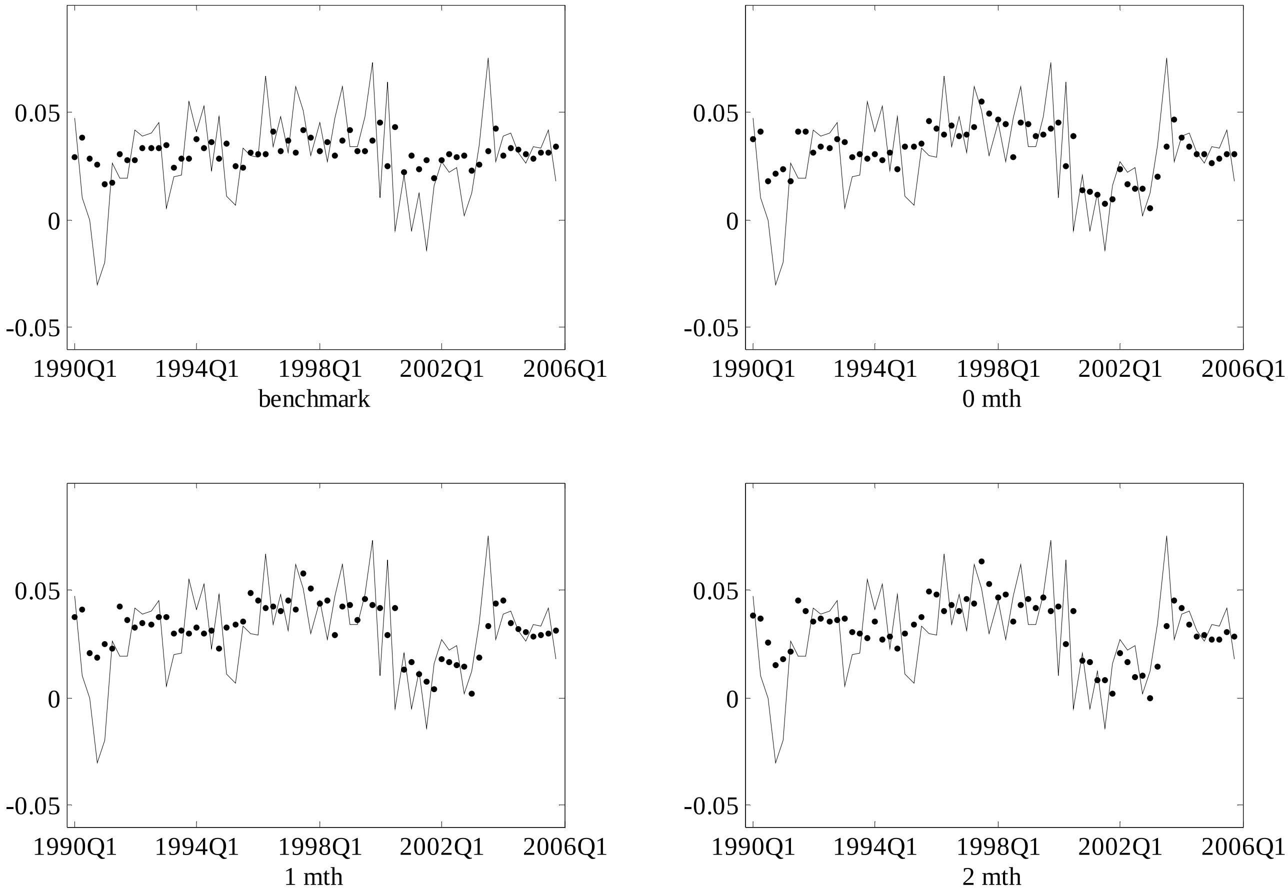
Note: Time series of forecasts (dots) from various models plotted against realizations of real output growth (solid line). The figure labeled 'benchmark' show forecasts from model (4) with $y^* = y(t_{k-2})$. The other figures show forecasts from model (2) for h = 70, 90, and 110 (3 month, 4 month, and 5 month horizons, resp.).

Figure 3(c) Realized Output with Functional Forecasts, Various Horizons



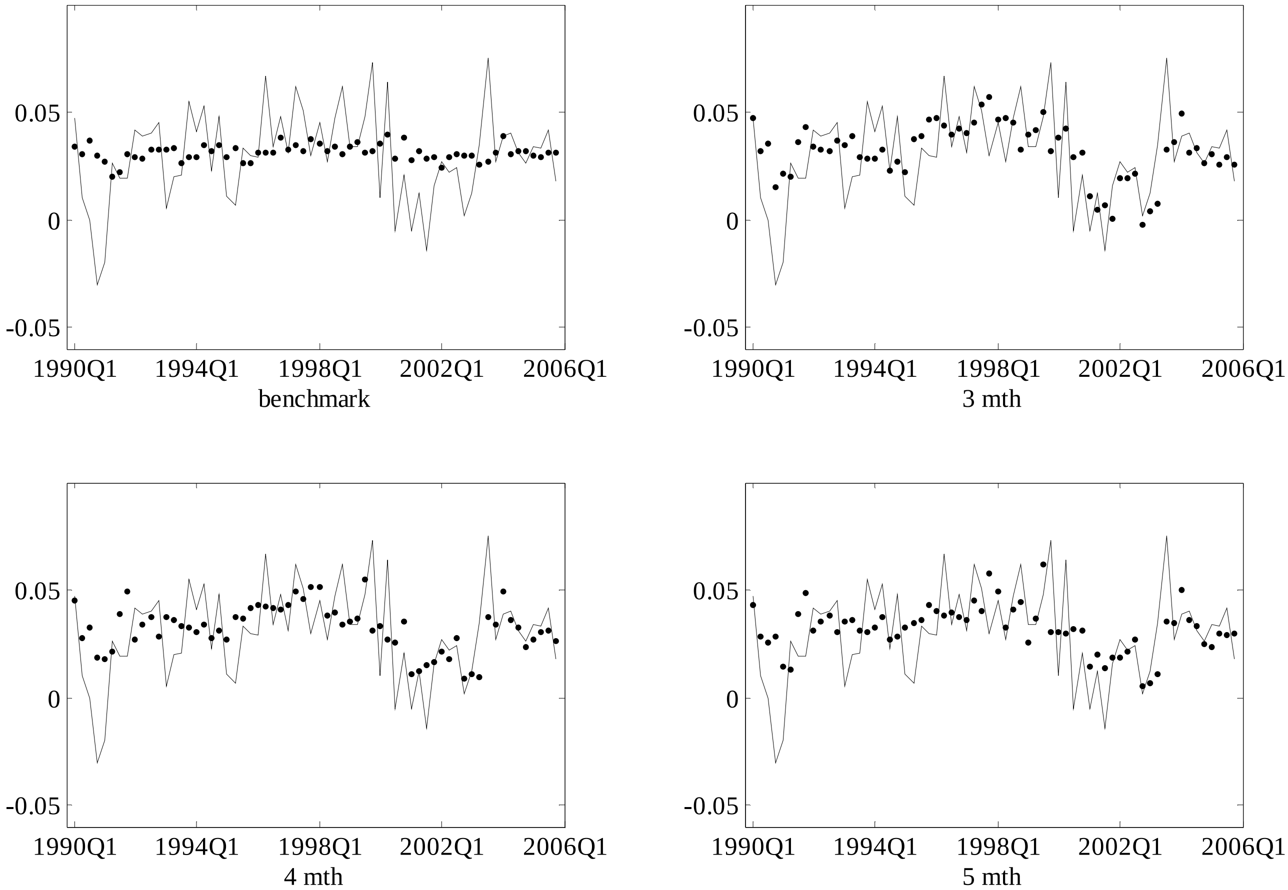
Note: Time series of forecasts (dots) from various models plotted against realizations of real output growth (solid line). The figure labeled 'benchmark' show forecasts from model (4) with $y^* = y(t_{k-3})$. The other figure show forecasts from model (2) for h = 130 (6 month horizon.)

Figure 4(a) Realized Output with Simple Regression Forecasts, Various Horizons



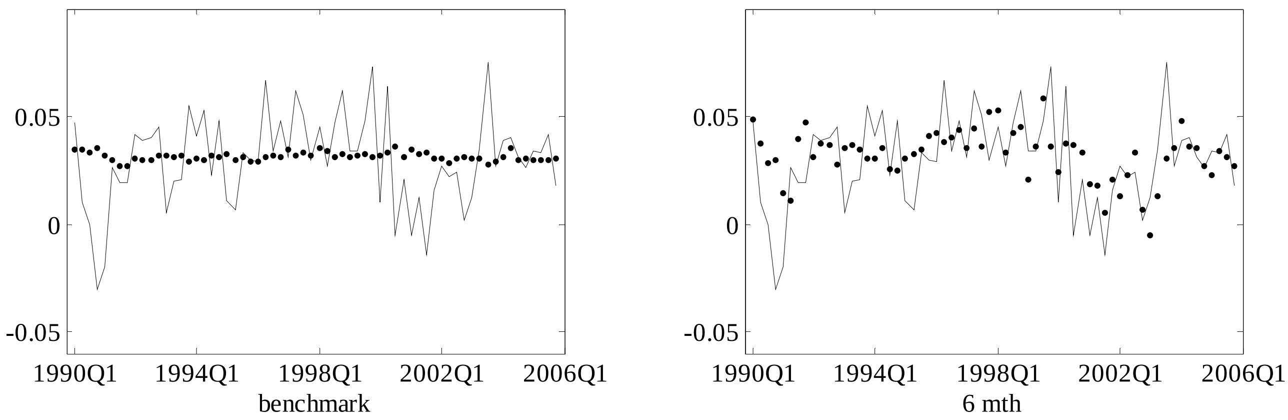
Note: Time series of forecasts (dots) from various models plotted against realizations of real output growth (solid line). The figure labeled 'benchmark' show forecasts from model (4) with $y^* = y(t_{k-1})$. The other figures show forecasts from model (3) for h = 0, 20, and 40 (0 month, 1 month, and 2 month horizons, resp.).

Figure 4(b) Realized Output with Simple Regression Forecasts, Various Horizons



Note: Time series of forecasts (dots) from various models plotted against realizations of real output growth (solid line). The figure labeled 'benchmark' show forecasts from model (4) with $y^* = y(t_{k-2})$. The other figures show forecasts from model (3) for h = 70, 90, and 110 (3 month, 4 month, and 5 month horizons, resp.).

Figure 4(c) Realized Output with Simple Regression Forecasts, Various Horizons



Note: Time series of forecasts (dots) from various models plotted against realizations of real output growth (solid line). The figure labeled 'benchmark' show forecasts from model (4) with $y^* = y(t_{k-3})$. The other figure show forecasts from model (3) for h = 130 (6 month horizon.)